

# CHELTENHAM GIRLS HIGH SCHOOL



YEAR 11  
COMMON TEST 4  
2010

## MATHEMATICS

*Time allowed: 90 minutes*

### DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are not of equal value.
- All **necessary working** should be shown in every question. Marks may be deducted for careless or badly arranged work
- Examination answers are to be completed in blue or black pen.
- Board approved calculators may be used.
- Each **question** is to be started on a **new page**.
- The marks allocated for each question are indicated

Name: \_\_\_\_\_

Class Teacher: \_\_\_\_\_

Question	1	2	3	4	Total	%
<i>Mark</i>	/14	/14	/15	/15	/58	



**Question 1 (14 marks) Start a new page****Mark**

- (a) Write 8539.25377 correct to 3 significant figures 1
- (b) Factorise completely:  $3x^2 - 12y^2$ . 2
- (c) Express  $\frac{\sqrt{32} - \sqrt{8}}{3\sqrt{2}}$  in the form of a simple fraction  $\frac{a}{b}$ , where  $a$  and  $b$  are rational numbers. 2
- (d) Solve the equation  $\frac{2x}{x-5} = \frac{3}{7}$  2
- (e) Sketch the graph of  $y = |3 - 2x|$  and find the set of values of  $x$  for which  $y \leq 3$ . 3
- (f) Given that  $3 \log_x 2 - \log_x 128 = 4$ , find the value of  $x$ . 2
- (g) Find the *EXACT* value of:
- (i)  $\sin 225^\circ$  1
- (ii)  $\sec 120^\circ$  1

**Question 2 (14 marks) Start a new page****Mark**

- (a) Differentiate:
- (i)  $2x^3 + \frac{1}{2x^3}$  2
- (ii)  $\frac{(x+2)^2}{x}$  2
- (iii)  $(5x+1)^5$  2
- (iv)  $\sqrt{3x^2 - 1}$  2
- (b) Calculate the gradient of the curve  $y = \frac{3x^2 - 4}{3 - 2x}$  at the point (2, -8) 3
- (c) The curve  $y = x^3 - 3x^2 - 9x + 7$  has two stationary points. Find the co-ordinates of the minimum turning point. 3

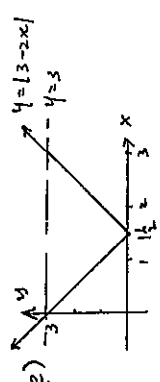
**Question 3 (15 marks) Start a new page****Mark**

- (a) If  $\alpha$  and  $\beta$  are the roots of  $3x^2 + 2x - 3 = 0$ , find the value of
- (i)  $\alpha + \beta$  1
  - (ii)  $\alpha\beta$  1
  - (iii)  $(\alpha + 1)(\beta + 1)$  2
- (b) Find the values of  $t$  for which the equation  $x^2 + tx + 2t - 3 = 0$  has real roots. 3
- (c) Solve the equation  $4^x - 9(2^x) + 8 = 0$  3
- (d) Find all values of  $\theta$  if  $0 \leq \theta \leq 360^\circ$  for  $\cos \theta = 0.5$  2
- (e) Find, by completing the square method, the centre and the radius of the circle with equation  $x^2 + y^2 = 8x - 4y + 29$ . 3

**Question 4 (15 marks) Start a new page****Mark**

- (a) Solve the equation  $5^y = 10$ , give your answer correct to 2 decimal places. 2
- (b) Given  $2x^2 + 9x - 1 \equiv a(x+1)^2 + b(x+1) + c$ , find the values of  $a$ ,  $b$  and  $c$ . 3
- (c) A parabola  $P$  has equation  $x^2 = -8(y - 4)$ .  
Draw a neat sketch of  $P$ . You must show clearly :
- (i) the co-ordinates of its vertex 1
  - (ii) the co-ordinates of its focus 1
  - (iii) the equation of its directrix 1
  - (iv) the  $x$ -intercepts 2
- (d)  $A$  and  $B$  are the points  $(4, 2)$  and  $(1, 5)$  respectively.
- (i) Find the equation of the locus of a point  $Q(x, y)$  which moves such that  $QA$  is perpendicular to  $QB$ . 3
  - (ii) Describe the locus of  $Q$  in part (i) geometrically. 1

*End of paper*

Q1. (a)  $8540$   
 (b)  $3(x^2 - 4y^2) = 3(x+y)(x-y)$   
 (c)  $\frac{4\sqrt{2} - 2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$   
 $\therefore a = 2$  &  $b = 3$   
 (d)  $\frac{2x}{x-5} = \frac{7}{14x = 3x - 15}$   
 $x = -\frac{15}{11}$   
 (e)   
 $0 \leq x \leq 3$

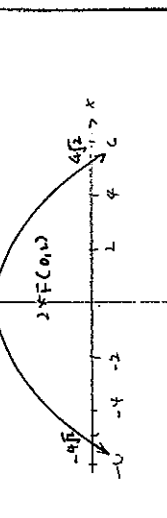
(f)  $\log_x \frac{2^3}{128} = 4$   
 $\log_x \frac{8}{128} = 4$   
 $x^4 = \frac{1}{16}$   
 $x = \frac{1}{2}$  only  
 (g) (i)  $\sin 225^\circ = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$   
 (ii)  $\sec 120^\circ = \frac{1}{\cos 60^\circ} = -2$

Q2. (a) (i)  $f' = 6x^2 - \frac{3}{2x^4}$   
 (ii)  $f' = 1 - \frac{4}{x^2}$   
 (iii)  $f' = 2.5(5x+1)^4$   
 (iv)  $f' = \frac{3x}{\sqrt{3x^2-1}}$   
 (b)  $y' = \frac{-6x^2 + 18x - 8}{(3-2x)^2}$   
 at  $x=2$ ,  $y' = \frac{-24+36-8}{1} = 4$

Q3. (a) (i)  $\alpha + \beta = -\frac{b}{a} = -\frac{2}{3}$   
 (ii)  $\alpha\beta = \frac{c}{a} = -1$   
 (iii)  $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1 = -1 - \frac{2}{3} + 1 = -\frac{2}{3}$   
 (b) For real roots:  $b^2 - 4ac \geq 0$   
 $t^2 - 8t + 12 \geq 0$   
 $(t-6)(t-2) \geq 0$   
 $t \geq 6$ ,  $t \leq 2$   
 (c) let  $u = 2^x$   
 $u^2 - 9u + 8 = 0$   
 $(u-8)(u-1) = 0$   
 $u = 8$  or  $u = 1$   
 $2^x = 8$  or  $2^x = 1$   
 $x = 3$  or  $x = 0$   
 (d)  $\cos \theta = 0.5$   
 $\theta = 60^\circ$  or  $300^\circ$   
 (e)  $x^2 - 5x + 4^2 + y^2 + 4y + 2 = 29 + 4 + 2^2$   
 $(x-4)^2 + (y-2)^2 = 49$   
 $\therefore$  Centre at  $(4, 2)$   
 Radius = 7 units.

Q4.  $5y = 10$   
 $\therefore y = \frac{10}{5} = 2$  (2 of A)  
 (b)  $2x^2 + 9x - 1 = a(x^2 + (5+a)x + 9) + b$   
 $a = 2$   
 $b = 5$   
 $c = -8$

(c) (i)  $V(0, 4)$   
 (ii)  $F(0, 2)$   
 (iii) directrix:  $y = 6$   
 (iv) let  $y = 0$ ,  $x^2 = 32$   
 $x = \pm\sqrt{32} = \pm 4\sqrt{2}$   
 at  $(4\sqrt{2}, 0)$  &  $(-4\sqrt{2}, 0)$   
 directrix:  $y = 6$



(d) (i)  $M_{PA} = \frac{y-2}{x-4}$ ,  $M_{PB} = \frac{y-5}{x-1}$   
 $M_{PA} \times M_{PB} = -1$   
 $\frac{y-2}{x-4} \times \frac{y-5}{x-1} = -1$   
 $(y-2)(y-5) = -(x-4)(x-1)$   
 $x^2 + y^2 - 5x - 7y + 11 = 0$   
 $(x - \frac{5}{2})^2 + (y - \frac{7}{2})^2 = 4\frac{1}{2}$   
 (ii) The locus of P is a circle  
 Centre  $(2\frac{1}{2}, 3\frac{1}{2})$   
 Radius  $\sqrt{4.5}$  units.

Q4. (a)  $5y = 10$   
 $\therefore y = \frac{10}{5} = 2$  (2 of A)  
 (b)  $2x^2 + 9x - 1 = a(x^2 + (5+a)x + 9) + b$   
 $a = 2$   
 $b = 5$   
 $c = -8$

