Name: ______________________________

Teacher: BMM RABS RPN

CRANBROOK SCHOOL

+ SOLUTIONS

Year 11 Mathematics

Preliminary Examination

September 2nd, 2008

Instructions

- Attempt Questions 1-7
- All questions are of equal value
- Answer each question in a new booklet
- All necessary working should be shown in every question
- Board approved calculators are allowed in all sections

Time Allowed: 2 hours **Total Marks: 84**

Question 1 (12 Marks) **Start a new booklet** **Marked by BMM**

(a) In a weekend garage sale, $\frac{3}{5}$ of the items were sold on Saturday. $\frac{2}{3}$ of the remaining items were then sold on Sunday. What percentage of the items remained unsold at the end of the weekend? **2**

(b) Simplify by removing the parentheses:

(i) $\sqrt{8}(\sqrt{2}-\sqrt{3})$ **1**

(ii) $\left(2\sqrt{5}-\sqrt{3}\right)^2$ **1**

(c) Solve: $x^2+9x=10$ **2**

(d) Factorise completely:

(i) m^3+10m^2+25m **1**

(ii) $x^3-\frac{1}{8}$ **1**

(e) Simplify: $\frac{1}{x+5}-\frac{1}{x-5}$ **2**

(f) Evaluate $255(0.0024)^{10}$ giving your answer in scientific notation correct to 3 significant figures. **2**

End of Question 1

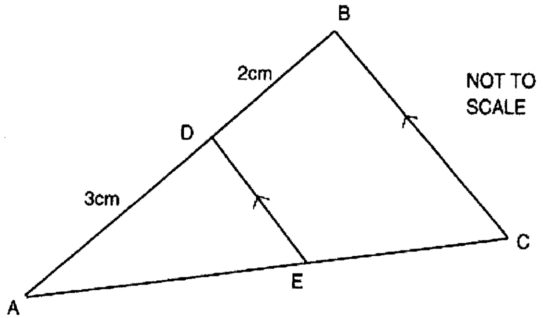
Question 2 (12 Marks)	Start a new booklet	Marked by BMM

(a) Solve: $|3-2x|>5$ **2**

(b) Solve: $\sin\beta - \frac{\sqrt{3}}{2} = 0$ for $0^\circ \le \beta \le 360^\circ$ **2**

(c) If $\frac{2}{3-\sqrt{7}} = a + b\sqrt{7}$, find the values of a and b. **2**

(d) Copy or trace the diagram into your booklet. **4**



In ΔABC, points D and E lie on lines AB and AC respectively, such that DE is parallel to BC. If AD = 3 cm and BD = 2 cm, find the ratio DE : BC, giving reasons.

(e) Solve the following pair of simultaneous equations: **2**

$$2x - y = -8$$
$$3x + 2y = -5$$

End of Question 2

Question 3 (12 Marks)	Start a new booklet	Marked by RABS

(a) Differentiate each of the following:

(i) $\frac{3x}{x^2 - 2x}$ **2**

(ii) $(2-3x)^7$ **1**

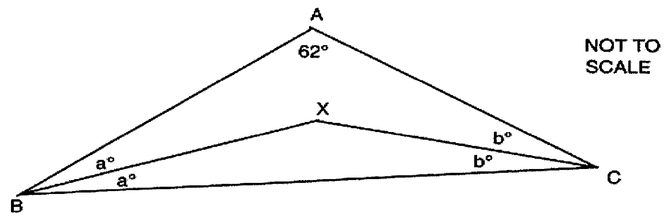
(iii) $x\sqrt{1-x}$ **2**

(iv) $\sqrt[3]{x} - \frac{1}{x}$ **2**

(b) Shade the region on the number plane where the following inequalities hold simultaneously: **2**

$$y \ge x^2 - 1$$
$$y \le x + 1$$

(c) Copy or trace the diagram into your booklet.



In the diagram above, XB and XC bisect $\angle ABC$ and $\angle ACB$ respectively. $\angle BAC = 62^\circ$. Find the size of $\angle BXC$. **3**

End of Question 3

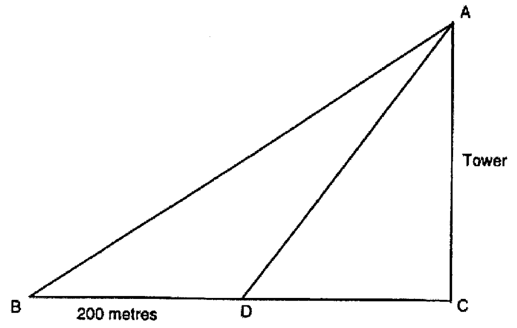
Question 4 (12 Marks) **Start a new booklet** **Marked by RABS**

(a) Solve, leaving your answers in exact form: **3**

$$x^6 + 6x^3 - 16 = 0$$

(b) Find the equation of the normal to the curve $y = x^3 - 3x + 2$ at the point (2, 4). **3**

(c)



NOT TO SCALE

The diagram above shows a vertical tower AC. Points B, C and D are in a straight line on level ground. The distance from B to D is 200m.

A surveyor found that the angle of elevation to the top of the tower from point B was 38°. She then moved to point D and measured the angle of elevation as 54°.

(i) Copy or trace the diagram and fill in all the information given. **1**

(ii) Show that the length $AD = \dfrac{200 \sin 38^\circ}{\sin 16^\circ}$ **2**

(iii) Calculate the height of the tower. **1**

(d) A circle has the equation $x^2 - 2x + y^2 + 6y = 6$. By writing the equation in the form $(x-a)^2 + (y-b)^2 = c$, or otherwise, find the radius of the circle and the co-ordinates of the centre. **2**

End of Question 4

Question 5 (12 Marks) **Start a new booklet** **Marked by RPN**

(a) Given that α and β are the roots of the quadratic equation $3x^2 - 5x - 1 = 0$, find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(b) A ship sails from point A on a bearing of $237^\circ T$ for a distance of 423 km. The ship then turns and sails due South to point C. The bearing of A from C is found to be $41^\circ T$.

(i) Draw a diagram showing the above information. **1**

(ii) Find the size of $\angle BAC$. **1**

(iii) Calculate the total distance sailed by the ship. **2**

(c) A parabola has the equation $4y = x^2 + 4x + 8$.

(i) Find its focal length. **1**

(ii) Find the coordinates of its vertex. **1**

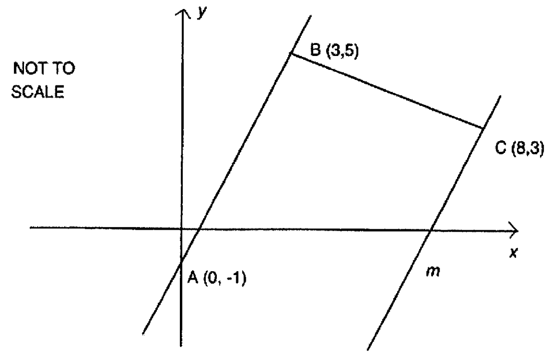
(iii) State the equation of the axis of symmetry. **1**

(iv) Draw a neat sketch of the parabola. **1**

End of Question 5

Question 6 (12 Marks) **Start a new booklet** **Marked by RPN**

(a)



In the diagram, A (0,-1), B (3, 5) and C (8,3) are points on the number plane. Copy or trace the diagram into your booklet.

(i) Find the length of AB. **1**

(ii) Find the equation of AB. **2**

(iii) Find the equation of the line m passing through C parallel to AB. **2**

(iv) The point D lies on m such that BC is parallel to AD. Find the coordinates of D. **2**

(v) What type of quadrilateral is $ABCD$? Give reasons. **2**

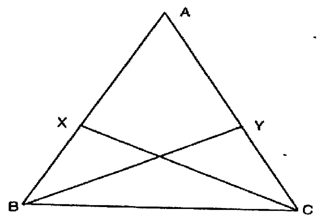
(vi) Find the perpendicular distance from C to AB. **2**

(vii) Find the area of ABCD. **1**

End of Question 6

Question 7 (12 Marks) **Start a new booklet** **Marked by SKB**

(a)



NOT TO SCALE

In the diagram above, ΔABC is isosceles with AB = AC. Points X and Y lie on AB and AC respectively such that AX = AY.
Copy or trace the diagram into your booklet.

(i) Prove that $\Delta BXC \equiv \Delta CYB$ **4**

(ii) Hence or otherwise prove that $\angle XBY = \angle YCX$ **1**

(b) A function *f(x)* if defined as follows:

$$f(x) = \begin{cases} \frac{1}{x} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ 2^x & \text{for } x < 0 \end{cases}$$

(i) Draw a neat sketch of the graph $y = f(x)$. **3**

(ii) Evaluate: $f(2) + f(0) + f(-2)$ **2**

(c) Simplify: $2\cos^2 B + 3\sin^2 B - 2$ **2**

End of Question 7
End of Examination

2008 YR. 11 PRELIM EXAM.

SOLUTIONS

QUESTION 1

(a) $\frac{3}{5} + \frac{2}{3} \times \frac{2}{5} = \frac{13}{15}$ ✓

UNSOLD $= \frac{2}{15} = 13.\dot{3}\%$ ✓

(b) (i) $\sqrt{16} - \sqrt{24} = 4 - 2\sqrt{6}$ ✓

(ii) $(2\sqrt{5} - \sqrt{3})(2\sqrt{5} - \sqrt{3})$

$= 4 \times 5 - 4\sqrt{15} + 3$

$= 23 - 4\sqrt{15}$ ✓

(c) $x^2 + 9x - 10 = 0$

$(x + 10)(x - 1) = 0$ ✓

$\therefore x = -10, 1$ ✓

(d) (i) $m(m^2 + 10m + 25)$

$= m(m+5)^2$ ✓

(ii) $x^3 - \frac{1}{8} = (x - \frac{1}{2})(x^2 + \frac{1}{2}x + \frac{1}{4})$ ✓

(e) $\frac{(x-5)}{(x+5)(x-5)} - \frac{(x+5)}{(x+5)(x-5)}$ ✓

$= \frac{-10}{x^2 - 25}$ ✓

(f) $= 1.62 \times 10^{-24}$ ✓ ✓

QUESTION 2

(a) $3 - 2x > 5$

$-2x > 2$

$x < -1$ ✓

$-(3 - 2x) > 5$

$-3 + 2x > 5$

$2x > 8$

$x > 4$ ✓

(b) $\sin\beta = \frac{\sqrt{3}}{2}$

$\beta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 60^\circ$

sin positive in 1st & 2nd Quadrants.

$\therefore \beta = 60^\circ$ & $120^\circ \rightarrow (180^\circ - \theta)$

✓ ✓

(c) $\frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{6+2\sqrt{7}}{9-7}$

$= \frac{6+2\sqrt{7}}{2}$

$= 3+\sqrt{7}$

$\therefore a = 3, \quad b = 1$ ✓ ✓

(d) IN $\triangle ADE$ & $\triangle ABC$ ...

- $\angle ADE = \angle ABC$ (corresponding $\angle$s on // lines) ✓
- $\angle AED = \angle ACB$ (corresponding $\angle$s on // lines) ✓
- $\angle DAE = \angle BAC$ (common)

$\therefore \triangle ADE \ ||| \ \triangle ABC$ (AAA) ✓

$\therefore DE : BC = AD : AB$

$= 3 : 5$ ✓

(e) $4x - 2y = -16$ —— ③

$3x + 2y = -5$ —— ②

③ + ② $\quad 7x = -21$

$x = -3$ ✓

Sub $x = -3$ into ③

$-12 - 2y = -16$

$-2y = -4$

$y = 2$ ✓

QUESTION 3

(a) (i) $u = 3x \qquad v = x^2 - 2x$

$u' = 3 \qquad v' = 2x - 2$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{3(x^2-2x) - 3x(2x-2)}{(x^2-2x)^2}$ ✓

$$= \frac{3x^2 - 6x - 6x^2 + 6x}{(x^2-2x)^2}$$

$$= \frac{-3x^2}{(x^2-2x)^2} ✓$$

(ii) $y' = 7(2-3x)^6 . (-3)$

$= -21(2-3x)^6$ ✓

(iii) $u = x \qquad v = \sqrt{1-x} = (1-x)^{1/2}$

$u' = 1 \qquad v' = \frac{1}{2}(1-x)^{-1/2} . -1$

$= -\frac{1}{2}(1-x)^{-1/2}$

$y' = vu' + uv'$

$= (1-x)^{1/2} + -\frac{x}{2}(1-x)^{-1/2}$ ✓

$= \sqrt{1-x} - \frac{x}{2\sqrt{1-x}}$

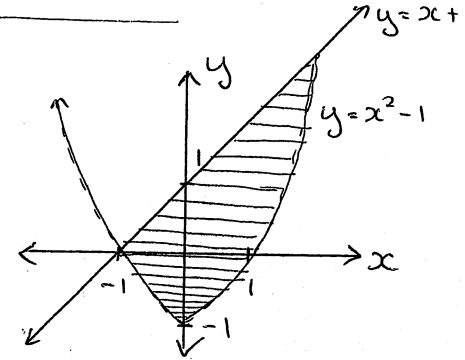
$= \frac{2(1-x) - x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$ ✓

(iv) $y = x^{1/3} - x^{-1}$

$y' = \frac{1}{3}x^{-2/3} + x^{-2}$ ✓

$= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{x^2}$ ✓

(b)



• correct graphs ✓

• regions ✓

(c) $62° + 2a + 2b = 180°$ ✓

$2a + 2b = 118°$

$a + b = 59°$ ✓

$\angle BXC + a + b = 180°$

$\angle BXC + 59° = 180°$

$\therefore \angle BXC = 121°$ ✓

QUESTION 4

(a) Let $X = x^3$

$\therefore X^2 + 6X - 16 = 0$ ✓

$(X+8)(X-2) = 0$

$\therefore X = -8$ or 2 ✓

$\therefore x^3 = -8$ or 2

$x = -2$ or $\sqrt[3]{2}$ ✓

(b) $y' = 3x^2 - 3$

Gradient $= 3 \times 2^2 - 3$ (sub in $x=2$)

$= 9$ ✓

$\therefore$ Normal $= -\frac{1}{m_1} = -\frac{1}{9}$ ✓

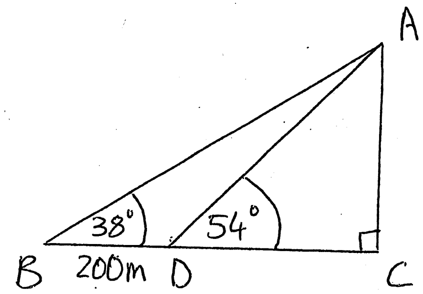
pt/grad. formula:

$y - 4 = -\frac{1}{9}(x-2)$

$9y - 36 = -x + 2$

$x + 9y - 38 = 0$ ✓

(c) (i)



✓

(ii) $54° = 38° + \angle BAD$ (ext. angle theorem)

$\therefore \angle BAD = 16°$ ✓

Sine rule:

$$\frac{AD}{\sin 38°} = \frac{200}{\sin 16°}$$

$$AD = \frac{200 \sin 38°}{\sin 16°}$$ ✓

(iii) In $\triangle ACD$,

$$\sin 54° = \frac{o}{h}$$

$$= \frac{AC}{AD} \Rightarrow AC = AD \sin 54°$$

$$\therefore AC = \frac{200 \sin 38° . \sin 54°}{\sin 16°}$$

$= 361.4028\ldots$ m ✓

(d) COMPLETE THE SQUARE:

$$x^2 - 2x + 1 \quad + \quad y^2 + 6y + 9 = 6 + 1 + 9$$

$$(x-1)^2 + (y+3)^2 = 16$$

$\therefore$ radius $= \sqrt{16} = \underline{4}$ ✓

centre $= \underline{(1, -3)}$ ✓

QUESTION 5

(a) (i) $\alpha + \beta = \dfrac{-b}{a} = \underline{\dfrac{5}{3}}$ ✓

(ii) $\alpha\beta = \dfrac{c}{a} = \underline{\dfrac{-1}{3}}$ ✓

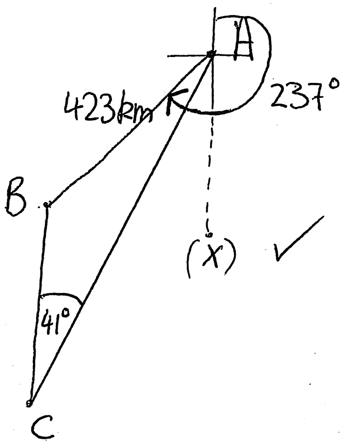
(iii) $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{5}{3}\right)^2 - 2\left(-\frac{1}{3}\right)$

$= \underline{3\frac{4}{9}}$ ✓

(b) (i)



✓

(ii) Add point X above to help.

$\angle BCA = \angle CAX$ (alternate $\angle$s on // lines)

$\therefore \angle CAX = 41°$

$\angle BAX = 237° - 180°$

$= 57°$

$\therefore \angle BAC = 57° - \angle CAX°$

$= \underline{16°}$ ✓

(iii) Sine rule:

$$\frac{BC}{\sin 16°} = \frac{423}{\sin 41°} \Rightarrow BC = \frac{423 \sin 16°}{\sin 41°}$$

$= \underline{177.7\text{km}}$ ✓

$\therefore$ TOTAL DISTANCE $= \underline{600.7\text{km}}$ ✓

(c) (i) $4y - 4 = x^2 + 4x + 4$

$4(y-1) = (x+2)^2$

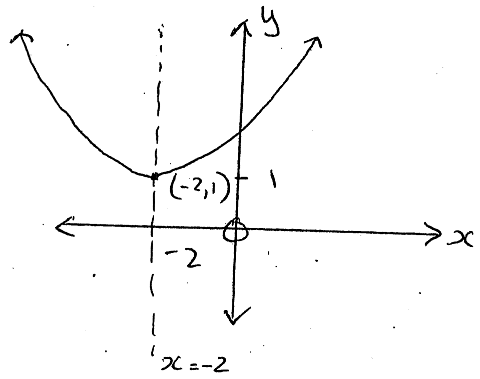
$\therefore a = \frac{4}{4} = 1$

(ii) FROM THE EQ'N:

$(-2, 1)$

(iii) AS x-COORD IS 2, THE AXIS OF SYM. IS $x = -2$

(iv)



QUESTION 6

(i) $AB = \sqrt{(3-0)^2 + (5+1)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$ UNITS OR $3\sqrt{5}$

(ii) AB gradient $= \frac{5+1}{3-0} = \frac{6}{3} = 2.$

USING PT./GRAD:

$y - 5 = 2(x-3)$

$y - 5 = 2x - 6$

$2x - y - 1 = 0$

(iii) AB gradient = m gradient (// lines)

USING PT./GRAD:

$y - 3 = 2(x-8)$

$y - 3 = 2x - 16$

$2x - y - 13 = 0$

(iv) BC gradient $= \dfrac{3-5}{8-3} = -\frac{2}{5}$

USING PT/GRAD ON AD:

$y+1 = -\frac{2}{5}(x-0)$

$5y+5 = -2x$

$\underline{2x+5y+5=0}$ ✓

SOLVE SIM. WITH EQ'N OF m:

$2x+5y+5=0$ —— ①

$2x-y-13=0$ —— ②

① − ②

$6y+18=0$ ⇒ CO-ORDS $(5,-3)$ ✓

$6y=-18$

$\underline{y=-3}$

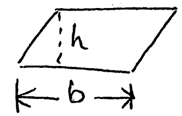
SUB INTO ① ∴ $\underline{x=5}$

(v) PARALLELOGRAM ✓ (2 PAIRS OF OPP. SIDES) ✓

(vi) $d = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \dfrac{|2\times(8)-(3)-1|}{\sqrt{2^2+1^2}}$

$\boxed{\text{USING } 2x-y-1 \text{ \& } (8,3)}$

$= \dfrac{12}{\sqrt{5}}$ UNITS ✓

(vii) AREA OF //GRAM $= b \times h$ 

$= 3\sqrt{5} \times \dfrac{12}{\sqrt{5}}$

FROM (i) FROM (vi)

$= \underline{36 \text{ UNITS}^2}$ ✓

QUESTION 7

(a) (i) • $\angle XBC = \angle YCB$ (base $\angle$s of isos. $\triangle$ are $=$.) ✓

• $AB - AX = AC - AY$

∴ $BX = YC$ (given) ✓

• BC is common

∴ $\triangle BXC \equiv \triangle CYB$ (SAS) ✓

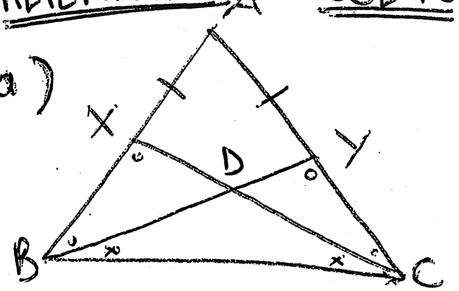
(ii) • $\angle XCB = \angle YBC$ (corresp. $\angle$s in congr. $\triangle$s) ✓

• $\angle XBC = \angle YBC$ (from above)

∴ $\angle XBC - \angle YBC = \angle YCB - \angle XCB$

∴ $\underline{\angle XBY = \angle YCX}$ ✓

7 (a)



(i) In Δs BXC and CYB

$\angle XBC = \angle YCB$ (base $\angle$s of isos Δ are equal)

$BX = YC$ (as $AB = AC$ (given), $AX = AY$ (given), $BX = AB - AX$, $YC = AC - AY$)

BC is common

$\therefore \Delta XBC \equiv \Delta CYB$ (SAS)

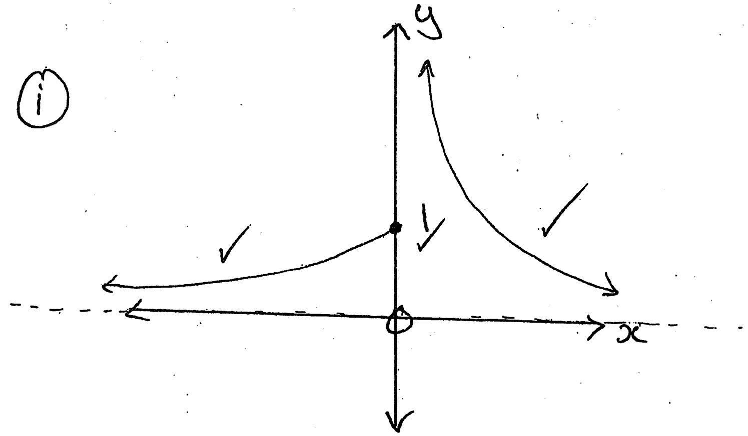
(ii) $\angle XCB = \angle YBC$ (corresponding sides of congruent Δs from (i) are equal)

But $\angle XBC = \angle YCB$ (base $\angle$s of isos Δ are equal)

$\therefore \angle XBC - \angle YBC = \angle YCB - \angle XCB$

ie $\angle XBY = \angle YCX$

7 (b) (i)



(ii) $f(2) + f(0) + f(-2)$

$= \frac{1}{2} + 1 + 2^{-2}$

$= 1\frac{3}{4}$

(c) $2\cos^2 B + 3\sin^2 B - 2$

$= 2\cos^2 B + 2\sin^2 B + \sin^2 B - 2$

$= 2(\cos^2 B + \sin^2 B) + \sin^2 B - 2$

$= 2 + \sin^2 B - 2$

$= \sin^2 B$

or (ii) In Δs XBD and YCD:

$\angle CYD = \angle BXD$ (corr. $\angle$s of congruent Δs in (i) are equal)

$\angle CDY = \angle BDX$ (vert. opp $\angle$s are equal)

$\therefore \angle YCD = \angle XBD$ ($\angle$s sum of $\Delta = 180°$)

ie $\angle YCX = \angle XBY$

7(a) (In Δs XBD and YCD:)

or (ii) $XB = YC$ (corr. sides of congruent Δs in (i) are equal)

$\angle XDB = \angle YDC$ (vert. opp $\angle$s are equal)

$\angle CXB = \angle BYC$ (corr. $\angle$s of congruent Δs in (i) are equal)

$\therefore \Delta XBD \equiv \Delta YCD$ (AAS)

$\therefore \angle XBY = \angle YCX$ (corr. angles of congruent Δs are equal)

or (ii) (In Δs ABY and ACX:)

$AX = AY$ (given)

$\angle A$ is common

$AB = AC$ (given)

$\therefore \Delta ABY \equiv \Delta ACX$ (SAS)

$\Rightarrow \angle XBY = \angle YCX$ (corr. angles of congruent Δs are equal)