CRANBROOK
SCHOOL

+ SOLUTIONS

Year 11 (2U) Mathematics

Preliminary Yearly Examination

Wednesday September 9, 2009

Time Allowed: 2 hours *plus* 5 minutes reading time

Total Marks: 84

There are 7 questions, each of equal value.
Start a new booklet for each question.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved calculators may be used.

Total marks - 84
Attempt Questions 1 - 7
ALL questions are of equal value

Start each question in a SEPARATE booklet

Question 1 (12 marks) **Marks**

(a) Factorise $x^2 - 7x + 12$ **2**

(b) Find correct to 3 significant figures places $\dfrac{5.31^2}{6.84 - 2.91}$ **2**

(c) Solve for x and y where

$3x - 2y = 7$ and $4x + 3y = -2$ **2**

(d) Evaluate $\lim\limits_{x \to 3}\left(\dfrac{x^2 - 9}{x - 3}\right)$ **2**

(e) Rationalise the denominator $\dfrac{2}{\sqrt{3} - 1}$. Simply your answer. **2**

(f) Simplify $\dfrac{2}{5} - \dfrac{x-2}{3}$. **2**

Question 2 (12 marks) START A NEW BOOKLET. **Marks**

(a) If $\sqrt{45}+\sqrt{20}=\sqrt{a}$ find the value of a. **3**

(b) Consider the equation of the parabola $x^2-4x-8y=8$.

(i) Write this equation in the form $(x-h)^2=4a(y-k)$. **2**

(ii) Hence, find the coordinates of the vertex and the focus. **2**

(c) A retailer increased the price for a pair of shoes by 6%. The new price of the shoes is $132.50. What was the old price? **2**

(d) Show that the points $A(-4,5)$, $B(2,-3)$ and $C(-1,1)$ are collinear. **3**

Question 3 (12 marks) START A NEW BOOKLET. **Marks**

(a) Find the exact area of an equilateral triangle with a side length of x cm. **2**

(b) Solve for x where $0^0 \le x \le 360^0$ **4**

$$2\sin x = 1$$

(c) Solve for x $|4-x|<2$ **2**

(d) Show that $f(x)=(x-x^7)^8$ is an even function. **1**

(e) Find the range of values of k for which the equation $x^2+3x+1=kx$ has real roots. **3**

Question 4 (12 marks) START A NEW BOOKLET. **Marks**

(a) In a triangle ABC, $\angle CAB = 38^0$, $BC = 16\,cm$ and $AC = 18\,cm$. Find the size of $\angle ABC$ to the nearest minute. **2**

(b) (i) Express $x^{-1} + y^{-1}$ as a single fraction. **2**

(ii) Hence show that

$$\frac{x^2 - y^2}{x^{-1} + y^{-1}} = xy(x - y)$$ **2**

(c) Show that $-3x^2 + 6x - 7 < 0$ for all x. **2**

(d) A function $y = f(x)$ is given by the rule

$$f(x) = 2x \text{ if } x \geq 2$$
$$= 4 \text{ if } x < 2$$

(i) Draw this graph from $x = -4$ to $x = 4$ **3**

(ii) Evaluate $2f(5) + f(2)$ **1**

Question 5 (12 marks) START A NEW BOOKLET. **Marks**

(a) The points A $(-1,-1)$, B $(1,3)$ and C $(5,1)$ form the triangle ABC.

i) Find the gradient of the sides of the triangle. **3**

ii) Hence, or otherwise, show that the triangle is right-angled. **1**

iii) Show that the co-ordinates of M, the midpoint of the hypotenuse, is $(2,0)$. **1**

iv) Show that a circle, centre M, passes through A, B and C. Find the equation of the circle. **3**

(b) Simplify $\dfrac{\sin(180^0 + \alpha)}{\cos(-\alpha)}$ **2**

(c) Solve for x: $2(4^x) - 3(2^x) + 1 = 0$ **2**

Question 6 (12 marks) START A NEW BOOKLET. **Marks**

(a) Differentiate with respect to x

(i) $2x+\sqrt{x}$. **1**

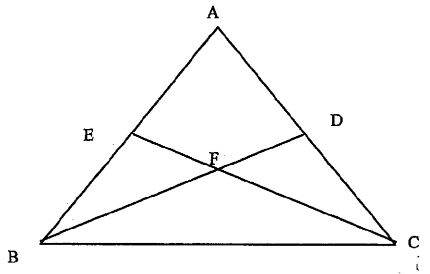
(ii) $\sqrt{2x-3}$ **2**

(iii) $\dfrac{2x+1}{x-2}$ **2**

(iv) $\dfrac{2}{\sqrt[3]{x}}$ **2**

(b) In ΔABC, altitudes BD, CE are equal. EC meets BD at F.

NOT TO SCALE



(i) Prove $\Delta BDC \equiv \Delta CEB$ **2**

(ii) Explain why $\angle ABC = \angle ACB$ **1**

(iii) Prove $\angle FBC = \angle FCB$ **2**

Question 7 (12 marks) START A NEW BOOKLET. **Marks**

(a) Express $3x^2+4x+1$ in the form $Ax(x-1)+B(x+3)+C$ **2**

(b) (i) Find the point P where $3x+5y-2=0$ cuts the y axis. **1**

(ii) Hence, or otherwise, find the perpendicular distance from P to $3x+5y+2=0$ **2**

(c) If $y=\dfrac{(3x+4)^{n-1}}{\sqrt{x^3}}$, where n is a constant, show that

$$\frac{dy}{dx}=\frac{3[3x+4]^{n-2}[x(2n-5)-4]}{2x^{\frac{5}{2}}}.$$ **4**

(d) If the smaller root of $ax^2+bx+c=0$ is one less than the larger root, show that

$$b^2-a^2=4ac$$ **3**

— 2U Preliminary Exam 2009 Solutions —

Question 1

(a) $x^2 - 7x + 12$

$(x-3)(x-4)$ ✓✓

(b) 7.174580153

$= 7.17$ (3 sig fig) ✓✓

(c) $3x - 2y = 7$ — ①

$4x + 3y = -2$ — ②

① × 4 ② × 3

$12x - 8y = 28$ — ③

$12x + 9y = -6$ — ④

③ − ④ $-17y = 34$

$y = -2$ ✓

sub $y = -2$ into ①

$3x - 2(-2) = 7$

$3x + 4 = 7$

$3x = 3$

$x = 1$ ✓

(d) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

$= \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$ ✓

$= \lim_{x \to 3} x + 3$

$= 6$ ✓

(e) $\frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \frac{2\sqrt{3} + 2}{3 - 1}$ ✓

$= \frac{2\sqrt{3} + 2}{2}$ ✓

$= \sqrt{3} + 1$

(f) $\frac{2}{5} - \frac{(x-2)}{3}$

$= \frac{6 - 5(x-2)}{15}$ ✓

$= \frac{6 - 5x + 10}{15}$

$= \frac{16 - 5x}{15}$ ✓

Marker's Notes (Q1) – BMM

(a) Lots of students lost a mark for turning this expression into an equation!! Factorise ≠ Solve!

(b) 3 sig fig ≠ 3 decimal places.

(c) OK done.

(d) fairly well done

(e) Lots of careless simplifying! Learn to simplify and not cancel. You simplify by factorising first.

(f) Careful when expanding brackets. As in step 3 above.

Question 2

a) $\sqrt{45} + \sqrt{20} = \sqrt{a}$

$3\sqrt{5} + 2\sqrt{5}$ ✓

$= 5\sqrt{5}$

$= \sqrt{25} . \sqrt{5}$ ✓

$= \sqrt{125}$

$\therefore a = 125$ ✓

b) $x^2 - 4x - 8y = 8$

i) complete the □

$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 8y + 8 + 4$

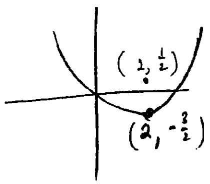
$(x-2)^2 = 8y + 12$ ✓

$(x-2)^2 = 8(y + \frac{3}{2})$ ✓

$(x-2)^2 = 4(2)(y + \frac{3}{2})$

ii) vertex @ $(2, -\frac{3}{2})$ ✓

Focus @ $(2, \frac{1}{2})$ ✓



(c) $106\% = \$132.50$

$1\% = (132.50 \div 106) = 1.25$ ✓

$100\% = (132.50 \div 106) \times 100$

$= \$125$ ✓

(d) collinear ∴ same gradient.

$m_{AB} = \frac{-3-5}{2+4} = \frac{-8}{6} = -\frac{4}{3}$ ✓

$m_{BC} = \frac{1+3}{-1-2} = \frac{4}{-3}$ ✓

$m_{AC} = \frac{1-5}{-1+4} = \frac{-4}{3}$ ✓

$m_{AB} = m_{BC} = m_{AC}$

∴ A, B, C are collinear.

Marker's Notes

(a) poorly done. Many students left answers as $5\sqrt{5}$ ∴ a = 5. This lost 2 marks.

(b) (i) Most students completed the □ correctly but did not factorise the RHS accurately. Always leave your coeff. of y as 1.

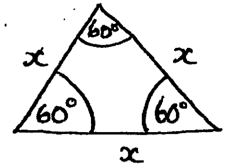
(ii) Marks were lost here for moving the x-coord instead of the y-coord. When in doubt, draw a diagram.

(c) Surprisingly poorly done. These sorts of questions are very common.

(d) very well done! ☺

QUESTION 3 SOL'NS.

a)



Area = $\frac{1}{2} ab \sin C$

$= \frac{1}{2} . x . x . \sin 60°$ ✓

$= \frac{x^2}{2} . \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3} x^2}{4}$ ✓

OR BY USING $\frac{1}{2} bh$.

b) $2 \sin x = 1$

$\sin x = \frac{1}{2}$ ✓

$\sin x$ is +ve in Q1 & Q2.

Q1: $x = 30°$ ✓

Q2: $180° - 30° = 150°$ ✓ ✓

c) $|4 - x| < 2$

$4 - x < 2$ $4 - x > -2$

$2 < x$ ✓ $6 > x$ ✓

OR $2 < x < 6$

d) EVEN F'N: $f(x) = f(-x)$

$f(-x) = (-x + x^7)^8$

$= [-(x - x^7)]^8$

$= (x - x^7)^8$

$= f(x)$ ✓

e) $x^2 + 3x + 1 = kx$

$x^2 + (3-k)x + 1 = 0$

$\Delta = b^2 - 4ac$, $\Delta \geq 0$ for real roots.

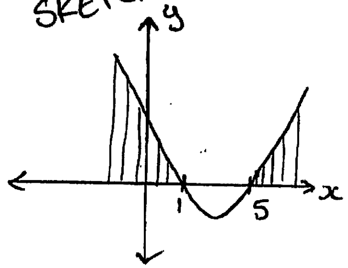
$(3-k)^2 - 4.1.1 \geq 0$

$k^2 - 6k + 9 - 4 \geq 0$ ✓

$k^2 - 6k + 5 \geq 0$

$(k-5)(k-1) \geq 0$ ✓

SKETCH!



$\therefore k \leq 1$, $k \geq 5$ ✓

Q4 & 5 (HRK)

4 (a)

$\frac{\sin\theta}{18}=\frac{\sin 38^0}{16}$ ✓ ✓ = 1 mark

$\sin\theta=\frac{18\sin 38^0}{16}$

$\sin\theta=.692619..$

$\theta=43^0 50'$ ✓ $136^0 10'$ NOTE: 2 possible ∠'s

(b) (i)

$x^{-1}+y^{-1}=\frac{1}{x}+\frac{1}{y}$ ✓ Many struggled with addition of fractions!! ☹

$=\frac{x+y}{xy}$ ✓

(ii)

$\frac{x^2-y^2}{x^{-1}+y^{-1}}=xy(x-y)$

$\text{LHS}=\frac{x^2-y^2}{x^{-1}+y^{-1}}=\frac{(x-y)(x+y)}{\frac{x+y}{xy}}$

$=\frac{xy(x-y)(x+y)}{x+y}$

$=xy(x-y)=\text{RHS}$

c)

$-3x^2+6x-7<0$ if a 0 and $\Delta<0$

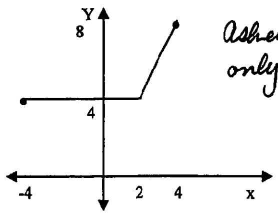
$a=-3\therefore$ true ✓

$\Delta=6^2-4\times-3\times-7=-48\therefore$ true ✓

Too many ignored $a<0$

(d) (i)

RTQ = Read the question! Asked to sketch only FOR $-4\le x\le 4$



(ii) $2f(5)+f(2)=2\times10+4=24$

5(a) (i)

$M_{AB}=\frac{3--1}{1--1}=2$

$M_{BC}=\frac{1-3}{5-1}=\frac{-1}{2}$

$M_{AC}=\frac{1--1}{5--1}=\frac{1}{3}$

(ii) $2\times\frac{-1}{2}=-1\therefore$ a right-angled triangle.

$\angle ABC=90^0$

(iii) Midpoint of AC $\left(\frac{-1+5}{2},\frac{-1+1}{2}\right)=M(2,0)$

5a) ctd

(iv) $D_{MA}=\sqrt{(2--1)^2+(0--1)^2}=\sqrt{10}$

$D_{MB}=\sqrt{(2-1)^2+(0-3)^2}=\sqrt{10}$

$D_{MC}=\sqrt{(2-5)^2+(0-1)^2}=\sqrt{10}$

$\therefore$ M is the centre of the circle

$(x-2)^2+y^2=10$

(b) $\frac{\sin(180^0+\alpha)}{\cos(-\alpha)}=\frac{-\sin\alpha}{\cos\alpha}$

$=-\tan\alpha$

(c) $2(4^x)-3(2^x)+1=0$

$=2(2^x)^2-3(2^x)+1=0$

let $u=2^x\therefore 2u^2-3u+1=0$

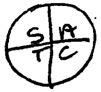
$(2u-1)(u-1)=0$

$u=2^x=\frac{1}{2},\ u=2^x=1$

$\therefore x=-1,\ x=0$

5a(iv) also accepted reference to right ∠ in semicircle.

b) Not well done!



needs revision!! ... serious revision this is VERY basic and important material that keeps coming into various applications.

2 UNIT EXAM – QUESTION 6 – 2009

MARKERS NOTES

6(a)(i) Answer can be left as $2+\frac{1}{2}x^{-\frac{1}{2}}$, $2+\frac{1}{2\sqrt{x}}$, $\frac{4\sqrt{x}+1}{2\sqrt{x}}$ or $\frac{4x+\sqrt{x}}{2x}$

(ii) Don't forget to multiply by the derivative of the function inside the brackets!

$y=(2x-3)^{\frac{1}{2}}$ $\quad\frac{dy}{dx}=\frac{1}{2}(2x-3)^{-\frac{1}{2}}\times 2=(2x-3)^{-\frac{1}{2}}$

(iii) Most students obtained $\frac{dy}{dx}=\frac{2(x-2)-(2x+1)}{(x-2)^2}=\frac{-5}{(x-2)^2}$

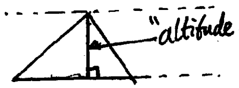
but forgot to put brackets around this so ended up with +1 on the numerator instead of −1.

Some remembered the quotient rule incorrectly or had the numerator terms the wrong way around.

(iv) $y=2x^{-\frac{1}{3}}$ $\quad\frac{dy}{dx}=-\frac{2}{3}x^{-\frac{4}{3}}$ or $\frac{-2}{3\sqrt[3]{x^4}}$ many made a meal out of this!

Make sure you clearly write "$y=$" and "$\frac{dy}{dx}=$" rather than "$2x^{-\frac{1}{3}}=-\frac{2}{3}x^{-\frac{4}{3}}$" which is not correct!

(b)(i) Very few students realised that "altitudes" of a triangle meet opposite sides at 90°

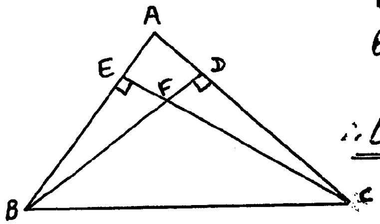


This was critical as the proof was conditional on RHS → BC is a common hypotenuse of ΔBDC and ΔCEB

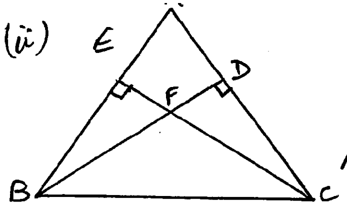
$B\hat{E}C=C\hat{D}B=90^\circ$ (altitudes of a triangle meet sides at 90°)

BD = CE (given)

∴ ΔBDC ≡ ΔCEB (RHS)



(b) (ii)



$E\hat{B}C = D\hat{C}B$ (corresponding angles in congruent triangles are equal)

$A\hat{B}C = E\hat{B}C$
$A\hat{C}B = D\hat{C}B$ } (Same angles).

$\therefore A\hat{B}C = A\hat{C}B$

NB, you need to do these bits!

(iii) The easiest method is: $D\hat{B}C = E\hat{C}B$ (corresponding angles in congruent triangles are equal)

$F\hat{B}C = D\hat{B}C$
$F\hat{C}B = E\hat{C}B$ } (same angles)

$\therefore F\hat{B}C = F\hat{C}B$ as required

NB: You can still do parts (ii) and (iii) even if you can't do (i) and use the result $\Delta BDC \equiv \Delta CEB$

7(a) $3x^2+4x+1 = Ax(x-1)+B(x+3)+C$

$= Ax^2 - Ax + Bx + 3B + C$

$\therefore 3 = A$ — 1

$4 = -A+B \therefore B = 7$
$1 = 3B+C \therefore C = -20$ } — 1

$\therefore 3x^2+4x+1 = 3x(x-1)+7(x+3)-20$

[2]

(b) (i) $3(0)+5y+2=0 \therefore y = \frac{2}{5} \therefore P\left(0,\frac{2}{5}\right)$ — 1

[1]

(ii) distance from $\left(0,\frac{2}{5}\right)$ to $3x+5y+2=0$ is

$$d = \frac{\left|3(0)+5\left(\frac{2}{5}\right)+2\right|}{\sqrt{3^2+5^2}} = \frac{4}{\sqrt{34}} = \frac{4}{\sqrt{34}}\times\frac{\sqrt{34}}{\sqrt{34}}$$ — 1 — 1

$$= \frac{4\sqrt{34}}{34} = \frac{2\sqrt{34}}{17}$$

[2]

(c) $y = \frac{(3x+4)^{n-1}}{\sqrt{x^3}}$

$$\frac{dy}{dx} = \frac{x^{\frac{3}{2}}(n-1)(3x+4)^{n-2}\times 3-(3x+4)^{n-1}\times\frac{3}{2}x^{\frac{1}{2}}}{x^3}$$ — 1

$$= \frac{3\sqrt{x}(3x+4)^{n-2}\left[x(n-1)-\frac{1}{2}(3x+4)\right]}{x^3}$$ — 1

$$= \frac{3(3x+4)^{n-2}\left[\frac{2x(n-1)-(3x+4)}{2}\right]}{x^{\frac{5}{2}}}$$

$$= \frac{3(3x+4)^{n-2}[2xn-2x-3x-4]}{2x^{\frac{5}{2}}}$$ — 1

$$= \frac{3(3x+4)^{n-2}[2xn-5x-4]}{2x^{\frac{5}{2}}}$$

$$= \frac{3(3x+4)^{n-2}[x(2n-5)-4]}{2x^{\frac{5}{2}}}$$ — 1

[4]

(d) let the roots be $\alpha, \alpha+1$

$\alpha+\alpha+1 = \frac{-b}{a} \therefore 2\alpha+1 = \frac{-b}{a}$ ← 1

$\therefore 2\alpha = \frac{-b}{a}-1 \therefore \alpha = \frac{-b-a}{2a}$ .....A

and $\alpha(\alpha+1) = \frac{c}{a}$ ....B

sub A into B and $\therefore \frac{-b-a}{2a}\left(\frac{-b-a}{2a}+1\right) = \frac{c}{a}$

$(-b-a)\left[\frac{-b-a}{2a}+1\right] = 2c$

$\frac{(-b-a)(-b-a+2a)}{2a} = 2c$

$(-b-a)(-b-a+2a) = 4ac$ ← 1

$-(a+b)(a-b) = 4ac$

$b^2-a^2 = 4ac$

[3]

Comments on Year 11 - 2 Unit Preliminary Exam, 2009

Question 7 – PSJ

(a) $3x^2+4x+1 \equiv Ax(x-1)+b(x+3)+c$

- ✓ Most students handled this question fairly well.
- ✓ No students found A, B, C by substitution.
- ✓ Many students made algebraic errors such as $-Ax+Bx=-(A+B)x$

(b) (i)

- ✓ Fairly well done, though a large number of students simply wrote $P=\frac{2}{5}$.
- ✓ Some students found both x and y intercepts and wrote $P=(\frac{2}{3},\frac{2}{5})$.

(ii)

- ✓ Fairly well done.
- ✓ But some students used the incorrect line from Part (i) ; viz.

  $3x+5y-2=0$ instead of $3x+2y+2=0$.
- ✓ Besides substituting the incorrect point, some students used the perpendicular distance formula incorrectly, viz.

$$d=\frac{|3(0)+5(\frac{2}{5})+2|}{\sqrt{0^2+(\frac{2}{5})^2}}$$

(c)

- ✓ Very poorly done.
- ✓ Most students struggled algebraically with the question.
- ✓ Most students could not combine fractional indices and could only manage the first step of the Quotient rule, if they quoted it properly.
- ✓ Factorising the numerator proved far too difficult for most.
- ✓ Only a few students gained 4 marks for this part.

(d)

- ✓ Many students were able to obtain the 2 equations :

  $2\alpha+1=-\frac{b}{a}$ and $\alpha(\alpha+1)=\frac{c}{a}$, but were unsuccessful in eliminating α.