Epping Boys High School
213 VIMIERA RD MARSFIELD

# Year 11 Preliminary Mathematics
# Yearly Exam 2012

Name: _____

| Tick your class/teacher | |
|---|---|
| 11M1 | |
| 11M2 | |
| 11M3 | |
| 11M4 | |
| 11M5 | |

Examiner:

Time Allowed:  2 hours plus 5 minutes reading time
Start each question in a new booklet (5 booklets)
All questions are of equal value
You must show all necessary working
Full marks may not be awarded for careless or badly arranged work
Only calculators approved by he Board of Studies may be used
Include your name and the question number on the front of each booklet
All students to remain in the exam centre until the conclusion of the examination

| | MC | Q1 | Q2 | Q3 | Q4 | Q5 | Total | % |
|---|---|---|---|---|---|---|---|---|
| Total Marks | 10 | 14 | 14 | 14 | 14 | 14 | 80 | % |

**Answer these multiple choice questions on the sheet provided at the back**

1    The correct factorisation of $8x^3 - 125$ is

    A    $(2x-5)^3$                               B    $(2x-5)(4x^2 + 10x + 25)$

    C    $(2x+5)(4x^2 - 10x + 25)$             D    $8(x-5)(x^2 - 5x + 25)$

2    The graph shown has equation



    A    $y = |x+3| + 1$                        B    $y = |x-3| + 1$

    C    $y = |x-3| - 1$                        D    $y = |x+3| - 1$

3    The value of $\sec 240°$ is

    A    2                                    B    $\dfrac{2}{\sqrt{3}}$

    C    $-2$                               D    $-\dfrac{2}{\sqrt{3}}$

4    A function is defined for $x = a$. It is defined for other values of $x$. $\displaystyle\lim_{x \to a} f(x) \neq f(a)$

    Therefore the function

    A    is continuous at $x = a$            B    is differentiable at $x = a$
    C    is not continuous at $x = a$        D    cannot be determined

5    In simplified form, the algebraic expression $\dfrac{x+5}{(x-3)(x+1)} - \dfrac{x-1}{x^2 - x - 2}$ can be written as

    A    $\dfrac{7x-13}{(x+1)(x-2)(x-3)}$           B    $\dfrac{-x-13}{(x+1)(x-2)(x-3)}$

    C    $\dfrac{7x-7}{(x+1)(x-2)(x-3)}$            D    $\dfrac{-x-7}{(x+1)(x-2)(x-3)}$

6    A function possesses the property that $f(-x) = -f(x)$. Which of the following graphs could represent the function?
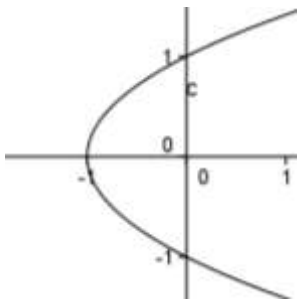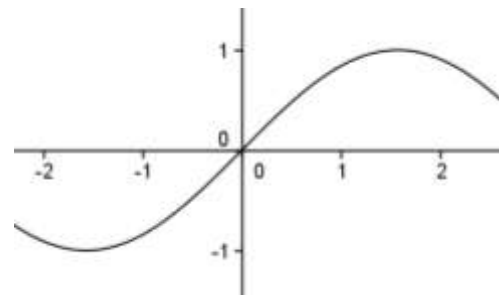
A
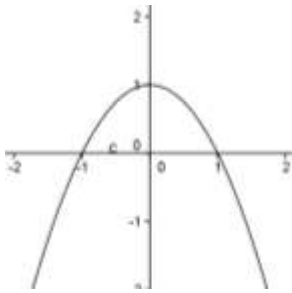


B
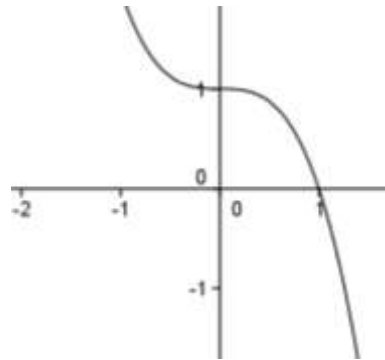


C



D



7    What is the equation for a circle of radius 9 units, which has its centre at (-2, 5)

A    $(x-2)^2 + (y+5)^2 = 81$            B    $(x+2)^2 + (y-5)^2 = 81$

C    $(x-2)^2 + (y+5)^2 = 3$             D    $(x+2)^2 + (y-5)^2 = 3$

8    Complete the following trigonometric identity $\cos ec^2 x =$

A    $1 + \cot^2 x$                      B    $\sec^2 x - 1$

C    $1 - \tan^2 x$                      D    $\cos^2 x - \sin^2 x$

9    A triangle $XYZ$ has $x = 14$cm, $y = 13$cm, $\angle Z = 105^*$. The side length $z$ can be calculated using

A    Cosine Rule                        B    Sine Rule

C    cannot be calculated               D    It is the ambiguous case

10   A triangle $MNP$ has $m = 12$cm, $n = 14$cm, $p = 10.5$cm. Its smallest angle to the nearest degree is

A    $46^0$                             B    $47^0$

C    $39^0$                             D    $40^0$

**Q11  Commence each answer in a fresh booklet**                                    **Marks**

*Outcome    P3 Performs arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities*

a)    Factorise:

    (i)    $x^2 - 11x$                                                                    **1**

    (ii)   $3x^2 - 4x - 7$                                                            **2**

    (iii)  $x^3 + 8$                                                                      **2**


(b)   Solve:

    (i)    $\dfrac{8}{x} + 2 = \dfrac{16}{x}$                                           **2**


    (ii)   $|2x - 6| \leq 4$                                                            **3**
        Graph your solution on the number line

(c)   Solve the simultaneous equations:                                            **2**
      $xy + y^2 = 54$
      $x + y = 9$


(d)   Find $0 \cdot 7\dot{1}$ as a fraction in simplest terms                         **2**

**Q12  Begin a fresh booklet for Q2**

*Outcome    P3 Performs arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities*

(a)   Find the value of                                                             **1**
    $27^{\frac{1}{3}} + 2^{-2}$

(b)   Give the value of $1.79^{1.35}$ correct to 3 decimal places                    **1**


(c)   Simplify the following $2\sqrt{20} + \sqrt{45}$                                **2**

(d)   Rationalise the denominator and simplify fully                                **2**
    $\dfrac{4 + \sqrt{2}}{\sqrt{5} + \sqrt{3}}$

(e)   Expand and simplify $(3 + 2\sqrt{5})(2 - \sqrt{5})$                            **2**


(f)   Find values for *a* and *b* by completing the square                          **2**

    $x^2 - 8x + 4 = (x - a)^2 - b$

(g)   Find the following trigonometric ratios in exact terms

    (i)    $\cos 240°$                                                              **1**

    (ii)   $\cot 330°$                                                              **1**

(h)   Show that $\cos^2(90-\theta)\cot\theta = \sin\theta\cos\theta$   **2**

**Q13  Begin a fresh booklet for Q13**
*Outcome P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques*
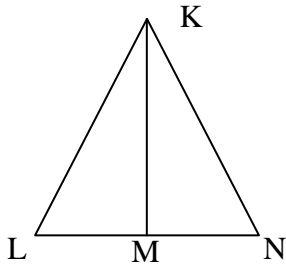
(a)   The diagram shows an isosceles triangle $\Delta KLN$
      With $KL = KN$. $KM$ is constructed to bisect $LN$.
      (i)   Complete the proof that $\Delta KMN \equiv \Delta KML$   **3**
            $KM$ common
            $KN = KL$ _____
            _____ = _____ by construction
            $\Delta KMN \equiv \Delta KML$ _____
      (ii)  Complete the proof that an isosceles triangle   **1**
            has base angles equal

(b)   BC//DE.
      (i)   Prove $\Delta ABC \, ||| \, \Delta ADE$   **3**
      (ii)  $AD = 7$, $BD = 3$, $CE = 4.5$. Find $AC$   `   **1**
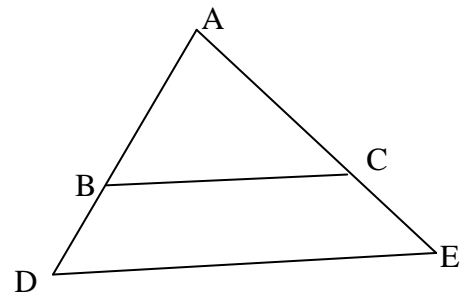
(c)   A tree is situated on a level field.   **3**
      The sun is at an angle of elevation of $31^0$. The length of the shadow is 47m.
      Draw a diagram to represent this information.
      Calculate the height of the tree.

(d)   Calculate the size of the angle A   **3**

      to the nearest whole degree.

      15.2m
      A
      11.3m
      $107^0$
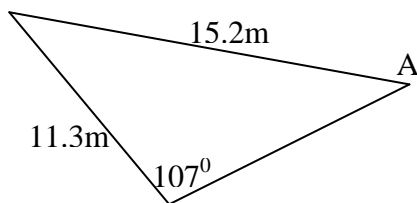
**Q14  Begin a fresh booklet for Q14**

*Outcome P5 Understands the concept of a function and the relationship between a function and its graph*

(a)    What is the natural domain and the range for  $y = \sqrt{9 - x^2}$      **3**
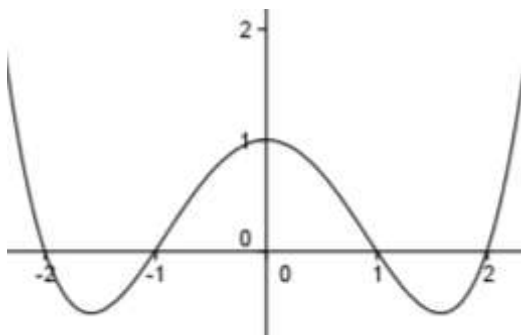


(b)    The accompanying graph shows      **3**
a function.    $y = \dfrac{x^4 - 5x^2 + 4}{4}$

Is it is odd, even or neither?

Give both a graphical and an algebraic reason

for your decision.

(c)    (i)    Graph the function  $y = \dfrac{6}{x}$  include at least 1 point.      **2**

(ii)    Is the function continuous? Be specific.      **1**

(d)    Shade the region specified by      **2**

$x^2 + y^2 \le 25$ **or**  $y \ge x$  on the number plane

(e)    (i)    Sketch the relation  $x = y^2$      **1**

(ii)    Is this relation a function? You must give a reason for your decision.      **2**

You may make marks on your sketch to help with your justification.

**Q15  Begin a fresh booklet for Q15**

*Outcome P6 Relates the derivative of a function and its graph*

(a)    $\dfrac{dy}{dx} = 3 - 2x$

(i)    Find  $f'(3)$      **1**

(ii)    For what value of $x$ does the tangent have a gradient of 5      **1**

(b)    Find  $f'(x)$  for each of the following

(i)    $f(x) = x^3 - 2x$      **1**

(ii)    $y = 2x - \dfrac{3}{x}$      **1**

(iii)    $y = (x^2 - 3x)^{-2}$      **2**

(c)    Find the equation of the tangent to  $y = x^2 - 3\sqrt{x}$  at the point where $x = 1$      **3**

(d)  (i)   Obtain the equation of the normal to $y = x^2 - 3$ at the point (2, 1)   **2**

     (ii)  Where does this line cut the $y$ axis?   **1**

(e)  Obtain the coordinates of the point on $y = x^2 - 3x + 2$   **2**

     where the tangent is parallel to the $x$ axis.

☺ **END OF PAPER** ☺

# Multiple Choice Answer Sheet

**Name:** _____

**Teacher's Name** _____

|  |  |  |  |  |
|--|--|--|--|--|
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Prelim Exam (2U) Mathematics        2012

| | |
|---|---|
| 1 B      2 B      3 C | |
| 4 C      5 A      6 D. | 1 each. |
| 7 B.      8 A.      9. A. | |
| 10. B. | /10 |

11 a) i)   $x(x-11)$         1

    ii)  $(3x-7)(x+1)$      2
                                  1 for progress

    iii) $(x+2)(x^2-2x+4)$   2
                                    1 for progress.

b) i)   $\dfrac{8}{x}+2=\dfrac{16}{x}$

        $8+2x=16$       2
           $x=4$         1 for progress.

   ii)  $|2x-6|\le 4$

    $-4\le 2x-6\le 4$      1
     $2\le 2x\le 10$
      $1\le x\le 5$        1


     0   1       5        1.

c)   $xy+y^2=54$  ----①
      $x+y=9$.   ----②.

11 c) contd,

from ② $y = 9 - x$. sub into ①

$x(9-x) + (9-x)^2 = 54$.

$9x - x^2 + 81 - 18x + x^2 = 54$

$81 - 9x = 54$.     (1 for progress)

$27 = 9x$

$3 = x$     2

$y = 6$.

d)   $x = 0.7\dot{1}$

$10x = 7.1\dot{1}$     1

$9x = 6.4$.

$x = \dfrac{64}{90} = \dfrac{32}{45}$.     1.    /14

12 a)   $3 + \dfrac{1}{4} = 3.25$.     1

b)   $2.195$     1

c)   $2 \times 2\sqrt{5} + 3\sqrt{5}$.     1

$7\sqrt{5}$.     1

d)   $\dfrac{4+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \dfrac{(\sqrt{5}-\sqrt{3})}{\sqrt{5}-\sqrt{3})}$     1

$\dfrac{4\sqrt{5} - 4\sqrt{3} + \sqrt{10} - \sqrt{6}}{2.}$     1

e) $(3 + 2\sqrt{5})(2 - \sqrt{5}) = 6 - 10 - 3\sqrt{5} + 4\sqrt{5}$     1

$= \sqrt{5} - 4$     1

f) $x^2 - 8x + 4 = x^2 - 8x + 16 - 12$.      1

$$= (x - 4)^2 - 12 \qquad a = 4$$
$$b = 12$$      1

g) $\cos 240 = -\cos 60 = -\dfrac{1}{2}$      1

$\cot 330 = \dfrac{-1}{\tan 30} = -\sqrt{3}$.      1.

h) $\cos^2(90 - \theta) \cot \theta$
$$= \sin^2 \theta \times \frac{\cos \theta}{\sin \theta}$$      1

$$= \sin \theta \cos \theta \qquad \text{as req}^d$$      1    $\dfrac{}{14}$

---

Q13.    given      1

   LM = MN      1.

   SSS rule      1

   $\angle L = \angle N$   corresponding $\angle s$ in      1
     congruent $\Delta s$.

   $\angle ABL = \angle ADE$.   corresponding $\angle s$      1.
     parallel lines.

   $\angle A$ common ( or $\angle ACB = \angle AED$. )      1
     corr $\parallel$ lines.

   $\Delta ABL \parallel\parallel\parallel \Delta ADE$.    equiangular      1

ii)    b      1

$$\frac{h}{47} = \tan 31$$

1

$$h = 47 \tan 31$$
$$= 28.24 \text{ m}.$$

1

d) $$\frac{\sin A}{11.3} = \frac{\sin 107}{15.2}$$

$$\sin A = .7109.....$$

1
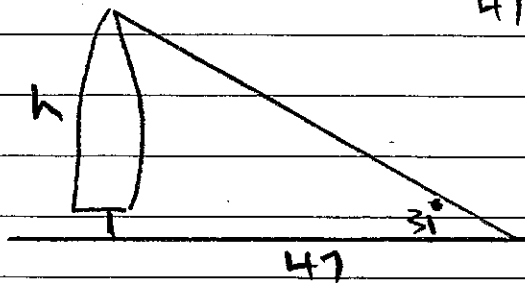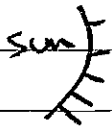
A = 45. or 135.

but 135 would be too big

∴ A = 45

Must be
sure about
obtuse.

1

1

/14.

14 a) Domain     $-3 \leq x \leq 3$.

2  allow 1 for
$x \leq 3$.

Range     $0 \leq y \leq 3$.

1

b) even

Symmetric about y axis.

$$f(-x) = \frac{x^4 - 5x^2 + 4}{4} = f(x)$$
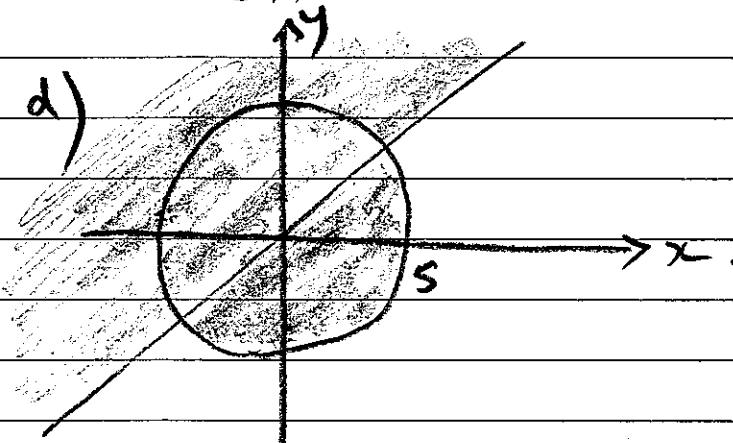
1

1.

1

c) i)     PTO

   ii)     Not continuous when
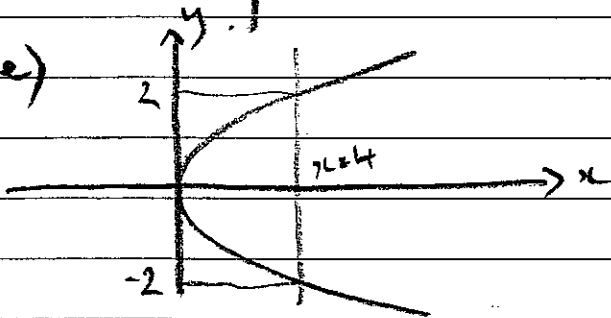           $x = 0$.

c) i)
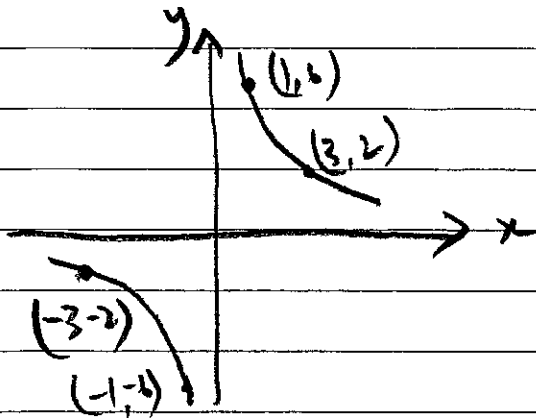


2

must have ≥ 1
point specified

d)



1 for ○ + /

1 for shading.

e)



1

ii)  Not function.
for example if x = 4 y = ±2.

1

1

15.    $f'(3) = 3 - 2 \times 3 = -2$

1

a)

$5 = 3 - 2x.$

$2x = -2$

$x = -1.$

1

b), 1)   $f'(x) = 3x^2 - 2$

1

15b) ii)  $f'(x) = 2 + 3x^{-2}$

$= 2 + \dfrac{3}{x^2}$

| | |
|---|---|
| | 1 |

iii)  $f'(x) = -2(x^2 - 3x)^{-3}$
$\times 2x - 3$

| | |
|---|---|
| | 1  1 |

$= \dfrac{-4x + 6}{(x^2 - 3x)^3}$

no need to render in simplified form
$(-1)$ if error made.

---

c)  $y = x^2 - 3x^{\frac{1}{2}}$
$y' = 2x - 1\frac{1}{2}x^{-\frac{1}{2}}$

| | |
|---|---|
| | 1 |

$f'(1) = 2 - 1\frac{1}{2} = \frac{1}{2}$.  $M_T$.

| | |
|---|---|
| | 1 |

$y_1 = 1 - 3 = -2$.

$y + 2 = \frac{1}{2}(x - 1)$

| | |
|---|---|
| | 1 |

$y = \frac{1}{2}x - 2\frac{1}{2}$

d) i)  $y' = 2x$     $f'(2) = 4$.
$M_N = -\frac{1}{4}$.

| | |
|---|---|
| | 1 |

$y - 1 = -\frac{1}{4}(x - 2)$

$4y - 4 = -x + 2$ / OR. $y = -\frac{1}{4}x + 1\frac{1}{2}$
$x + 4y - 6 = 0$ /

| | |
|---|---|
| 1 | must be simplified into a form |

y axis   $x = 0$.   $y = 1\frac{1}{2}$.

| | |
|---|---|
| | 1 |

e) $y' = 2x - 3$    if $y' = 0$  $x = 1\frac{1}{2}$
$f 1\frac{1}{2} = -\frac{1}{4}$         $(1\frac{1}{2}, -\frac{1}{4})$

| | |
|---|---|
| | 1 |
| | 1 |