GIRRAWEEN HIGH SCHOOL

YEAR 11 YEARLY EXAMINATION

**2005**

# MATHEMATICS

# 2 UNIT

*Time allowed – Two Hours*
*(Plus 5 minutes reading time)*

**DIRECTIONS TO CANDIDATES**

- Attempt ALL questions.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a new page.

**Question 1. (21 marks)** **Marks**

(a) Find the exact value of:

(i) $\frac{1}{4}(x^3 - x^2 + 4)$ when $x = -2$ 1

(ii) $49^{-\frac{1}{2}} \times 8^{\frac{1}{3}}$ 2

(b) Rationalise the denominator and express in simplest surd form. 3

$$\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

(c) Solve the equation $\frac{5x+1}{3x+1} = \frac{5x+2}{3x-2}$ 3

(d) Simplify $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ 3

(e) Factorise fully $4x^3 - 32$ 2

(f) Express $0.\dot{2}\dot{3}$ as a fraction in simplest form. 2

(g) Find all the solutions for:

(i) $|2x-1| = 3x+5$ 3

(ii) $|x-3| < 2$ 2

**Question 2.** **(26 marks)** (Start on a new page)

(a) Find the derivative of:

(i) $x^5 + 4x^2 - 7$ 2

(ii) $\dfrac{2x+1}{3x-2}$ 3

(iii) $(2x^2 - 3)^5$ 2

(iv) $\sqrt{x^3}$ 2

(b) Find, from first principles, the slope of the tangent to the curve $y = x^2 + 2$ at the point (-1, 3). 3

(c) For the curve $y = ax^2 + bx + c$ where a, b, c are constants, it is given that when x = 1, y = 1 and $\dfrac{dy}{dx} = 1$. Show that a = c. 3

(d) Evaluate:

(i) $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5}$ 2

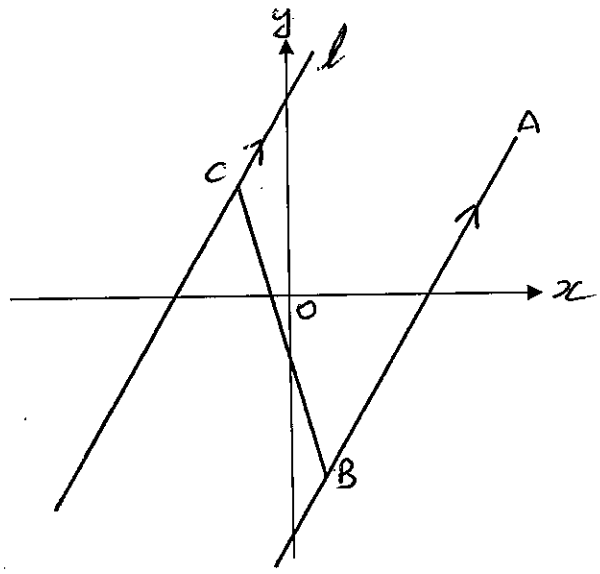
(ii) $\lim\limits_{x \to \infty} \dfrac{2x^2 + 5}{x^2 + 3x}$ 2

(e) Find the equation of the normal to the curve $y = x^3 - 3x^2 - 9x + 30$ at the point (2,8). 4

(f) Find the coordinates of the points where the line $y = 30 - 9x$ intersects the curve $y = x^3 - 3x^2 - 9x + 30$ 3

**Question 3. (18 marks)** (Start on a new page)

(a)




The line $l$ passes through C(-1,2) and has equation $y = 2x + 4$.

The point B has coordinates (1,-6) and the line AB is parallel to $l$.

(i) Find the length of the interval BC. 2

(ii) Find the mid-point of the interval BC. 2

(iii) Write down the slope of the line $l$ and use your calculator to find the acute angle $l$ makes with the x-axis. Give your answer to the nearest degree. 3

(iv) Show that AB has equation $y = 2x - 8$. 2

(v) If P is a point which lies on AB, and on the line $y = 2$, find the coordinates of P. 1

(vi) Find the length of PC. 1

(vii) Find the area of triangle PBC. 2

(b) Two perpendicular lines $3x + 2y = 5$ and $4x + ay = b$ intersect in the point (1,1). Find the values of $a$ and $b$. 3

(c) Find the perpendicular distance from the point A(5,7) to the line $3x + 4y - 18 = 0$. 2

**Question 4. (21 marks)** (Start on a new page)

(a) Find the exact value of the following without using a calculator:

(i) $\tan 150^{0}$. 1

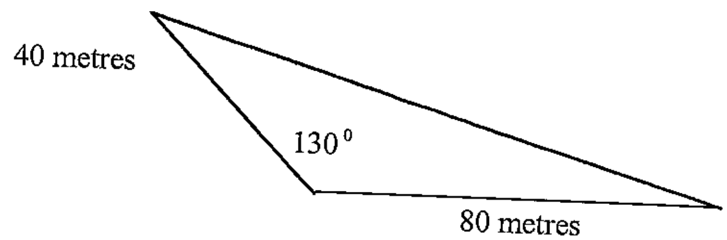
(ii) $\sin^2 30^{0} + \cos^2 30^{0}$. 2

(b) If $\sin\theta = -\frac{\sqrt{3}}{2}$, find all the values of $\theta$ for $0^{0} \leq \theta \leq 360^{0}$. 2

(c) Given $\theta$ is in the second quadrant and $\cos\theta = -\frac{1}{2}$, 3
find the exact value for $\sin\theta + \tan\theta$

(d) Simplify fully $\dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta}$. 3

(e)

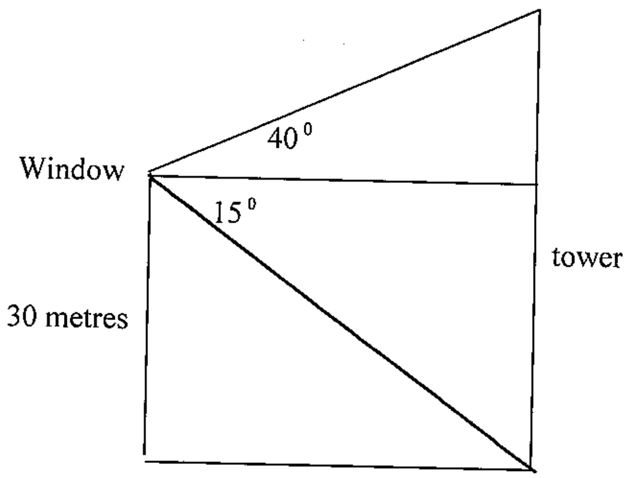



A field is triangular, with two sides of 80 metres and 40 metres enclosing an angle of $130^{0}$.

(i) Calculate the area of the field. 2

(ii) Use the cosine rule to calculate the length of the third side to the nearest metre. 3

(f)




From a window 30m above the ground, the angle of elevation of the top of a church tower is found to be $40^0$, and the angle of depression of the bottom of the tower to be $15^0$.

(i) Find the horizontal distance from the observer to the tower. 2

(ii) Find the height of the tower. 3

**Question 5. (17 marks)** (Start on a new page)

(a) If $\alpha$ and $\beta$ are the roots of the equation $2x^2 - 5x - 1 = 0$, find an expression for:

(i) $\alpha + \beta$. 1

(ii) $\alpha\beta$. 1

(iii) $\alpha^2 + \beta^2$. 2

(b) For what values of k does the expression $x^2 + (k-3)x + k = 0$ have real roots? 3

(c) For what values of k is the expression $x^2 + (k-3)x + k$ positive for all values of ~~k~~ $x$? 3

(d) Solve $4^x - 3(2^x) + 2 = 0.$ 4

(e) Express $4x^2 + x - 1$ in the form $A + B(x+1) + Cx(x+1)$. 3

**Question 6.** **(18 marks)** (Start on a new page)

(a) A function is defined as: $f(x) = \begin{cases} 2x - 1 \; for \; 0 \leq x \leq 3 \\ \frac{1}{3}x + 4 \; for \; 3 < x \leq 5 \end{cases}$

(i) Find the domain of the function. 1

(ii) Find the range of the function. 1

(iii) Find the value of $f(4) - f(2)$. 2

(b) For the function $y = \sqrt{9 - x^2}$,

(i) Find the domain. 2

(ii) Find the range. 1

(c) On the same set of axes, sketch the graph of:

(i) $y = 2 - x^2$. 2

(ii) $y = x$. 1

(iii) Shade the region where $y \leq 2 - x^2$ and $y \geq x$. 2

(d) Draw a sketch graph for each of the following functions:

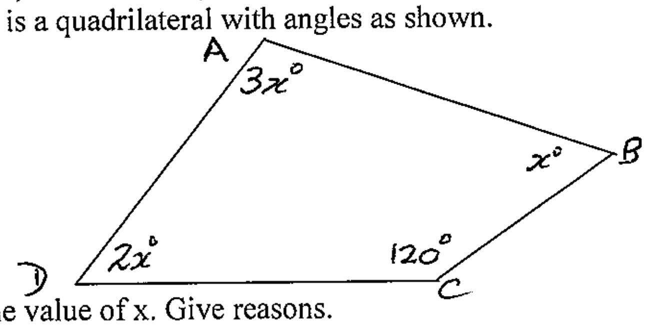
(i) $xy = -9$. 2

(ii) $y = 2^x$. 2

(iii) $x^2 + y^2 = 4$ 2

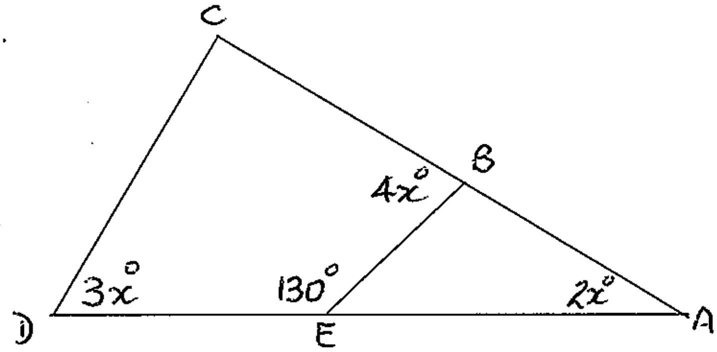
**Question 7. (18 marks)** (Start on a new page)

(a) ABCD is a quadrilateral with angles as shown. 3




Find the value of x. Give reasons.

(b)



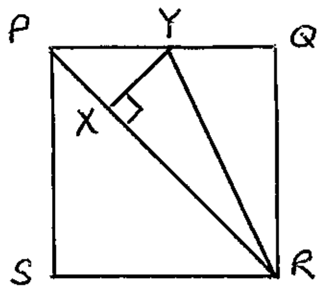

In $\Delta$ ACD, the points B and E lie on the sides AC and AD respectively.

(i) Find the value of x. 3

(ii) Hence find $\angle$ ACD in degrees. 2

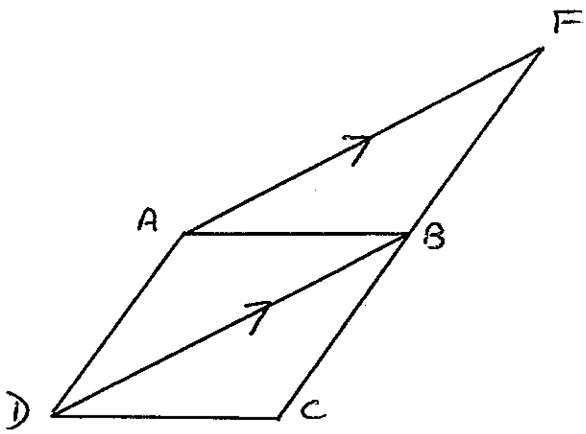
Give reasons for your answers.

(c)




PQRS is a square. YR bisects $\angle$ PRQ.
X lies on PR such that XY $\perp$ PR.
Prove that $\Delta$ YQR and $\Delta$ YXR are congruent. 4

(d)




ABCD is a rhombus in which AD = 6cm and BD = 9cm.
FA∥BD and the points C, B, F are collinear.
Find, giving reasons:

(i) the area of rhombus ABCD, correct to 2 decimal places. 3

(ii) the area of the figure AFCD, correct to 2 decimal places. 3

2005 Year 11 2 Unit Solutions.

Question 1 (21 marks)

(a) (i) $\frac{1}{4}\left((-2)^3-(-2)^2+4\right)$

$= \frac{1}{4}(-8-4+4)$

$= \frac{1}{4}\times -8$

$= -2$ (1)

(ii) $49^{-\frac{1}{2}}\times 8^{\frac{1}{3}}$

$= \frac{1}{\sqrt{49}}\times\sqrt[3]{8}$

$= \frac{2}{7}$ (2)

(b) $\frac{2\sqrt3+\sqrt2}{\sqrt3+\sqrt2}\times\frac{\sqrt3-\sqrt2}{\sqrt3-\sqrt2}$

$= \frac{6-2\sqrt6+\sqrt6-2}{3-2}$

$= 4-\sqrt6$ (3)

(c) $\frac{5x+1}{3x+1}=\frac{5x+2}{3x-2}$

$(5x+1)(3x-2)=(5x+2)(3x+1)$

$15x^2+3x-10x-2=15x^2+6x+5x+2$

$-7x-2=11x+2$

$-4=18x$

$x=-\frac{2}{9}$ (3)

(d) $\frac{x+1}{x^2-x}-\frac{x-1}{x^2+x}$

$= \frac{(x+1)^2-(x-1)^2}{x(x-1)(x+1)}$

$= \frac{(x^2+2x+1)-(x^2-2x+1)}{x(x-1)(x+1)}$

$= \frac{4x}{x(x-1)(x+1)}$ (2)

$= \frac{4}{(x-1)(x+1)}$

(e) $4x^3-32$

$= 4(x^3-8)$ (2)

$= 4(x-2)(x^2+2x+4)$

(f) Let $x = 0.2323\ldots$

$100x = 23.2323$

$99x = 23$ (2)

$x=\frac{23}{99}$

(g) (i) $|2x-1| = 3x+5$

$2x-1=3x+5$ OR $-(2x-1)=3x+5$

$-6=x$ $\qquad -2x+1=3x+5$ (3)

$|13|\neq -13$ $\qquad -4=5x$

$x=-6$ is not a solution $\qquad -\frac{4}{5}=x$

(ii) $|x-3|<2$

$-2<x-3<2$ (2)

$1<x<5$

Question 2 (26 marks)

(a) (i) $\frac{d}{dx}(x^5 + 4x^2 - 7)$

$= 5x^4 + 8x$ (2)

(ii) $\frac{d}{dx}\left(\frac{2x+1}{3x-2}\right)$

$= \frac{(3x-2)\times 2 - 3(2x+1)}{(3x-2)^2}$

$= \frac{6x - 4 - 6x - 3}{(3x-2)^2}$ (3)

$= -\frac{7}{(3x-2)^2}$

(iii) $\frac{d}{dx}\left[(2x^2-3)^5\right]$

$= 5(2x^2-3)^4 \times 4x$

$= 20x(2x^2-3)^4$ (2)

(iv) $\frac{d}{dx}(\sqrt{x^3})$

$= \frac{d}{dx}(x^{3/2})$

$= \frac{3}{2}x^{1/2}$ (2)

$= \frac{3}{2}\sqrt{x}$

(b) $\frac{[(x+h)^2+2] - [x^2+2]}{h}$

$= \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h}$

$= \frac{2xh + h^2}{h}$

$= 2x + h$

$\lim_{x \to 0} (2x+h)$

$= 2x.$

At $x = -1$, $\frac{dy}{dx} = -2.$ (3)

Slope of tangent $= -2$

(c) $y = ax^2 + bx + c$ — ①

$\frac{dy}{dx} = 2ax + b$ — ②

Sub $x = 1$, $y = 1$ in ①

$1 = a + b + c$

Sub $x = 1$, $\frac{dy}{dx} = 1$ in ②

$1 = 2a + b$

By subtraction (3)

$a - c = 0$

$a = c$

(d) (i) $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

$= \lim_{x \to 5} x + 5$

$= 10$ (2)

(ii) $\lim_{x \to \infty} \frac{2x^2 + 5}{x^2 + 3x}$

$= \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} + \frac{5}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2}}$

$= \lim_{x \to \infty} \frac{2 + \frac{5}{x^2}}{1 + \frac{3}{x}}$

$= 2$ (2)

(e) $y = x^3 - 3x^2 - 9x + 30$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

At $x = 2$, $\frac{dy}{dx} = 3\times 4 - 6\times 2 - 9 = -9$

Gradient of normal $= \frac{1}{9}$

Equation of normal:

$(y - 8) = \frac{1}{9}(x - 2)$

$9y - 72 = x - 2$

$x - 9y + 70 = 0.$ (4)

(f) $x^3 - 3x^2 - 9x + 30 = 30 - 9x$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$ (3)

$x = 0$ or $x = 3$

$y = 3$

Question 3 (18 marks)

(a) (i) $BC = \sqrt{(1+1)^2 + (-6-2)^2}$

$= \sqrt{2^2 + (-8)^2}$

$= \sqrt{4+64}$

$= \sqrt{68}$ units (2)

(ii) Midpoint of $BC = \left(\frac{-1+1}{2}, \frac{2-6}{2}\right)$

$= \left(\frac{0}{2}, \frac{-4}{2}\right)$

$= (0, -2)$ (2)

(iii) Slope $= 2$

$\tan\theta = 2$

$\theta = 63^\circ$ (3)

(iv) $AB \parallel l$

$\therefore$ Gradient $AB = 2$

$B(1,-6)$ lies on $AB$

Equation $AB$

$(y+6) = 2(x-1)$

$y = 2x-2-6$

$y = 2x-8$ (2)

(v) If $y=2$, $2 = 2x-8$

$10 = 2x$

$5 = x$

$P$ is the point $(5,2)$ (1)

(vi) Length $PC = \sqrt{(5+1)^2+(2-2)^2}$

$= 6$ units (1)

(b) $3x+2y=5$

$2y = -3x+5$

$y = \frac{-3}{2}x + \frac{5}{2}$

$4x + ay = b$

$ay = -4x+b$

$y = \frac{-4}{a}x + \frac{b}{a}$

For perpendicular lines: $-\frac{3}{2} \times -\frac{4}{a} = -1$

$\frac{6}{a} = -1$

$a = -6$

Sub $x=1$, $y=1$, $a=-6$ in

$4x + ay = b$

$4 - 6 = b$

$-2 = b$ (3)

(c) Perpendicular distance

$= \left|\frac{3\times5 + 4\times7 - 18}{\sqrt{3^2+4^2}}\right|$

$= \left|\frac{15+28-18}{\sqrt{25}}\right|$

$= \left|\frac{25}{5}\right|$

$= 5$ units (2)

Question 4 (21 marks)

(a) (i) $\tan 150^\circ = -\frac{1}{\sqrt{3}}$ ①

(ii) $\sin^2 30^\circ + \cos^2 30^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$ ②

$= \frac{1}{4} + \frac{3}{4} = 1$

(b) $\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = (180+60)^\circ$ OR $(360-60)^\circ$

$= 240^\circ$ OR $300^\circ$. ②

(c) $\cos\theta = -\frac{1}{2}$

$\theta = (180-60)^\circ = 120^\circ$

$\sin 120^\circ + \tan 120^\circ$

$= \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{1}\right)$

$= \frac{-\sqrt{3}}{2}$ ③

(d) $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$

$= \frac{1+\cos\theta + 1-\cos\theta}{1-\cos^2\theta}$

$= \frac{2}{\sin^2\theta}$ ③

$= 2\operatorname{cosec}^2\theta$

(e) (i) Area $= \frac{1}{2} \times 40 \times 80 \times \sin 130^\circ$

$= 1226 \text{ m}^2$ (to nearest m$^2$) ②

(ii) $l^2 = 40^2 + 80^2 - 2 \times 40 \times 80 \times \cos 130^\circ$

$= 1600 + 6400 + 4113.84$

$= 12113.84$

$l = \sqrt{12113.84}$ ③

$= 110$ m (to nearest metre)

(f) (i) $\tan 15^\circ = \frac{30}{\text{distance}}$

distance $= \frac{30}{\tan 15^\circ}$ ②

$= 112$ m to nearest m.

(ii) $\tan 40^\circ = \frac{h}{112}$

$h = 112 \tan 40^\circ$ ③

$= 94$ m to nearest m.

Height of tower $= 94 + 30$

$= 124$ m.

Question 5 (17 marks)

(a) (i) $\alpha + \beta = -\frac{-5}{2} = \frac{5}{2}$ ①

(ii) $\alpha\beta = -\frac{1}{2}$ ①

(iii) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{5}{2}\right)^2 - 2 \times -\frac{1}{2}$

$= \frac{25}{4} + 1$

$= \frac{29}{4}$ ②

$= 7\frac{1}{4}$

(b) $\Delta \geq 0$

$(k-3)^2 - 4 \times 1 \times k \geq 0$

$k^2 - 6k + 9 - 4k \geq 0$

$k^2 - 10k + 9 \geq 0$

$(k-9)(k-1) \geq 0$

$k \leq 1,\ k \leq 9$ OR $k \geq 1,\ k \geq 9$

$k \leq 1$ $\quad$ $k \geq 9$ ③

(c) $\Delta < 0$

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0$ ③

$k-9 > 0$ and $k-1 < 0$ OR $k-9 < 0$ and $k-1 > 0$

$k > 9$ and $k < 1$ $\quad$ $k < 9$ and $k > 1$

No solution $\quad$ $\therefore 1 < k < 9$.

(d) $4^x - 3(2^x) + 2 = 0$

Let $U = 2^x$

$U^2 - 3U + 2 = 0$

$(U-2)(U-1) = 0$

$\therefore U = 2$ or $U = 1$

$2^x = 2$ $\quad$ $2^x = 1$ ④

$x = 1$ $\quad$ $x = 0$

(e) $4x^2 + x - 1 \equiv A + B(x+1) + Cx(x+1)$

$\equiv A + Bx + B + Cx^2 + Cx$

$\equiv (A+B) + (B+C)x + Cx^2$

$\therefore C = 4$

$B + C = 1$

$\therefore B = -3$

$A + B = -1$

$\therefore A = 2$ ③

$4x^2 + x - 1 \equiv 2 - 3(x+1) + 4x(x+1)$

Question 6 (18 marks)

(a) (i) Domain $0 \le x \le 5$ ①

(ii) Range When $x = 0$, $y = -1$

When $x = 5$, $y = \frac{5}{3} + 4 = 5\frac{2}{3}$

Range $-1 \le y \le 5\frac{2}{3}$ ①

(iii) $f(4) - f(2)$

$= (\frac{1}{3} \times 4 + 4) - (2 \times 2 - 1)$

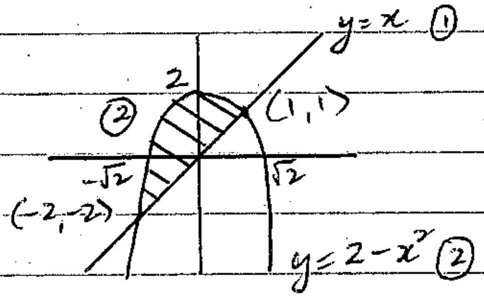
$= 5\frac{1}{3} - 3$

$= 2\frac{1}{3}$ ②

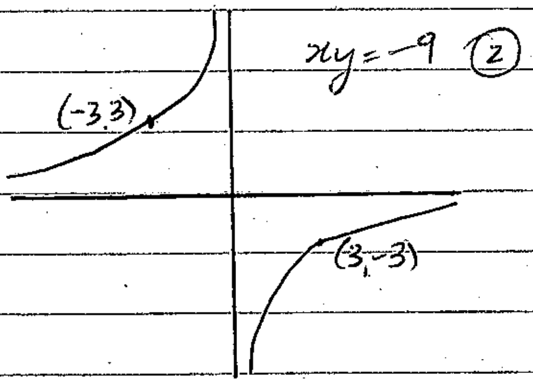
(b) (i) Domain $-3 \le x \le 3$ ②

(ii) Range $0 \le y \le 3$ ①

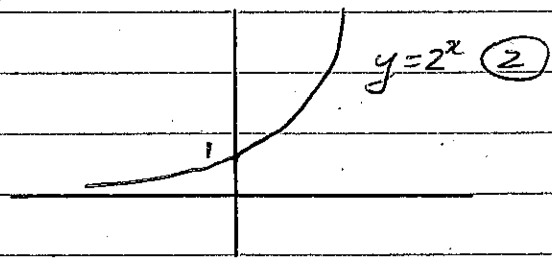
(c)



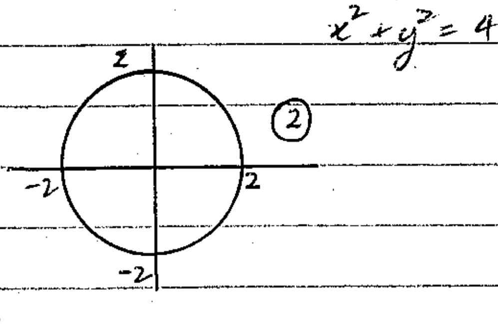

(d) (i)




(ii)




(iii)



Question 7 (18 marks)

(a) $2x + 3x + x + 120 = 360$ (angle sum of a quadrilateral)

$6x + 120 = 360$

$6x = 240$

$x = 40$ (3)

(b) (i) $\angle EBA = 180 - 4x$ (supplementary angles)

$\angle DEB = \angle EBA + \angle BAE$ (exterior angle of triangle is the sum of the two interior opposite angles)

$130 = 180 - 4x + 2x$

$130 = 180 - 2x$

$2x = 50$

$x = 25$ (3)

(ii) $2x + 3x + \angle ACD = 180$ (angle sum of a triangle)

$5x + \angle ACD = 180$

$125 + \angle ACD = 180$

$\angle ACD = 55^\circ$ (2)

(c) In $\Delta YQR$ and $\Delta YXR$

$\angle YQR = \angle YXR$ (angles of a square, data)

$\angle YRQ = \angle YRX$ ($YR$ bisects $\angle PRQ$)

$YR$ is common

$\therefore \Delta YQR \equiv \Delta YXR$ (AAS) (4)

(d) (i) Let $AC$ meet $BD$ in $E$.

$DE^2 + EA^2 = AD^2$ (diagonals of a rhombus bisect each other at right angles)

$4.5^2 + EA^2 = 6^2$

$EA^2 = 6^2 - 4.5^2$

$= 15.75$

$EA = \sqrt{15.75}$

Area of rhombus $= \frac{1}{2} \times 9 \times 2 \times \sqrt{15.75}$

$= 35.72\ cm^2$ (3)

(ii) $AD \parallel CBF$ (opposite sides of rhombus)

$AF \parallel BD$ (data)

$\therefore AFBD$ is a parallelogram.

AB divides the parallelogram into two congruent triangles. (3)

$\therefore$ Area $AFCD = \frac{3}{2} \times 9 \times \sqrt{15.75}$

$= 53.58\ cm^2$.