GIRRAWEEN HIGH SCHOOL

YEAR 11 YEARLY EXAMINATION

2006

MATHEMATICS

Time allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question.
  Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question 1 , Question 2 , etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

Preliminary Learning Outcomes. Mathematics

P2 provides reasoning to support conclusions which are appropriate to the context.

P3 performs routine arithmetic and algebraic manipulations involving surds simple rational expressions and trigonometric identities.

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.

P5 understands the concept of a function and the relationship between a function and its graph.

P6 relates the derivative of a function to the slope of its graph.

P7 determines the derivative of a function through the routine application of the rules of differentiation.

P8 understands and uses the language and notation of calculus.

Question 1(15 marks)

a) Find, correct to two decimal places: **2**

$$\frac{2.48^2}{\sqrt{3.75-2.93}}$$

b) Factorise fully: **4**

(i) x^3-2x^2-15x (ii) x^3+8

c) By expressing $\frac{4}{2+\sqrt{5}}-\frac{1}{9-4\sqrt{5}}$ in its simplest form show that it is a rational number. **3**

d) Express as a single fraction, leaving the denominator in factorised form: **3**

$$\frac{x+5}{x^2+x}+\frac{x+4}{x^2-x}$$

e) Solve the following pair of simultaneous equations: **3**

$$7a+3b=36$$
$$5a+2b=25$$

Question 2(12 marks)

a) Solve $|3x-4|+2\le 7$ **2**

b) Express $0.\dot{6}\dot{4}$ as a fraction in its simplest form. **2**

c) Find the exact value of: **2**

$$49^{\frac{-1}{2}}\times 27^{\frac{2}{3}}$$

d) Solve: **2**

$$x^4-10x^2+21=0$$

e) Find the value of x if: **2**

$$8^{2x-1}=16^{x+2}$$

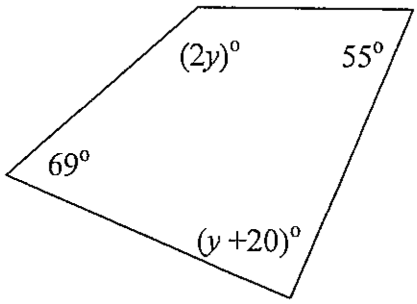
f) Simplify: **2**

$$(\sqrt{2}+1)^2+(2\sqrt{3})^2$$

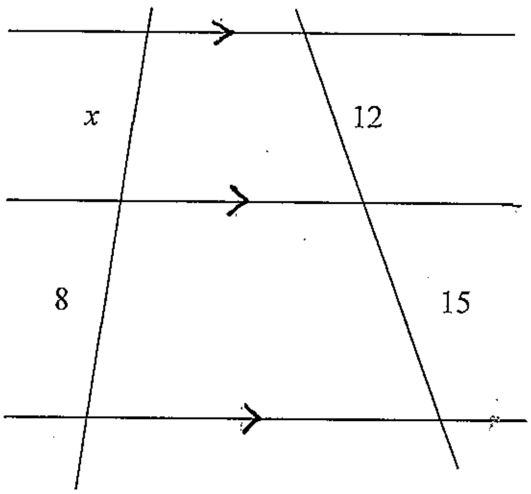
Question 3(12 marks)

a) Find the size of each interior angle of a regular hexagon. **2**

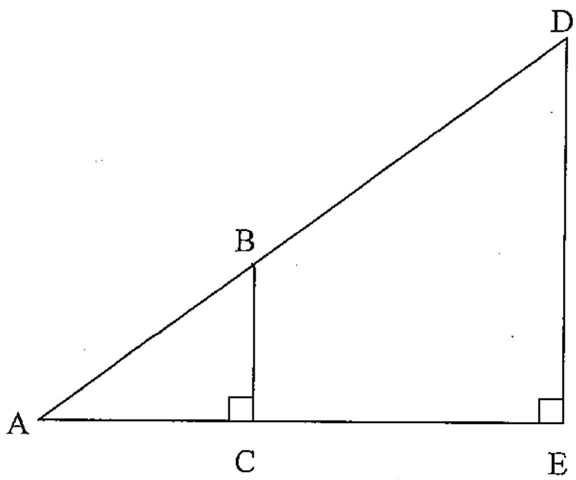
b) Find the value of y, giving reasons **2**



c) Find the value of x, giving reasons. All measurements are in cm. **3**



d) **5**



AC = 10cm, CE = 15cm, BC = 8cm

(i) Prove that $\Delta ABC \ ||| \ \Delta ADE$.

(ii) Find the length of DE.

Question 4(21 marks)

a) Find the derivative of:

(i) $5x^3+4x-7+\frac{3}{x^2}$ 2

(ii) $(3x^2-7)^{10}$ 2

(iii) $\frac{2x-5}{3x+7}$ 3

(iv) $\sqrt{4-x^2}$ 2

(v) $3x^4(x-4)^5$ 3

b) Find, from first principles, the gradient of the tangent to the curve $y=16-x^2$ at the point where $x=-3$ 3

c) If $S(t)=5t^3-4t^2+5t$, find the value of $S'(-2)$ 2

d) Find the equation of the normal to the curve $y = 3 + 6x - 2x^2$ at the point (1, 7). 4

Question 5(22 marks)

a) A(4, 10), B(-3, 1) and C(5, 7) are the vertices of triangle ABC. Plot the points on a number plane. Find

(i) the coordinates of M, the midpoint of BC 1

(ii) the equation of BC 3

(iii) the perpendicular distance of A from BC 2

(iv) the area of ΔABC 2

b) Prove that the points A(3, -2), B(-1, -7) and C(11, 8) are collinear. 2

c) Find the exact value of sin 315° 2

d) Solve: $\sqrt{3}\tan\theta = 1$ for $0° \leq \theta \leq 360°$ 2

e) Prove that $\tan\theta+\cot\theta=\sec\theta\,\mathrm{cosec}\,\theta$ 3

f) ABCD is a parallelogram in which AB = 4cm, BC = 3cm, $\angle$ ABC = 120°. Find, in exact form

(i) the area of the parallelogram 2

(ii) the length of the diagonal AC. 3

Question 6 (20 marks)

a) Draw separate sketches of the following, showing the important features:

(i) $y = |x-3|$ **2**

(ii) $y = 2^x$ **2**

(iii) $y = x^2 - 4$ **2**

b) Indicate clearly on a diagram the region determined by the inequality:

(i) $x + 3y - 3 \leq 0$ **3**

(ii) $y < x^2 + 2x$ **3**

c) Draw a clear sketch of the region which satisfies the inequalities

$x^2 + y^2 \leq 9$ and $y \geq x + 3$ **4**

d) (i) Sketch the graph of $y = \dfrac{1}{2x-1}$ clearly showing any special features. **2**

(ii) State the domain and range of the function. **2**

Question 7 (17 marks)

a) Find the values of a, b and c if

$2x^2 + 3x - 9 \equiv ax(x-1) + b(x-1) + c$ for all x. **3**

b) For the parabola

$y = 5x^2 + 10x - 2$, find:

(i) the equation of the axis of symmetry **1**

(ii) the coordinates of the vertex **1**

c) The quadratic equation $2x^2 + 4x - 3 = 0$ has roots α and β. Find:

(i) $\alpha + \beta$ (ii) $\alpha\beta$ **2**

(iii) $\alpha^2 + \beta^2$ (iv) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **4**

d) For what values of k is the parabola $y = 2x^2 - kx + 8$ positive definite? **2**

e) (i) Show that there are two values of k for which the equation

$x^2 + (k-6)x + 2k = 0$ has equal roots. **2**

(ii) Determine these roots for each value of k. **2**

Year 11 Yearly 2006

Mathematics Solutions

Question 1 (15 marks)

a) 6.79 ②

b) i) x^3-2x^2-15x

$= x(x^2-2x-15)$

$= x(x-5)(x+3)$ ②

ii) x^3+8

$= (x+2)(x^2-2x+4)$ ②

c) $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}}$

$= \frac{4(9-4\sqrt{5})-(2+\sqrt{5})}{(2+\sqrt{5})(9-4\sqrt{5})}$

$= \frac{34-17\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= 34\sqrt{5}+68-85-34\sqrt{5}$

$= -17$ ③

d) $\frac{x+5}{x^2+x} + \frac{x+4}{x^2-x}$

$= \frac{(x+5)(x-1)+(x+4)(x+1)}{x(x+1)(x-1)}$

$= \frac{x^2+4x-5+x^2+5x+4}{x(x+1)(x-1)}$

$= \frac{2x^2+9x-1}{x(x+1)(x-1)}$ ③

e) $7a+3b=36$ —① ×2

$5a+2b=25$ —② ×3

$14a+6b=72$ —③

$15a+6b=75$ —④

④−③ $a=3$

Sub $a=3$ into ①

$7(3)+3b=36$

$b=5$

$\therefore a=3, b=5$ ③

Question 2 (12 marks)

a) $|3x-4|+2\leq 7$

$|3x-4|\leq 5$

$-5\leq 3x-4\leq 5$

$-1\leq 3x\leq 9$

$-\frac{1}{3}\leq x\leq 3$ ②

b) $0.\dot{6}\dot{4}$

$x=0.646464\ldots$

$100x=64.646464\ldots$

$99x=64$

$x=\frac{64}{99}$ ②

c) $49^{-1/2}\times 27^{2/3}$

$=\frac{1}{\sqrt{49}}\times(3^3)^{2/3}$

$=\frac{1}{7}\times 9=\frac{9}{7}$ ②

d) $x^4-10x^2+21=0$

Let $u=x^2$

$u^2-10u+21=0$

$(u-7)(u-3)=0$

$u=7, 3$

But $u=x^2$

$\therefore x^2=7$ or $x^2=3$

$x=\pm\sqrt{7}$ or $x=\pm\sqrt{3}$ ②

e) $8^{2x-1}=16^{x+2}$

$(2^3)^{2x-1}=(2^4)^{x+2}$

$\therefore 6x-3=4x+8$

$x=5\frac{1}{2}$ ②

f) $(\sqrt{2}+1)^2+(2\sqrt{3})^2$

$=2+2\sqrt{2}+1+12$

$=15+2\sqrt{2}$ ②

Question 3 (12 marks)

a) $\angle$ sum of hexagon $= 4 \times 180$

$= 720^\circ$

$\therefore$ vertex angle $= \frac{720}{6}$

$= 120^\circ$ (2)

b) $2y + y + 20 + 69 + 55 = 360$

($\angle$ sum of quad $= 360^\circ$)

$3y + 144 = 360^\circ$

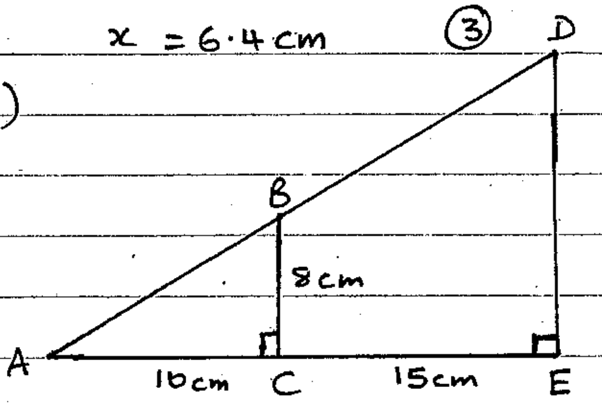
$y = 72^\circ$ (2)

c) $\frac{x}{8} = \frac{12}{15}$ (intercept theorem)

$15x = 96$

$x = 6.4$ cm (3)

d)



i) $\angle BAC = \angle DAE$ (common $\angle$)

$\angle BCA = \angle DEA$ (given)

$\therefore \Delta ABC \parallel\!\mid \Delta ADE$ (equiangular)

(3)

ii) $\frac{DE}{8} = \frac{25}{10}$

$DE = 20$ cm. (2)

Question 4 (21 marks)

a) i) $y = 5x^3 + 4x - 7 + 3x^{-2}$

$\frac{dy}{dx} = 15x^2 + 4 - \frac{6}{x^3}$ (2)

ii) $y = (3x^2 - 7)^{10}$

$\frac{dy}{dx} = 60x(3x^2 - 7)^9$ (2)

iii) $y = \frac{2x - 5}{3x + 7}$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{2(3x+7) - 3(2x-5)}{(3x+7)^2}$

$= \frac{6x + 14 - 6x + 15}{(3x+7)^2}$

$= \frac{29}{(3x+7)^2}$ (3)

iv) $y = (4 - x^2)^{1/2}$

$y' = \frac{1}{2}(4 - x^2)^{-1/2} \cdot -2x$

$= \frac{-x}{\sqrt{4 - x^2}}$ (2)

v) $y = 3x^4(x - 4)^5$

$y' = vu' + uv'$

$= 12x^3(x-4)^5 + 15x^4(x-4$

$= 3x^3(x-4)^4(4(x-4) + 5x$

$= 3x^3(x-4)^4(9x - 16)$ (3

b) $f(x) = 16 - x^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{16 - x^2 - 2xh - h^2 - 16 + x}{h}$

$= \lim_{h \to 0} -2x - h$

$= -2x$

when $x = -3$

$f'(x) = -2(-3) = 6$

$\therefore$ gradient of tangent at $x = -3$ is 6. (3)

4 c) $S(t) = 5t^3 - 4t^2 + 5t$

$S'(t) = 15t^2 - 8t + 5$

$S'(-2) = 15(-2)^2 - 8(-2) + 5$

$= 81$ ②

d) $y = 3 + 6x - 2x^2$

$\frac{dy}{dx} = 6 - 4x$

At $x = 1$

$m_{tangent} = 2$

$m_{normal} = -\frac{1}{2}$ pt $(1,7)$

Equation of normal:

$y - y_1 = m(x - x_1)$

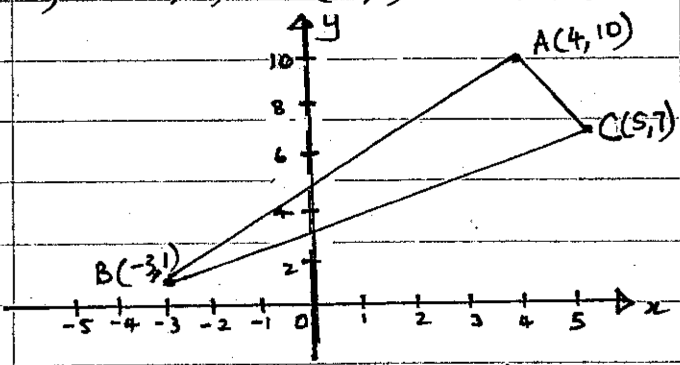
$y - 7 = -\frac{1}{2}(x - 1)$

$2y - 14 = -x + 1$ ④

$x + 2y - 15 = 0$

Question 5 (22 marks)

a) $A(4,10)$ $B(-3,1)$ $C(5,7)$



i) $M_{BC} = \left(\frac{-3+5}{2}, \frac{1+7}{2}\right)$ ①

$= (1, 4)$

ii) $m_{BC} = \frac{6}{8} = \frac{3}{4}$ pt $(-3,1)$

Equation of BC

$y - 1 = \frac{3}{4}(x + 3)$

$4y - 4 = 3x + 9$

$3x - 4y + 13 = 0$ ③

iii) $d_{perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|3(4) - 4(10) + 13|}{\sqrt{3^2 + 4^2}}$

$= 3$ units ②

iv) $BC = \sqrt{8^2 + 6^2}$

$= 10$ units

$A_{\Delta ABC} = \frac{1}{2} BC \cdot d_{perp}$

$= \frac{1}{2} \times 10 \times 3$ ②

$= 15\ u^2$

b) $A(3,-2)$ $B(-1,-7)$ $C(11,8)$

$m_{AB} = \frac{-7+2}{-1-3} = \frac{5}{4}$

$m_{BC} = \frac{8+7}{11+1} = \frac{15}{12} = \frac{5}{4}$ ②

$\therefore$ A, B and C are collinear

c) $\sin 315^\circ = \sin(360^\circ - 45^\circ)$

$= -\sin 45^\circ$

$= -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ ②

d) $\sqrt{3} \tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ, (180 + 30^\circ)$

$\theta = 30^\circ, 210^\circ$ ②

e) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

LHS: $\tan\theta + \cot\theta$

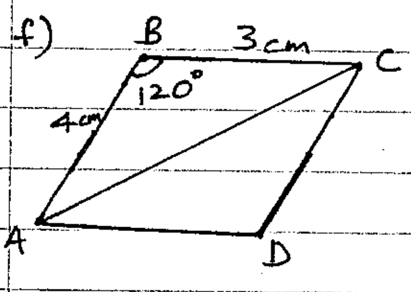
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$

② $=$ RHS

f)



i) $A = 2\left[\frac{1}{2} \times 4 \times 3 \sin 120^\circ\right]$

$= 6\sqrt{3}\ cm^2$ (2)

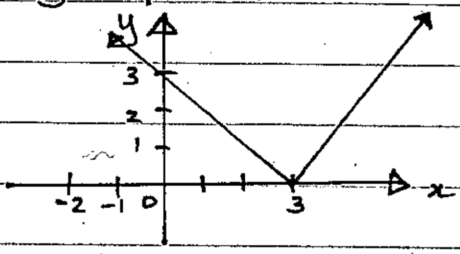
ii) $a^2 = b^2 + c^2 - 2bc \cos A$

$AC^2 = 3^2 + 4^2 - 2(3)(4)\cos 120^\circ$

$AC = \sqrt{37}$ cm (3)

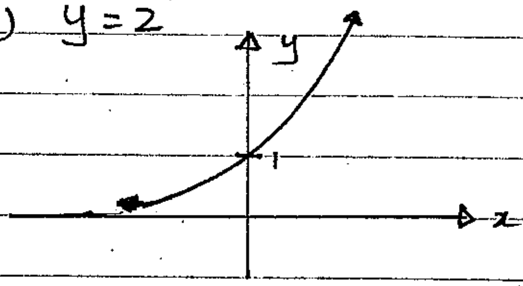
Question 6 (20 marks)

a) i) $y = |x-3|$



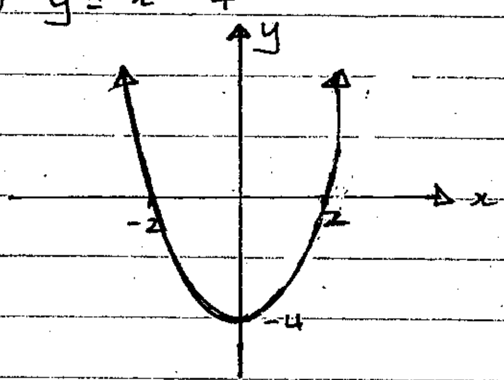
(2)

ii) $y = 2^x$



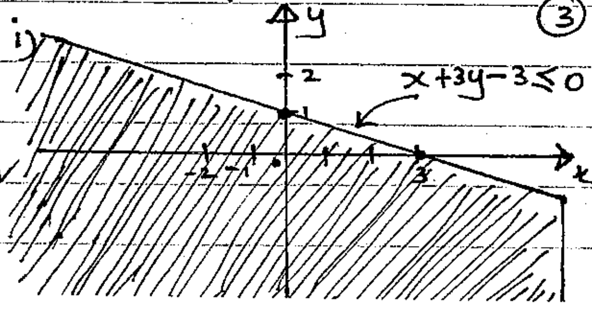
(2)

iii) $y = x^2 - 4$



(2)

b) i)

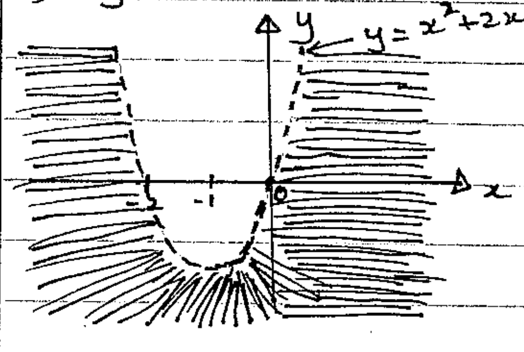


(3)

Test (0,0)

$0+0-3<0$ ✓

ii) $y < x^2 + 2x$

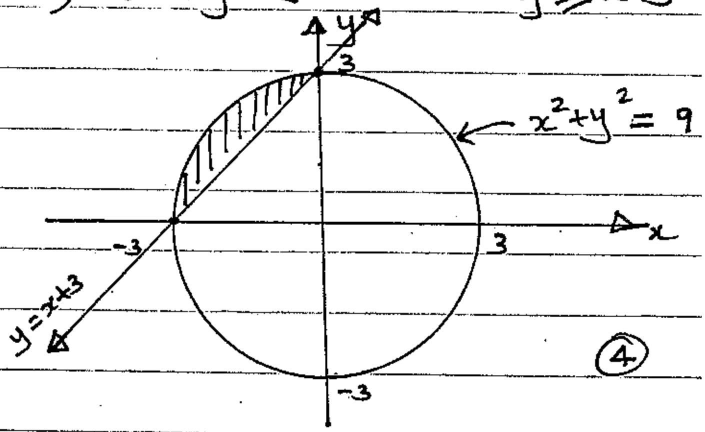


Test (0,1)

$1 < 0$

(3)

c) $x^2 + y^2 \leq 9$ and $y \geq x+3$

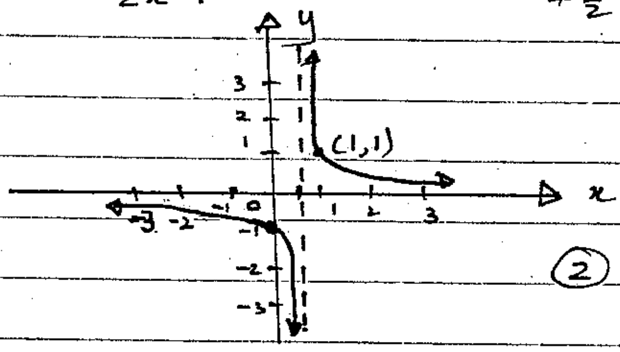


(4)

d) i) $y = \frac{1}{2x-1}$

$2x - 1 \neq 0$

$x \neq \frac{1}{2}$



(2)

ii) Domain : $x \in R,\ x \neq \frac{1}{2}$

Range : $y \in R,\ y \neq 0$ (2)

Question 7 (17 marks)

a) $2x^2+3x-9 \equiv ax(x-1)+b(x-1)+c$

$\equiv ax^2-ax+bx-b+c$

$\equiv ax^2+(b-a)x+c-b$

$\therefore a=2$

$b-a=3$

$b=5$

$c-b=-9$

$c=-4$ (3)

$a=2,\ b=5,\ c=-4$

b) $y=5x^2+10x-2$

i) axis of sym: $x=\frac{-b}{2a}$

$x=-1$ (1)

ii) when $x=-1$

$y=5(-1)^2+10(-1)-2$

$y=-7$

$\therefore$ vertex $=(-1,-7)$ (1)

c) $2x^2+4x-3=0$

i) $\alpha+\beta=\frac{-b}{a}$

$=-2$ (1)

(ii) $\alpha\beta=\frac{c}{a}$

$=-\frac{3}{2}$ (1)

iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-2)^2-2\left(\frac{-3}{2}\right)$

$=7$ (2)

iv) $\frac{1}{\alpha}+\frac{1}{\beta}$

$=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{-2}{-3/2}$

$=\frac{4}{3}$ (2)

d) $y=2x^2-kx+8$

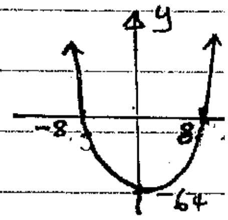
positive definite : $a>0,\ b^2-4ac<$

$(-k)^2-4(2)(8)<0$

$k^2-64<0$

$(k-8)(k+8)<0$

$\therefore -8<k<8$



(2)

e) i) $x^2+(k-6)x+2k=0$

equal roots : $b^2-4ac=0$

$(k-6)^2-4(1)(2k)=0$

$k^2-12k+36-8k=0$

$k^2-20k+36=0$

$(k-2)(k-18)=0$

$\therefore k=2$ or 18 (2)

$\therefore$ there are two values of k

ii) when $k=2$

$x^2-4x+4=0$

$(x-2)^2=0$

$\therefore x=2$

when $k=18$

$x^2+12x+36=0$

$(x+6)^2=0$

$\therefore x=-6$ (2)

$\therefore$ the roots are 2 and -6