GIRRAWEEN HIGH SCHOOL
YEAR 11 YEARLY EXAMINATION

2007

MATHEMATICS

2 UNIT

Time allowed- Two Hours
(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a new page.
- Write your name on each piece of paper.

Question 1 (18 marks) **Marks**

(a) Find, correct to 3 significant figures: $\dfrac{6.4^2 - 3.4^3}{2 \times 6.4 \times 3.4}$ 2

(b) Factorise fully: 4

(i) $2x^2 - 7x - 15$ (ii) $12x^2 - 3y^2$

(c) Find the exact value: 2

(i) $\dfrac{3^{-2}}{5^{-3}}$ (ii) $9^{\frac{1}{2}} \times 100^{\frac{-3}{2}}$

(d) Solve: $3 - (4 - x) = 5x$ 2

(e) Rationalise the denominator: $\dfrac{\sqrt{5} - \sqrt{2}}{3\sqrt{2} - \sqrt{5}}$ 4

(f) Solve and graph on a number line: $|2x - 1| \geq 3$ 4

Question 2 (15 marks)

(a) The points A(-4, 2), B(1, 3) and C(5, -4) are the vertices of a triangle. 9

(i) Plot the points on a number plane showing the above information.

(ii) Find the gradient of AC.

(iii) Find the equation of AC.

(iv) Find the equation of the line perpendicular to AC and passing through B.

(b) Find the perpendicular distance between the parallel lines $3x - y + 1 = 0$ and $3x - y + 5 = 0$. 3

(c) The points (4, -1), (5, 2) and $(7, k)$ are collinear. Find the value of k. 3

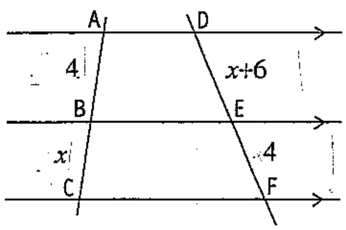
Question 3 (14 marks)

(a) Each interior angle of a regular polygon is $162°$. Find 3

(i) the number of sides.

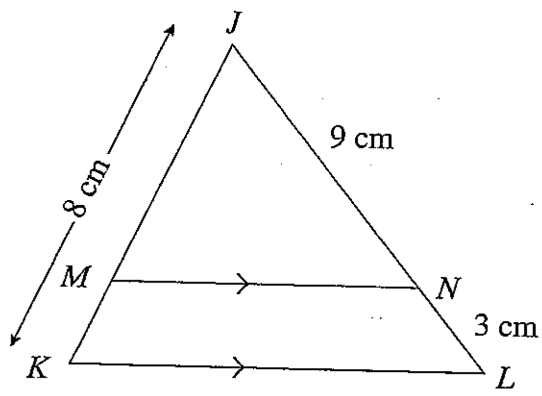
(ii) the sum of its interior angles.

(b) Find the value of x. Give reasons. 3



(c) In the diagram given below, ΔJMN is similar to ΔJKL.

Find the length of MK. 2



(d) Sketch the region: 6

(i) $y < \sqrt{16 - x^2}$

(ii) $y \le 2 - x^2$ and $y \ge |x|$

Question 4 (28 marks)

(a) For each of the following curves 16

(α) $y=(x+1)^2$ (β) $y=|x|-3$

(γ) $y=\dfrac{1}{x-4}$ (δ) $y=\sqrt{2-x}$

(i) Draw a neat sketch showing important features.

(ii) State the domain and range.

(b) Find the domain of $y=\dfrac{1}{\sqrt{25-x^2}}$. 2

(c) Is the function $f(x)=\dfrac{x}{x^2-1}$ odd, even or neither? Justify your answer. 3

(d) A function is defined by the rule

$$f(x)=\begin{cases} x^2-1 & x\geq -1 \\ x+1 & x<-1 \end{cases}$$

(i) Sketch the curve. 3

(ii) State the domain and range. 2

(iii) Find the value of $f(2)+f(-2)$. 2

Question 5 (23 marks)

(a) Differentiate: 11

(i) $y=3\sqrt[3]{x}+5x-4$ (ii) $y=(7x^3-1)(5x^2-x)$

(iii) $y=\dfrac{4x^2-2}{x^2+5}$ (iv) $y=\sqrt{x^2-2x}$

(b) Find the derivative of $y=\dfrac{1}{x}$ from first principles. 3

(c) Find the equation of the normal to the parabola $y=2x^2-5x+1$ at the point on the parabola where the gradient is 3. 4

(d) Evaluate the following limits: 5

(i) $\lim\limits_{x\to 5}\dfrac{x^3-125}{x^2-7x+10}$ (ii) $\lim\limits_{x\to\infty}\dfrac{x^2+1}{2x^2+5}$

Question 6 (19 marks)

(a) The quadratic equation $4x^2 - 9x + 1 = 0$ has roots α and β. Find: 10

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\alpha^2 + \beta^2$ (iv) $(\alpha + 2)(\beta + 2)$

(b) Find the values of k for which $y = kx^2 + 3kx + 6$ is positive definite. 3

(c) The line $y = 3x - p + 1$ is a tangent to the parabola $y = x^2$. Find the value of p. 3

(d) Find the values of A, B, C if

$4x^2 - 9x - 2 \equiv Ax(x+1) + B(x-1) + C$ for all x. 3

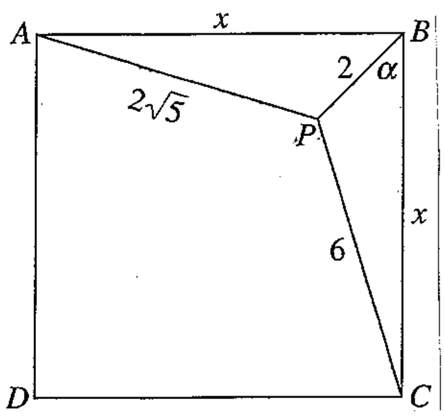
Question 7 (21 marks)

(a) Solve for $0° \le \theta \le 360°$. Write answer correct to the nearest minute. 6

(i) $2\sin\theta = \sqrt{3}$ (ii) $\cos 2\theta = \dfrac{-1}{\sqrt{2}}$

(b) Prove: $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$ 3

(c) The diagram shows a square $ABCD$ of side x cm. $PC = 6$ cm, $PB = 2$ cm and $AP = 2\sqrt{5}$ cm.



(i) Using the cosine rule in ΔPBC, show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$. 2

(ii) By considering ΔPBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$. 2

(iii) Show that the value of x is a solution of

$x^4 - 56x^2 + 640 = 0$ (Hint: $\sin^2\alpha + \cos^2\alpha = 1$). 3

(iv) Solve $x^4 - 56x^2 + 640 = 0$ and determine the length AB of the square. Give reason. 5

Year 11 Mathematics Yearly 2007 - Solutions

Question 1

(a) 0.0381 2

(b) (i) $2x^2 - 7x - 15$

$= (x-5)(2x+3)$ 2

(ii) $12x^2 - 3y^2$

$= 3(4x^2 - y^2)$

$= 3(2x+y)(2x-y)$ 2

(c) (i) $\dfrac{3^{-2}}{5^{-3}} = \dfrac{1}{9} \times \dfrac{125}{1}$

$= \dfrac{125}{9}$ 1

$= 13\frac{8}{9}$

(ii) $9^{\frac{1}{2}} \times 100^{-\frac{3}{2}}$

$= 3 \times \dfrac{1}{100^{\frac{3}{2}}}$ 1

$= \dfrac{3}{1000}$

(d) $3 - (4 - x) = 5x$

$3 - 4 + x = 5x$

$4x = -1$ 2

$x = \dfrac{-1}{4}$

(e) $\dfrac{\sqrt{5}-\sqrt{2}}{3\sqrt{2}-\sqrt{5}} = \dfrac{(\sqrt{5}-\sqrt{2})(3\sqrt{2}+\sqrt{5})}{(3\sqrt{2}-\sqrt{5})(3\sqrt{2}+\sqrt{5})}$

$= \dfrac{3\sqrt{10}+5-6-\sqrt{10}}{(3\sqrt{2})^2-(\sqrt{5})^2}$

$= \dfrac{2\sqrt{10}-1}{18-5} = \dfrac{2\sqrt{10}-1}{13}$ 4

(f) $|2x-1| \geq 3$

$2x-1 \geq 3$ or $-(2x-1) \geq 3$

$2x-1 \geq 3$ or $2x-1 \leq -3$

$2x \geq 4$ or $2x \leq -2$

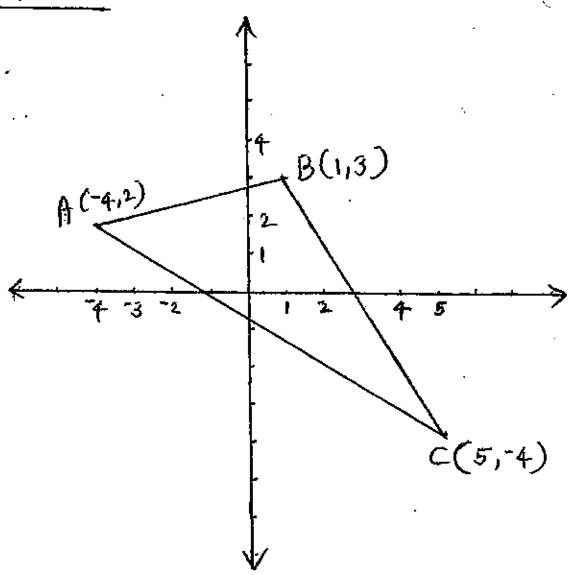
$x \geq 2$ or $x \leq -1$ 4

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

Question 2

(a)

(i)



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(ii) $m_{AC} = \dfrac{2+4}{-4-5} = \dfrac{6}{-9} = \dfrac{-2}{3}$ 2

(iii) $y - 2 = \dfrac{-2}{3}(x+4)$

$3y - 6 = -2(x+4)$ 3

$3y - 6 = -2x - 8$

$2x + 3y + 2 = 0$ OR $y = \dfrac{-2}{3}x - \dfrac{2}{3}$

(iv) Gradient of the perpendicular $= \frac{3}{2}$

Equation of the perpendicular line is

$$y - 3 = \frac{3}{2}(x-1)$$
$$2y - 6 = 3(x-1)$$
$$2y - 6 = 3x - 3 \qquad (3)$$
$$\underline{\underline{3x - 2y + 3 = 0}}$$

OR

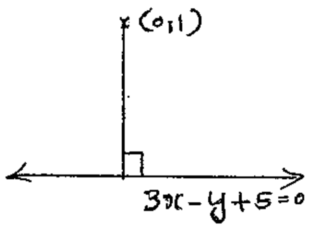
$$\underline{\underline{y = \frac{3}{2}x + \frac{3}{2}}}$$

(b) $3x - y + 1 = 0$

when $x = 0$, $1 - y = 0$

$1 = y$

$(0,1)$ is a point on $3x-y+1=0$



(3)

$$d = \frac{|3\times 0 - 1 + 5|}{\sqrt{3^2 + (-1)^2}}$$
$$= \underline{\underline{\frac{4}{\sqrt{10}} \text{ or } \frac{2\sqrt{10}}{5}}}$$

(c) $A(4,-1)$ $B(5,2)$ $C(7,k)$

$$m_{AB} = m_{BC}$$
$$3 = \frac{k-2}{2} \qquad (3)$$
$$k - 2 = 6$$
$$\underline{\underline{k = 8}}$$

Question 3

(a) (i) Let n be the number of sides

$$\frac{(n-2)\times 180}{n} = 162$$
$$180(n-2) = 162n$$
$$180n - 360 = 162n \qquad (2)$$
$$18n = 360$$
$$n = 20$$

(ii) Sum of interior angles $= (n-2)\times 180$

$$= 18 \times 180 \qquad (1)$$
$$= 3240$$

(b) $\frac{4}{x} = \frac{x+6}{4}$ (A family of parallel lines cutting transversals, preserves the ratio of sides on each transversal)

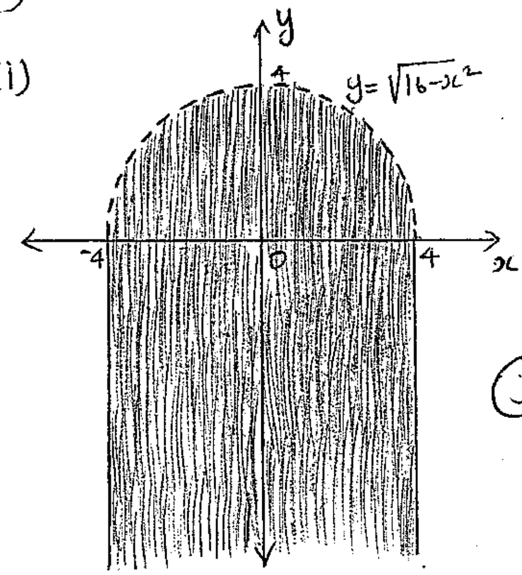
$$x(x+6) = 16$$
$$x^2 + 6x - 16 = 0 \qquad (3)$$
$$(x+8)(x-2) = 0$$
$$x = -8 \text{ or } 2$$

$\underline{\underline{x = 2}}$ (negative values can't be the length)

(c) $$\frac{JM}{JK} = \frac{JN}{JL}$$
$$\frac{JM}{8} = \frac{9}{12}$$
$$JM = \frac{9\times 8}{12} = 6 \qquad (2)$$
$$MK = 8 - 6$$
$$\underline{\underline{= 2 \text{ cm}}}$$

(d)

(i)



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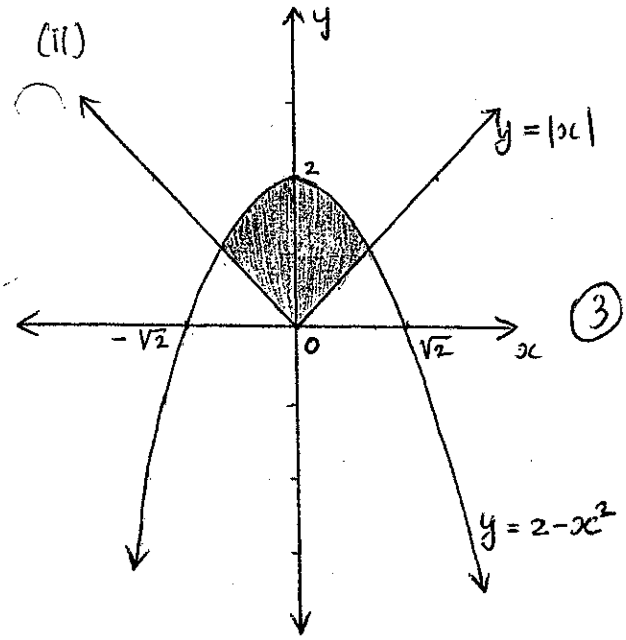
(ii)



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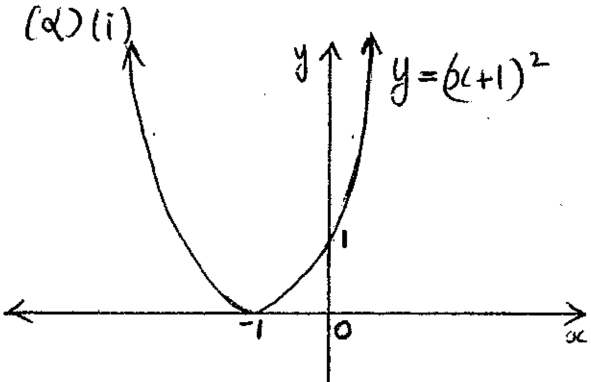
Question 4

(α)(i)



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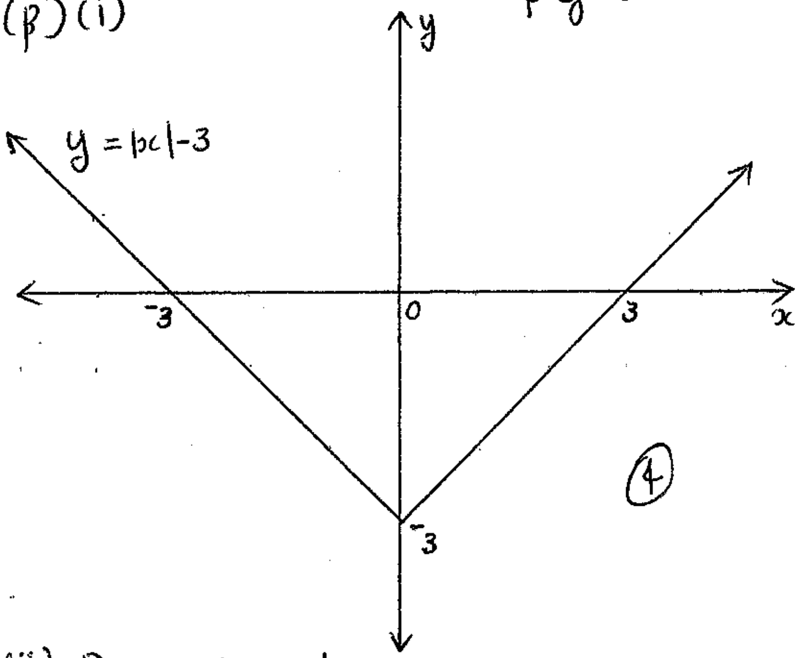
(ii) D : all real x

R : $y \geq 0$, y real

(β)(i)



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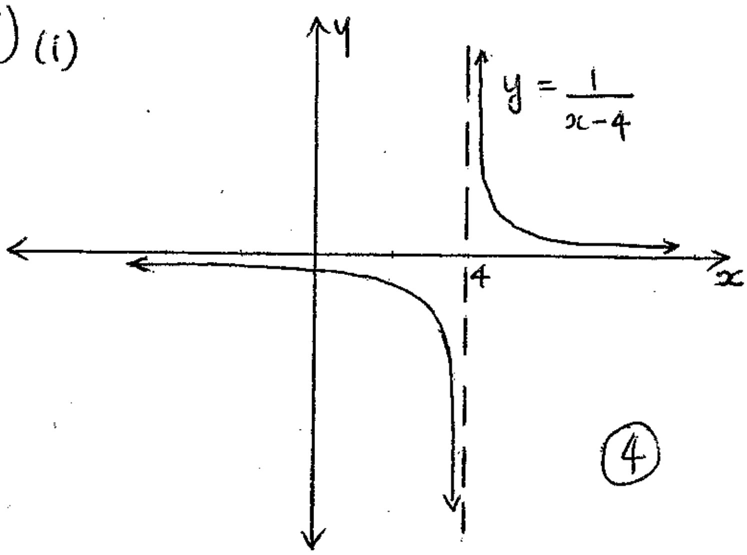
(ii) D : all real x

R : $y \geq 3$, $y \in R$

(γ)(i)



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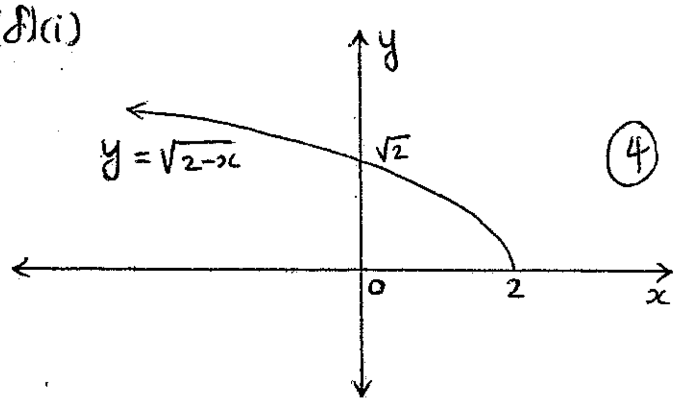
(ii) D : all real x, $x \neq 4$

R : all real y, $y \neq 0$

(δ)(i)



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(ii) D : $x \leq 2$

R : $y \geq 0$

(b) $y = \dfrac{1}{\sqrt{25-x^2}}$

$\sqrt{25-x^2}$ exists when

$25 - x^2 \geq 0$

$25 \geq x^2$

$x^2 \leq 25$ (2)

ie $-5 \leq x \leq 5$

D : $\underline{\underline{-5 < x < 5}}$

(c) $f(-x) = \dfrac{-x}{(x)^2-1}$

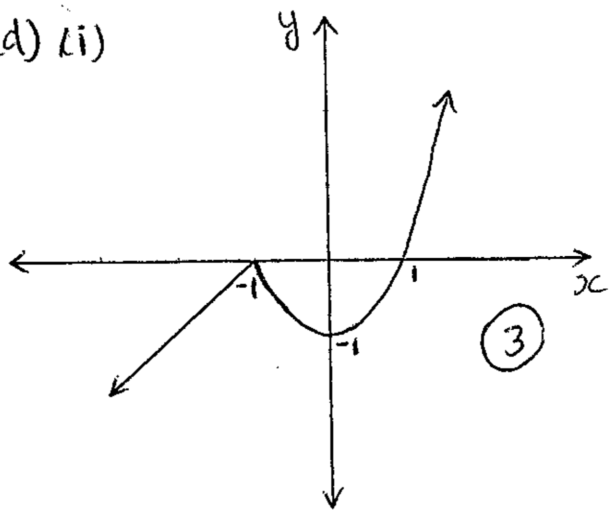
$= \dfrac{-x}{x^2-1}$

$= -\left(\dfrac{x}{x^2-1}\right)$ (3)

$= -f(x)$

$\therefore$ $\underline{\underline{f(x) \text{ is an odd function.}}}$

(d) (i)



(3)

(ii) Domain : R (2)

Range : R

(iii) $f(2) + f(-2)$

$= (4-1) + (-2+1)$ (2)

$= 3 - 1 = \underline{\underline{2}}$

Question 5

(a) (i) $y = 3(x)^{\frac{1}{3}} + 5x - 4$

$y' = 3 \times \frac{1}{3} x^{\frac{1}{3}-1} + 5$ (2)

$= x^{-\frac{2}{3}} + 5 = \underline{\underline{\sqrt[3]{x^{-2}} + 5}}$

(ii) $y = (7x^3-1)(5x^2-x)$

$y' = (7x^3-1)(10x-1) + (5x^2-x)21x^2$

$= \underline{\underline{175x^4 - 28x^3 - 10x + 1}}$ (3)

(iii) $y = \dfrac{4x^2-2}{x^2+5}$

$y' = \dfrac{(x^2+5)8x - (4x^2-2)2x}{(x^2+5)^2}$

$= \dfrac{8x^3 + 40x - 8x^3 + 4x}{(x^2+5)^2}$ (3)

$= \underline{\underline{\dfrac{44x}{(x^2+5)^2}}}$

(iv) $y = \sqrt{x^2-2x}$

$y' = \dfrac{1}{2\sqrt{x^2-2x}} \times 2x - 2 = \dfrac{2(x-1)}{2\sqrt{x^2-2x}}$ (3)

$= \underline{\underline{\dfrac{x-1}{\sqrt{x^2-2x}}}}$

(b) $f(x) = \dfrac{1}{x}$ $\qquad f(x+h) = \dfrac{1}{x+h}$

$f(x+h) - f(x) = \dfrac{1}{x+h} - \dfrac{1}{x}$

$= \dfrac{x - x - h}{x(x+h)} = \dfrac{-h}{x(x+h)}$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{x(x+h)}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} \quad (3)$$

$$= \frac{-1}{x^2}$$

(c) $y = 2x^2 - 5x + 1$

$y' = 4x - 5$

$4x - 5 = 3$

$x = 2$

when $x = 2$, $y = -1$

Equation of normal

$y + 1 = \frac{-1}{3}(x - 2)$ (4)

$3y + 3 = -x + 2$

$x + 3y + 1 = 0$

(d) (i) $\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 7x + 10}$

$$= \lim_{x \to 5} \frac{(x-5)(x^2+5x+25)}{(x-5)(x-2)}$$

$$= \lim_{x \to 5} \frac{x^2+5x+25}{x-2} \quad (3)$$

$$= 25$$

(ii) $\lim_{x \to \infty} \frac{x^2+1}{2x^2+5}$

$$= \lim_{x \to \infty} \frac{1 + \frac{1}{x^2}}{2 + \frac{5}{x^2}} \quad (2)$$

$$= \frac{1}{2} \quad \left(\because \lim_{x \to \infty} \frac{1}{x} = 0\right)$$

Question 6

(a) $4x^2 - 9x + 1 = 0$

(i) $\alpha + \beta = \frac{-b}{a}$ (2)

$= \frac{-(-9)}{4} = \frac{9}{4}$

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{1}{4}$ (2)

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{9}{4}\right)^2 - 2 \times \frac{1}{4}$

$= \frac{73}{16}$ (3)

(iv) $(\alpha+2)(\beta+2) = \alpha\beta + 2(\alpha+\beta) + 4$

$= \frac{1}{4} + 2 \times \frac{9}{4} + 4$

$= \frac{35}{4} = 8\frac{3}{4}$ (3)

(b) $y = kx^2 + 3kx + 6$ is positive definite

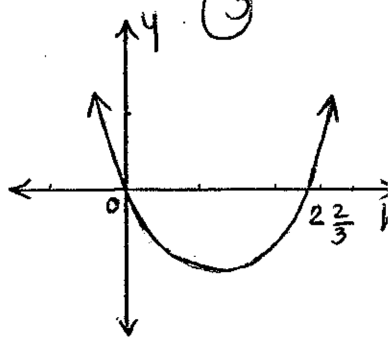
$\Longrightarrow a > 0$ and $\Delta < 0$

$\therefore k > 0$ and $\Delta = (3k)^2 - 4k \times 6 < 0$

$9k^2 - 24k < 0$ (3)

$k(9k - 24) < 0$

ie $0 < k < 2\frac{2}{3}$



(c) $y = x^2$ — (1)

$y = 3x - p + 1$ — (2)

Solving (1) and (2) simultaneously we get $x^2 - 3x + p - 1 = 0$

Given that (2) is a tangent to (1)

$\therefore \Delta = 0$

$\Delta = 9 - 4(p-1)$ (3)

$= 13 - 4p$

$13 - 4p = 0$ $\quad p = 3\frac{1}{4}$

(d) $4x^2 - 9x - 2 \equiv Ax(x+1) + B(x-1) + C$

$4x^2 - 9x - 2 \equiv Ax^2 + Ax + Bx - B + C$

Equating coefficients of x^2, x and Constant term we get

3

$A = 4$

$A + B = -9$

$B = -13$

$-B + C = -2$

$C = -2 + B$

$= -2 + {}^{-}13$

$= -15$

$A = 4,\ B = {}^{-}13,\ C = {}^{-}15$

Question 7

(a) (i) $2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

2

$\theta = 60°, 120°$

(ii) $\cos 2\theta = \frac{-1}{\sqrt{2}}$

Let $u = 2\theta$

$\cos u = \frac{-1}{\sqrt{2}},\ 0 \leq u \leq 720°$

4

$u = 135°, 225°, 495°, 585°$

$\theta = 67°30', 112°30', 247°30', 292°30'$

(b) $LHS = (\cot\theta + \operatorname{cosec}\theta)^2$

$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$

3

$= \frac{(1+\cos\theta)^2}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1+\cos\theta}{1-\cos\theta}$

$= RHS$

(c) (i) In ΔPBC

$\cos\alpha = \frac{x^2 + 2^2 - 6^2}{2 \times x \times 2}$

$= \frac{x^2 + 4 - 36}{4x}$

2

$= \frac{x^2 - 32}{4x}$

(ii) In ΔPBA

$\cos(90-\alpha) = \frac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times x \times 2}$

$= \frac{x^2 + 2^2 - 20}{4x}$

$= \frac{x^2 - 16}{4x}$

2

$\cos(90-\alpha) = \sin\alpha$

$\sin\alpha = \frac{x^2 - 16}{4x}$

(iii) $\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{x^2 - 32}{4x}\right)^2 + \left(\frac{x^2 - 16}{4x}\right)^2 = 1$

$\frac{x^4 - 64x^2 + 1024 + x^4 - 32x^2 + 256}{16x^2} = 1$

$2x^4 - 96x^2 + 1280 = 16x^2$

$2x^4 - 112x^2 + 1280 = 0$

3

$x^4 - 56x^2 + 640 = 0$

(IV) $x^4 - 56x^2 + 640 = 0$

Let $u = x^2$

$u^2 - 56u + 640 = 0$

$$u = \frac{56 \pm \sqrt{3136 - 4 \times 1 \times 640}}{2}$$

$$= \frac{56 \pm \sqrt{576}}{2}$$

$$= \frac{56 \pm 24}{2}$$

$= 40$ or 16 5

$x^2 = 40$ or $x^2 = 16$

$x = \pm\sqrt{40}$ or $x = \pm 4$

$x = \sqrt{40}$ or $x = 4$ (x can't be negative)

when $x = 4$, ΔPBC does not exist (The sum of two sides of a Δ must be greater than the third side)

$\therefore x = \underline{\underline{\sqrt{40} \text{ cm}}}$

or

$= \underline{\underline{2\sqrt{10} \text{ cm}}}$