GIRRAWEEN HIGH SCHOOL
YEAR 11 YEARLY EXAMINATION

2008

MATHEMATICS

2 UNIT

Time allowed – Two Hours

(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every questions.
- Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a new page.
- Write your name on each piece of paper.

Question 1 (16 marks)

(a) Find, correct to two decimal places: $\sqrt[3]{\dfrac{64.9\times 13.4}{862\times 3.69}}$ **1**

(b) Express $\dfrac{x-6}{x^2+4}+\dfrac{x+2}{x^2-4}$ as a single fraction and simplify. **3**

(c) Factorise fully:

(i) $4x^2+16x-9$ (ii) $27-y^3$ **3**

(d) Show that $\dfrac{1}{3-\sqrt{2}}+\dfrac{1}{3+\sqrt{2}}$ is a rational number. **2**

(e) Solve the following pair of simultaneous equations. **3**

$6x+3y=0$
$2x-5y=6$

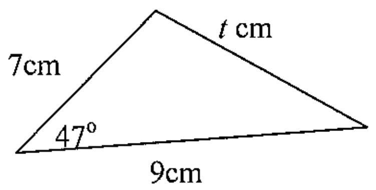
(f) Solve $|3x-1|>5$ and graph the solution on a number line. **4**

Question 2 (21 marks)

(a) Find the exact value of:

(i) $\cos 150^o$ (ii) $\tan 300^o$ **4**

(b)



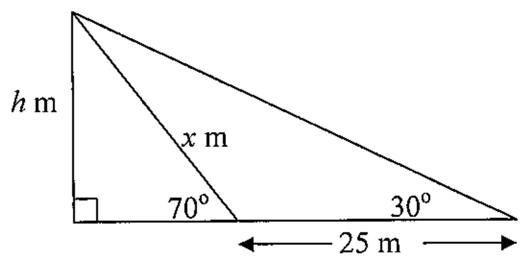
Find:

(i) the value of *t*, correct to one decimal place. **2**
(ii) area of the triangle, correct to one decimal place. **2**

(c) Solve the following for $0^o < \theta < 360^o$.

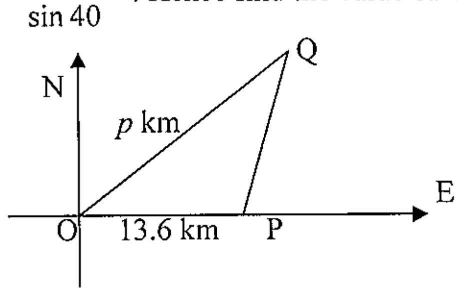
(i) $2\cos\theta + 1 = 0$ (ii) $\tan^2\theta = 3$ **5**

(d)



Prove that $x = \dfrac{25\sin 30}{\sin 40}$. Hence find the value of *h*, correct to one decimal place. **3**

(e)

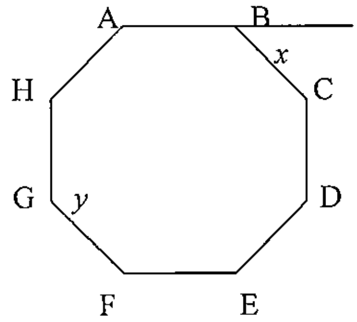


P is 13.6 km due east of O. The bearings of Q from O and P are 053° and 027° respectively. Find *p*, correct to one decimal place. **3**

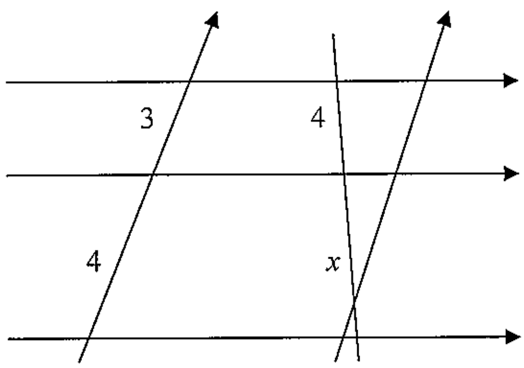
(f) Simplify $\dfrac{1-\cos^2\theta}{1-\sin^2\theta}$ **2**

Question 3 (11 marks)

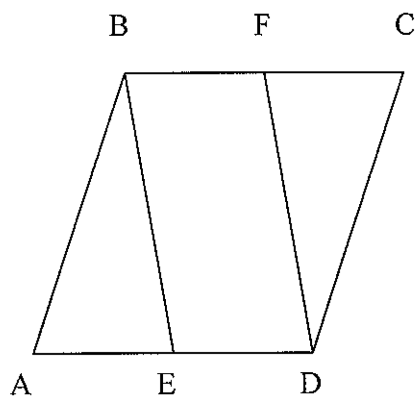
(a) ABCDEFGH is a regular octagon.
Find the values of x and y, giving reasons. **4**



(b) Find the value of x, giving reasons. **3**



(c) ABCD is a rhombus. E and F are midpoints of AD and BC respectively.
Prove that EBFD is a parallelogram. **4**



Question 4 (24 marks)

(a) Sketch the following functions, showing any intercepts with coordinate axes and stating their domain and range.

(i) $y = x^2 - 4$ **4**

(ii) $y = \frac{2}{x}$ **3**

(iii) $y = \sqrt{5 - x}$ **4**

(b) Show that the following function is odd:

$$f(x) = x^3 + 4x$$

Hence, or otherwise, evaluate $f(2) + f(-2)$ **3**

(c) Shade the region corresponding to each of the following inequalities on separate number planes.

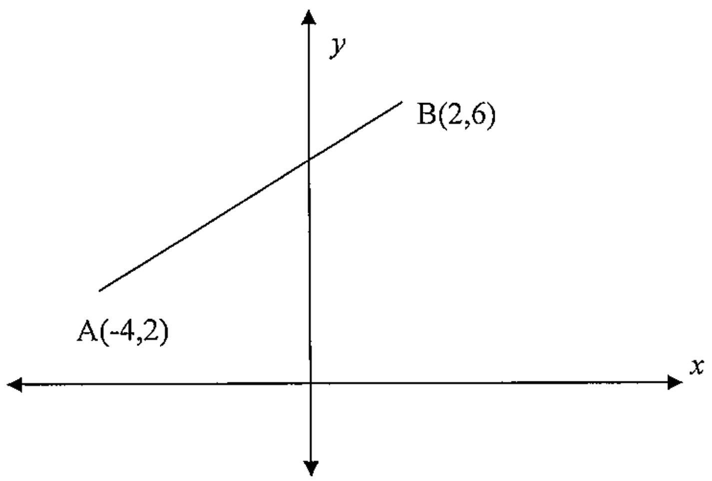
(i) $y > x^2 - 2$ **3**

(ii) $xy \leq 1$ **4**

(d) Sketch $y = \begin{cases} 2 - x^2 & x < 0 \\ -x & x \geq 0 \end{cases}$ **3**

Question 5 (20 marks)

(a) The diagram shows two points A(-4,2) and B(2,6) on the number plane.



Copy the diagram onto your answer sheet:

(i) Find the coordinates of the midpoint M of AB. **1**

(ii) Show that the equation of the perpendicular bisector of AB is $3x+2y-5=0$. **3**

(iii) Find the coordinates of the point C that lies on the x-axis and is equidistant from A and B. **2**

(iv) The point D lies on the intersection of the line $y=1$ and $3x+2y-5=0$. Find the coordinates of D and mark the position of D on your diagram. **3**

(b) Find the perpendicular distance of the point (-1, -2) from the line $6x+5y=30$. **3**

(c) Show that the lines $2y-3x=6$ and $6x+9y=14$ are perpendicular. **3**

(d) Find the equation of the line which passes through the point of intersection of $2y-3x=6$ and $4x+6y=5$, which is perpendicular to the line $3x-5y=15$. **5**

Question 6 (23 marks)

(a) Differentiate:

(i) $4x^3 - 6x^2 + 3$ **1**

(ii) $x - \dfrac{1}{4x^2}$ **2**

(iii) $\left(\sqrt{x} + 1\right)^2$ **3**

(iv) $\dfrac{x}{x^2 + 3}$ **2**

(v) $(x-1)(x^2 + x + 1)$ **2**

(b) Evaluate:

(i) $\lim\limits_{x \to 1} \dfrac{x-1}{x^2 + x - 2}$ **3**

(ii) $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x}{2x^2 - 3x + 7}$ **2**

(c) Differentiate by first principles $y = x^2 + 4x - 6$. **4**

(d) Show that the graph of $y = x^2 + 4x - 12$ crosses the x axis at two points. Find the equations of the normals at these points. **4**

Question 7 (24 marks)

(a) If α and β are the roots of $2x^2+5x-1=0$ find the values of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

(iv) $(\alpha+1)(\beta+1)$ **2**

(v) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

(b) For the equation $x^2+kx+(2k-3)=0$:

(i) find the two values of k for which the roots are equal **3**

(ii) solve the equation for each of these values of k. **2**

(c) Solve:

(i) $x^4-x^2-12=0$ **4**

(ii) $4^{2x}-5\times 4^x+4=0$ **3**

(d) Find the values of k for which $kx^2+(k+3)x+4$ is positive definite and explain why there are no values of k for which it is negative definite. **4**

YEAR 11 YEARLY SOLUTIONS.

Q1 a) $\sqrt[3]{\dfrac{64.9 \times 13.4}{862 \times 3.69}} = 0.65$

b) $\dfrac{x-6}{x^2+4} + \dfrac{x+2}{x^2-4}$

$= \dfrac{x-6}{x^2+4} + \dfrac{\cancel{x+2}}{(x-2)\cancel{(x+2)}}$

$= \dfrac{(x-6)(x-2) + (x^2+4)}{(x^2+4)(x-2)}$

$= \dfrac{x^2-8x+12+x^2+4}{(x^2+4)(x-2)}$

$= \dfrac{2x^2-8x+16}{(x^2+4)(x-2)}$

c) i) $4x^2+16x-9$

$= (4x^2-2x)+(18x-9)$

$= 2x(2x-1)+9(2x-1)$

$= (2x+9)(2x-1)$

ii) $27-y^3$

$= (3-y)(9+3y+y^2)$

d) $\dfrac{1}{3-\sqrt{2}} + \dfrac{1}{3+\sqrt{2}}$

$= \dfrac{3+\sqrt{2} + 3-\sqrt{2}}{7}$

$= \dfrac{6}{7}$

e) $6x+3y = 0$ (1)
$2x-5y = 6$ (2) $\times 3$

$6x+3y = 0$
$6x-15y = 18$

$-18y = -18$

$y = -1$

Sub $y=-1$ into (1)

$6x + 3(-1) = 0$

$6x - 3 = 0$

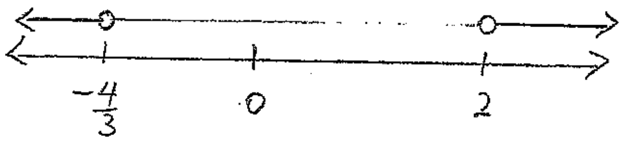
$6x = 3$

$x = \frac{1}{2}$

$\therefore x = \frac{1}{2}$ $y = -1$.

f) Solve $|3x-1| > 5$

$3x-1 > 5$ or $3x-1 < -5$
$3x > 6$ $\quad 3x < -4$
$x > 2$ $\quad x < -\frac{4}{3}$



QUESTION 2.

a) i) $\cos 150° = -\cos 30°$

$= -\dfrac{\sqrt{3}}{2}$

ii) $\tan 300° = -\tan 60°$

$= -\sqrt{3}$

b) i) $t^2 = 7^2+9^2-2(7)(9)\cos 47°$

$t^2 = 44.068$

$\therefore t = 6.6$ cm

ii) $A = \frac{1}{2}(7)(9)\sin 47°$

$= 23.0$ cm^2.

c) i) $2\cos\theta + 1 = 0$

$2\cos\theta = -1$

$\cos\theta = -\dfrac{1}{2}$

$\therefore \theta = 120°, 240°$

c ii) $\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\theta = 60°, 120°, 240°, 300°$

d) $\frac{x}{\sin 30°} = \frac{25}{\sin 40°}$

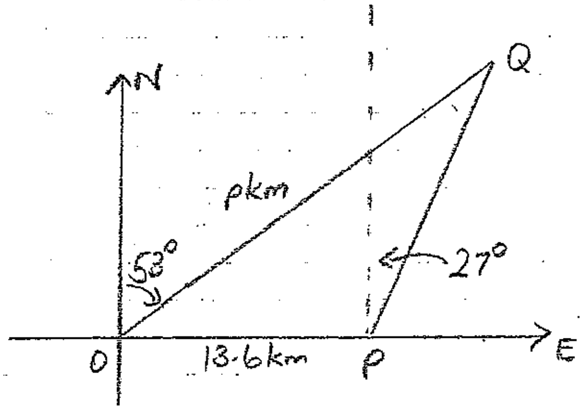
$\therefore x = \frac{25\sin 30°}{\sin 40°}$

$\sin 70° = \frac{h}{\frac{25\sin 30°}{\sin 40°}}$

$h = \frac{25\sin 30°}{\sin 40°} \times \sin 70°$

$h = 18 \cdot 3m$

e)



$\angle QPO = 27° + 90° = 117°$

$\angle QOP = 90° - 53° = 37°$

$\angle OQP = 180° - 117° - 37° = 26°$

$\frac{p}{\sin 117°} = \frac{13.6}{\sin 26°}$

$p = \frac{13.6 \times \sin 117°}{\sin 26°}$

$p = 27.6$ km.

f) $\frac{1-\cos^2\theta}{1-\sin^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

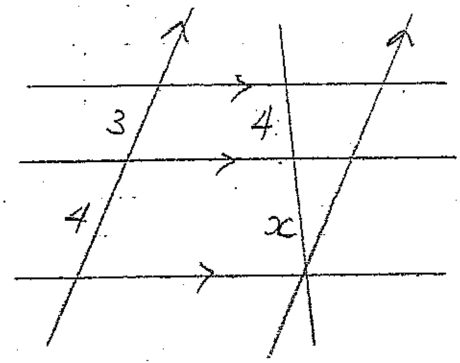
QUESTION 3.

a) $x = \frac{360°}{8} = 45°$ (exterior angle of n sided regular polygon).

$y = \frac{180(n-2)°}{n}$

$= \frac{180 \times 6°}{8} = 135°$ (Interior angle of n sided regular polygon).

b)

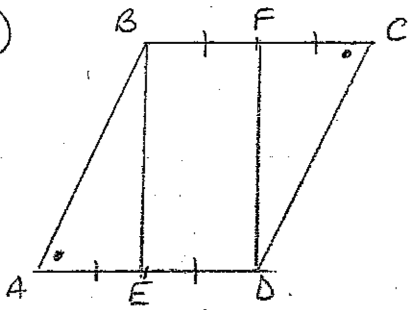


$\frac{x}{4} = \frac{4}{3}$ (ratio of intercepts)

$3x = 16$

$x = 5\frac{1}{3}$

c)



In ΔABE and ΔCDF

$AB = CD$ (sides of a rhombus equal,

$AE = CF$ (midpoint of sides on a rhombus)

$\angle BAE = \angle DCF$ (opposite angles of a rhombus)

$\therefore \Delta ABE \equiv \Delta CDF$ (SAS)

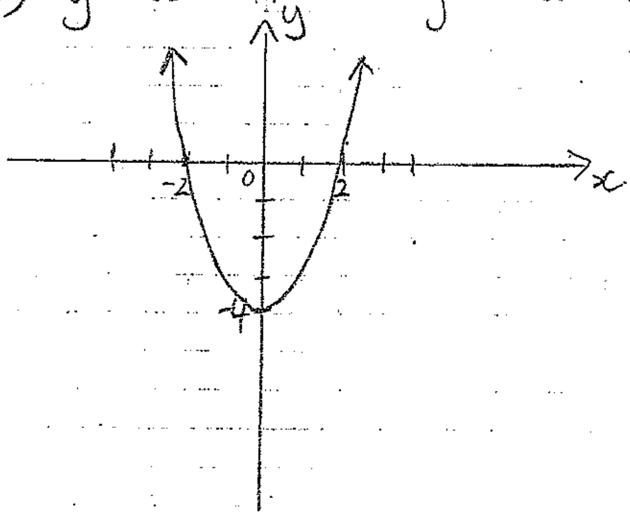
$BF = ED$ (midpoint of sides of rhombus)

$BE = FD$ (matching sides in congruent Δ's ABE and CDF,

$\therefore$ EBFD is a parallelogram.

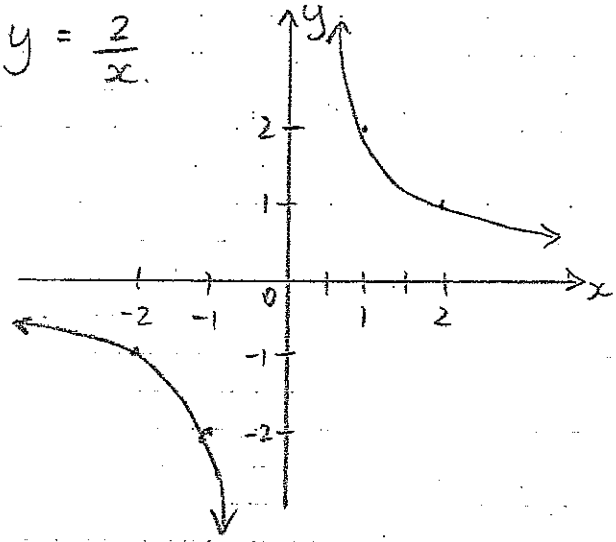
QUESTION 4

a) i) $y = x^2 - 4$ $y = 0$ $x = \pm 2$.



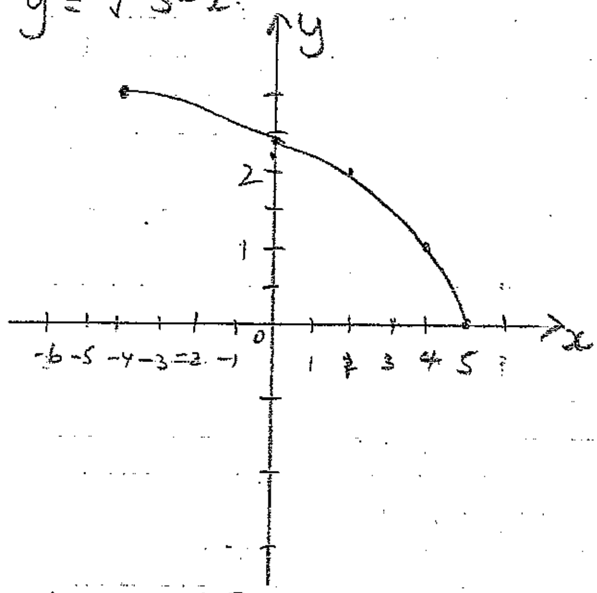
Domain: all real x.
Range: $y \geq -4$.

ii) $y = \frac{2}{x}$.



Domain: all x $x \neq 0$
Range: all y $y \neq 0$.

iii) $y = \sqrt{5 - x}$.



Domain: $x \leq 5$.
Range: $y \geq 0$.

b) $f(x) = x^3 + 4x$

$f(-x) = (-x)^3 + 4(-x)$
$= -x^3 - 4x$.
$= -(x^3 + 4x) = -f(x)$.

$\therefore f(-x) = -f(x)$

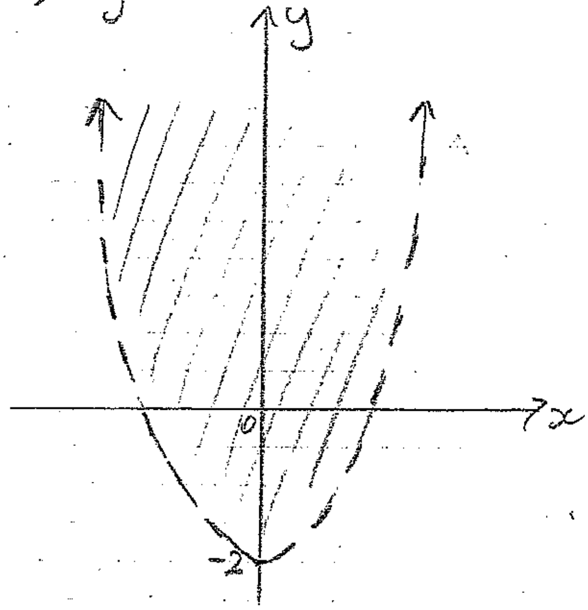
$\therefore$ The function is odd.

$f(2) = 2^3 + 4(2)$
$= 8 + 8$
$= 16$.

$f(-2) = (-2)^3 + 4(-2)$
$= -8 - 8$
$= -16$.

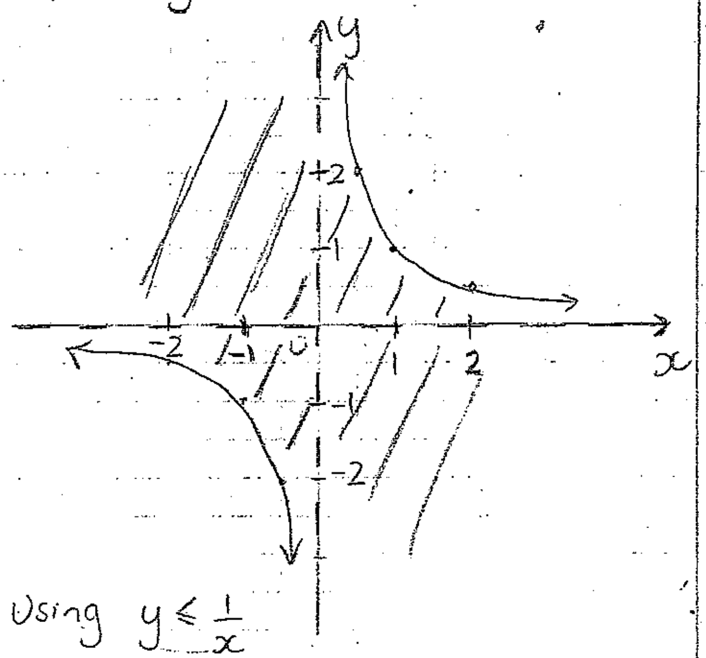
$\therefore f(2) + f(-2) = 16 - 16 = 0$.

c) i) $y > x^2 - 2$.



Test $(0,0)$ $0 > 0^2 - 2$
$0 > -2$

c ii) $xy \leq 1$



Using $y \leq \frac{1}{x}$

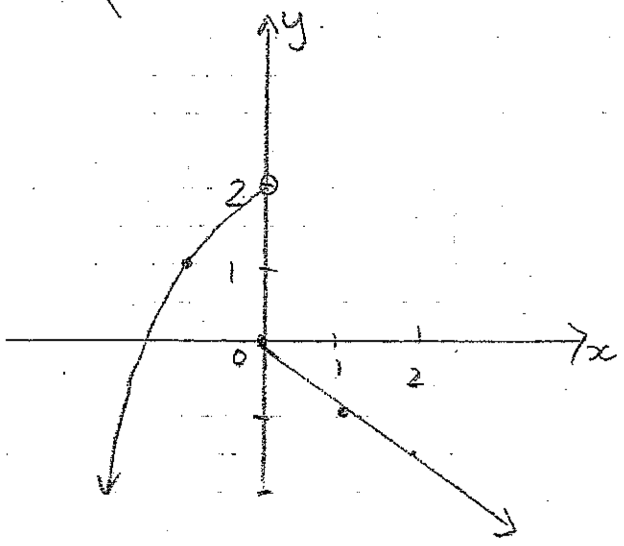
Test $(0,0)$ $\quad 0 \leq \frac{1}{0}$ undefined

Using $xy \leq 1$

Test $(2,1)$ $\quad 2 \leq 1$ not satisfied

Test $(-2,-2)$ $\quad 4 \leq 1$ not satisfied

d) $y = \begin{cases} 2-x^2 & x<0 \\ -x & x \geq 0 \end{cases}$



QUESTION 5

a i) $M_{AB} = \left(\frac{-4+2}{2}, \frac{2+6}{2}\right)$

$= \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$

ii) $m_1 = \frac{6-2}{2+4} = \frac{4}{6}$

$m_2 = -\frac{6}{4} = -\frac{3}{2}$

Equation of the perpendicular bisector of AB

$y-4 = -\frac{3}{2}(x+1)$

$y-4 = -\frac{3}{2}x - \frac{3}{2}$

$2y-8 = -3x-3$

$\therefore 3x+2y-5=0$

iii) C lies on the x axis.

so $y=0$.

$3x+2(0)-5=0$

$3x=5$

$x = \frac{5}{3}$

$\therefore C\left(\frac{5}{3}, 0\right)$

iv) $y=1 \quad 3x+2y-5=0$

$3x+2-5=0$

$3x-3=0$

$3x=3$

$x=1$

Sub $x=1$ into $3x+2y-5=0$.

$3+2y-5=0$

$2y-2=0$

$2y=2$

$y=1$

$\therefore D(1,1)$

b) $(-1,-2)$ $\quad 6x+5y=30$

$$d=\frac{|-6-10-30|}{\sqrt{6^2+5^2}}$$

$$=\frac{|-46|}{\sqrt{36+25}} = \frac{46}{\sqrt{61}}$$

c) $2y-3x=6$, $6x+9y=14$

$2y=3x+6$ $\quad 9y=-6x+14$

$y=\frac{3}{2}x+3$ $\quad y=-\frac{2}{3}+\frac{14}{9}$

$m_1=\frac{3}{2}$ $\quad m_2=-\frac{2}{3}$

$\therefore m_1\times m_2 = \frac{3}{2}\times -\frac{2}{3} = -1$

$\therefore$ $2y-3x=6$ is perpendicular to $6x+9y=14$

d) $2y-3x=6$, $4x+6y=5$

$6y+4x=5$ (1)
$2y-3x=6$ (2) $\times 3$

$6y+4x=5$
$6y-9x=18$

$13x=-13$
$x=-1$ $\quad$ sub in (1)

$6y-4=5$

$6y=9$

$y=\frac{3}{2}$

$\therefore$ Point of intersection $(-1,\frac{3}{2})$

Perpendicular gradient of

$3x-5y=15$

$-5y=-3x+15$
$y=\frac{3}{5}x-3$

$m_1=\frac{3}{5}$ $\quad m_2=-\frac{5}{3}$

Point gradient form

$y-\frac{3}{2}=-\frac{5}{3}(x+1)$

$y-\frac{3}{2}=-\frac{5}{3}x-\frac{5}{3}$

$3y-\frac{9}{2}=-5x-5$

$6y-9=-10x-10$

$\therefore 10x+6y+1=0$

$\therefore$ The equation of the line is $10x+6y+1=0$

QUESTION 6

a) i) $4x^3-6x^2+3$

$\frac{dy}{dx}=12x^2-12x$

ii) $x-\frac{1}{4x^2} = x-\frac{1}{4}x^{-2}$

$\frac{dy}{dx}=1+\frac{1}{2}x^{-3}$

$=1+\frac{1}{2x^3}$

iii) $(\sqrt{x}+1)^2$

$\frac{dy}{dx}=2(\sqrt{x}+1)\times\frac{1}{2\sqrt{x}}$

$=\frac{2(\sqrt{x}+1)}{2\sqrt{x}}$

$=\frac{\sqrt{x}+1}{\sqrt{x}}$

iv) $\frac{x}{x^2+3}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$u = x \qquad u' = 1$

$v = x^2+3 \qquad v' = 2x$

$$\frac{dy}{dx} = \frac{(x^2+3)\cdot 1 - x(2x)}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2}{(x^2+3)^2}$$

$$= \frac{3-x^2}{(x^2+3)^2}$$

v) $(x-1)(x^2+x+1)$

$$\frac{dy}{dx} = vu' + uv'$$

$u = x-1 \qquad u' = 1$

$v = x^2+x+1 \qquad v' = 2x+1$

$$\frac{dy}{dx} = (x^2+x+1)\cdot 1 + (x-1)(2x+1)$$

$$= (x^2+x+1) + (2x^2-x-1)$$

$$= x^2+x+1+2x^2-x-1$$

$$= 3x^2$$

bi) $\lim\limits_{x\to 1} \frac{x-1}{x^2+x-2}$

$$\lim_{x\to 1} \frac{\cancel{x-1}\ 1}{\cancel{(x-1)}(x+2)}$$

$$\lim_{x\to 1} \frac{1}{x+2}$$

$$= \frac{1}{3}$$

ii) $\lim\limits_{x\to\infty} \frac{3x^2+2x}{2x^2-3x+7}$

$$\lim_{x\to\infty} \frac{\frac{3x^2}{x^2} + \frac{2x}{x^2}}{\frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{7}{x^2}}$$

$$\lim_{x\to\infty} \frac{3+\frac{2}{x}}{2-\frac{3}{x}+\frac{7}{x^2}}$$

$$= \frac{3}{2}$$

c) $y = x^2+4x-6$

$f(x) = x^2+4x-6$

$f(x+h) = (x+h)^2+4(x+h)-6$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2+4(x+h)-6 - (x^2+4x-6)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{4x}+4h\cancel{-6}\cancel{-x^2}\cancel{-4x}\cancel{+6}}{h}$$

$$\lim_{h\to 0} \frac{2xh+h^2+4h}{h}$$

$$\lim_{h\to 0} 2x+h+4$$

$$= 2x+4$$

d) $y = x^2 + 4x - 12$

$y = 0 \quad x^2 + 4x - 12 = 0.$

$(x+6)(x-2) = 0.$

$x = -6$ or $x = 2.$

$\frac{dy}{dx} = 2x + 4 \quad$ When $x = -6 \ \frac{dy}{dx} = -8$

When $x = 2 \ \frac{dy}{dx} = 8.$

At $(-6,0)$ Gradient of normal $= \frac{1}{8}.$

Equation of normal at $(-6,0)$

$y - 0 = \frac{1}{8}(x+6).$

$y = \frac{x}{8} + \frac{6}{8}$ or $x - 8y + 6 = 0.$

At $(2,0)$ Gradient of normal $= -\frac{1}{8}.$

Equation of normal at $(2,0)$

$y - 0 = -\frac{1}{8}(x-2)$

$y = -\frac{x}{8} + \frac{1}{4}$ or $x + 8y - 2 = 0$

QUESTION 7.

a) $2x^2 + 5x - 1 = 0.$

$a = 2 \quad b = 5 \quad c = -1.$

i) $\alpha + \beta = \frac{-b}{a} = -\frac{5}{2}.$

ii) $\alpha\beta = \frac{c}{a} = -\frac{1}{2}.$

iii) $\alpha^2 + \beta^2 = (\alpha^2 + 2\alpha\beta + \beta^2) - 2\alpha\beta$

$= (\alpha+\beta)^2 - 2\alpha\beta.$

$= \left(\frac{-5}{2}\right)^2 - 2\left(-\frac{1}{2}\right)$

$= \frac{25}{4} + 1 \quad = \frac{29}{4}$

iv) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1.$

$= \alpha\beta + (\alpha+\beta) + 1.$

$= -\frac{1}{2} + \left(\frac{-5}{2}\right) + 1.$

$= -\frac{1}{2} - \frac{5}{2} + 1$

$= -3 + 1 \quad = -2.$

v) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}.$

$= \frac{-\frac{5}{2}}{-\frac{1}{2}} = -\frac{5}{2} \times \frac{-2}{1} = \frac{10}{2} = 5$

b) $x^2 + kx + (2k-3) = 0.$

i) $\Delta = 0 \quad b^2 - 4ac = 0$

$a = 1 \quad b = k \quad c = (2k-3)$

$\Delta = k^2 - 4(1)(2k-3)$

$= k^2 - 4(2k-3)$

$= k^2 - 8k + 12.$

When $k^2 - 8k + 12 = 0.$

$(k-6)(k-2) = 0.$

$\therefore \ k = 6$ or $k = 2.$

ii) When $k = 6$

$x^2 + 6x + 9 = 0.$

$(x+3)^2 = 0.$

$\therefore x = -3.$

When $k = 2$

$x^2 + 2x + 1 = 0.$

$(x+1)^2 = 0.$

$\therefore x = -1.$

c) i) $x^4 - x^2 - 12 = 0.$

Let $u = x^2$

$u^2 - u - 12 = 0.$

$(u-4)(u+3) = 0.$

$u = 4$ or $-3.$

$\therefore x^2 = 4$ or $x^2 = -3.$

as $x^2 \geq 0 \quad x^2 = -3$ has no real solution.

$\therefore x = \pm 2$ is the only solution.

ii) $4^{2x} - 5 \times 4^{x} + 4 = 0$

Let $u = 4^{x}$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$\therefore u = 4$ and 1

$4^{x} = 4$ and $4^{x} = 1$

$\therefore x = 1$ and 0.

d) $kx^2 + (k+3)x + 4$

Positive definite $a > 0$, $\Delta < 0$.

$\therefore b^2 - 4ac < 0$

$(k+3)^2 - 4(k)(4) < 0$

$(k+3)^2 - 16k < 0$

$k^2 + 6k + 9 - 16k < 0$

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0$

$1 < k < 9$.

There are no values of k for which it is negative definite as all values of k are positive therefore $a > 0$.