GIRRAWEEN HIGH SCHOOL
YEAR 11 YEARLY EXAMINATION

2009

MATHEMATICS

2 UNIT

Time allowed- Two Hours
(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a new page.
- Write your name on each piece of paper.

Question 1 (18 marks) **Marks**

(a) Find, correct to 3 decimal places: $\dfrac{\sqrt{421.6}}{17.93+4.9}$ 2

(b) Factorise fully:

(i) $5x^2-2x-3$ (ii) x^4-y^4 4

(c) Simplify: $\dfrac{2}{3}-\dfrac{x-1}{4}$ 3

(d) Solve: $\dfrac{3}{x+2}=\dfrac{5}{2x-1}$ 3

(e) Solve using quadratic formula: $x^2+2=-8x$ 3

(f) Find the value of the integers a, b and c if:

$$\frac{6}{2\sqrt{7}-5}=a+b\sqrt{c}$$ 3

Question 2 (21 marks)

(a) Describe (write the equation and name the geometrical shape) the locus of the following: 3

(i) a point that moves so that it is always 2 units above the x axis.

(ii) a point that moves so that it is always 5 units from the point (1, -2).

(b) Find the equation of the locus of a point P that moves so that PA is twice the distance of PB where $A=(0,3)$ and $B=(4,7)$. 4

(c) Find the equation of the locus of a point which moves in a plane so that its distance from the point $S(-2,4)$ is equal to its distance from the line $y=6$. 4

(d) For the following parabolas find 10

(i) the coordinates of the focus.

(ii) the equation of the directrix.

(iii) Sketch the parabola showing the above features.

(α) $y^2=6x$ (β) $x^2=-8y$

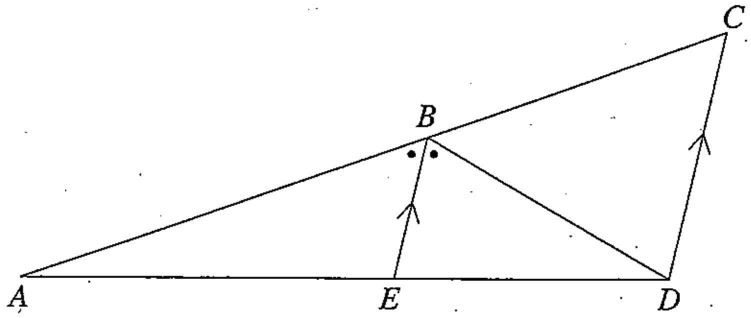
Question 3 (16 marks)

(a) A regular polygon has 9 sides. Find 2

(i) the angle sum.

(ii) the size of each interior angle.

(b) In the diagram, $BE \parallel CD$ and BE bisects $\angle ABD$.



NOT TO SCALE

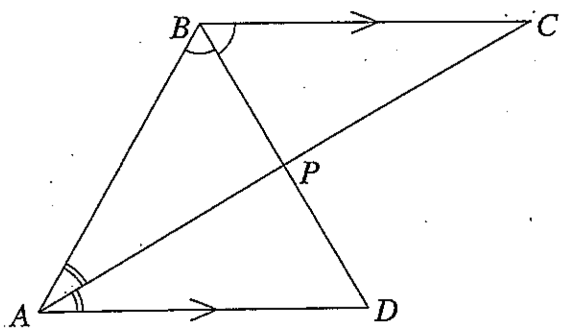
Copy the diagram into your writing booklet.

(i) Explain why $\angle EBD = \angle BDC$. 2

(ii) Prove that ΔBCD is isosceles. 2

(iii) Hence show that $AE : ED = AB : BD$ 2

(c) In the diagram below AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P.



Copy the diagram into your writing booklet.

(i) Prove that $\angle BAC = \angle BCA$. 2

(ii) Prove that $\Delta APB \equiv \Delta CBP$. 3

(iii) Prove that $ABCD$ is a rhombus. 3

Question 4 (30 marks)

(a) For each of the following curves

(α) $y=-x^2-3$ (β) $y=|x+1|$

(γ) $y=\dfrac{1}{x+2}$ (δ) $y=\sqrt{7+x}$ 16

(i) Draw a neat sketch showing important features.

(ii) State the domain and range.

(b) Find the domain of $y=\dfrac{1}{\sqrt{64-x^2}}$. 2

(c) Is the function $f(x)=x^4+2x$ odd, even or neither? Justify your answer. 3

(d) A function is defined by the rule

$$g(x)=\begin{cases} x^2-4 & x\ge -2 \\ x+2 & x<-2 \end{cases}$$

(i) Sketch the curve. 3

(ii) Find the value of $g(2)$, and $g(-3)$. 2

(e) Shade the region given by $x^2+y^2\le 9$ and $x-y>2$. 4

Question 5 (23 marks)

(a) Differentiate: 11

(i) $y=4x^2-7x+3+5x^{-2}$ **(ii)** $y=(2x+5)(x^2-1)^4$

(iii) $y=\dfrac{5x}{x+1}$ **(iv)** $y=\sqrt{2x-7}$

(b) Evaluate the following limits: 4

(i) $\lim\limits_{x\to 4}\dfrac{x-4}{x^2-16}$ **(ii)** $\lim\limits_{x\to\infty}\dfrac{4x^2+1}{3x^2}$

(c) If α and β are the roots of the quadratic equation $3y^2-8y+3=0$, find the value of

(i) $\alpha+\beta$ **(ii)** $\alpha\beta$ **(iii)** $\alpha^2+\beta^2$ 4

(d) For what values of k is the expression $k(x+1)(x-8)-(x-4)$ negative definite? 4

Question 6 (12 marks)

(a) Find the perpendicular distance from (-2, 3) to the line $5x-12y+20=0$. 2

(b) **(i)** Find the equation of the straight line l through (-1, 2) that is perpendicular to the line $3x+6y-7=0$. 3

(ii) Line l cuts the x axis at P and y axis at Q. Find the coordinates of P and Q. 2

(iii) Find the area of ΔOPQ where O is the origin. 2

(c) Show that the lines $x-5y-17=0$, $3x-2y-12=0$ and $5x+y-7=0$ are concurrent. 3

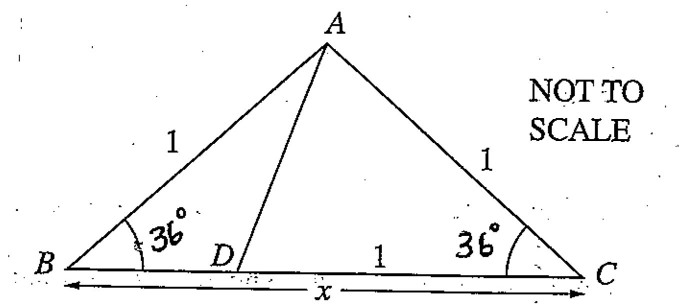
Question 7 (16 marks)

(a) Solve for $0° \le \theta \le 360°$. Write answer correct to the nearest minute. 5

(i) $\cos\theta = \dfrac{-1}{\sqrt{2}}$ **(ii)** $\tan 2\theta = \sqrt{3}$

(b) Prove: $(\sin\theta+\cos\theta)^2-(\sin\theta-\cos\theta)^2 = 4\sin\theta\cos\theta$ 3

(c) In the diagram, ABC is an isosceles triangle where $\angle BAC = 108°$ and $AB = AC = 1$. The point D is chosen on BC such that $CD = 1$. Let $BC = x$.



(i) Show that $\angle ADC = 72°$ and hence show that triangles DBA and ABC are similar. 3

(ii) Hence deduce that $x^2-x-1=0$. 2

(iii) By using the cosine rule, deduce that $\cos 36° = \dfrac{1+\sqrt{5}}{4}$ 3

Year 11 Mathematics Yearly 2009 - Solutions.

Question 1 (18 marks)

(a) 0.899 (1)

(b) (i) $5x^2-2x-3$

$-5, 3$ $\quad pq=-15$, $p+q=-2$

$= 5x^2-5x+3x-3$

$= 5x(x-1)+3(x-1)$

$= (x-1)(5x+3)$ (2)

(ii) x^4-y^4

$= (x^2)^2-(y^2)^2$ (3)

$= (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

(c) $\frac{2}{3}-\frac{x-1}{4}$

$= \frac{8-3(x-1)}{12}$ (3)

$= \frac{8-3x+3}{12} = \frac{11-3x}{12}$

(d) $\frac{3}{x+2}=\frac{5}{2x-1}$

$3(2x-1)=5(x+2)$

$6x-3=5x+10$

$6x-5x=13$ (3)

$x=13$

(e) $x^2+2=-8x$

$x^2+8x+2=0$

$x=\frac{-8\pm\sqrt{64-4\times1\times2}}{2}$

$=\frac{-8\pm\sqrt{64-8}}{2}=\frac{-8\pm\sqrt{56}}{2}$

$=\frac{-8\pm\sqrt{14\times4}}{2}=\frac{-8\pm2\sqrt{14}}{2}$ (3)

$=\frac{2(-4\pm\sqrt{14})}{2}=-4\pm\sqrt{14}$

(f) $\frac{6}{2\sqrt{7}-5}=\frac{6}{2\sqrt{7}-5}\times\frac{2\sqrt{7}+5}{2\sqrt{7}+5}$

$=\frac{6(2\sqrt{7}+5)}{(2\sqrt{7}-5)(2\sqrt{7}+5)}$

$=\frac{12\sqrt{7}+30}{(2\sqrt{7})^2-5^2}=\frac{12\sqrt{7}+30}{28-25}$

$=\frac{12\sqrt{7}+30}{3}$ (3)

$=4\sqrt{7}+10$

$=10+4\sqrt{7}$

$a=10, \quad b=4, \quad c=7$

Question 2 (21 marks)

(a) (i) $y = 2$, straight line ①

(ii) $(x-1)^2 + (y+2)^2 = 25$, circle ②

(b) $P(x,y)$ $A(0,3)$ $B(4,7)$

$PA = 2 \times PB$

$$\sqrt{x^2 + (y-3)^2} = 2 \times \sqrt{(x-4)^2 + (y-7)^2}$$

$$x^2 + (y-3)^2 = 4\left[(x-4)^2 + (y-7)^2\right]$$ ④

$$x^2 + y^2 - 6y + 9 = 4\left[x^2 - 8x + 16 + y^2 - 14y + 49\right]$$

$$x^2 + y^2 - 6y + 9 = 4x^2 - 32x + 64 + 4y^2 - 56y + 196$$

$$4x^2 - x^2 + 4y^2 - y^2 - 32x - 56y + 260 - 9 = 0$$

$$\underline{\underline{3x^2 + 3y^2 - 32x - 50y + 251 = 0}}$$

(c) Let $P(x,y)$ be the variable point. Let M be the foot of the perpendicular from P to the line $y = 6$

Given that $PS = PM$

$P(x,y)$ $S(-2,4)$

④

$$\sqrt{(x+2)^2 + (y-4)^2} = |y-6|$$

$$(x+2)^2 + (y-4)^2 = |y-6|^2 = (y-6)^2 \quad (\because |x|^2 = x^2)$$

$$x^2 + 4x + 4 + y^2 - 8y + 16 = y^2 - 12y + 36$$

$$x^2 + 4x = -4y + 16$$

$$\underline{\underline{(x+2)^2 = -4(y-5)}}$$

(d) (α) (i) $y^2 = 6x$

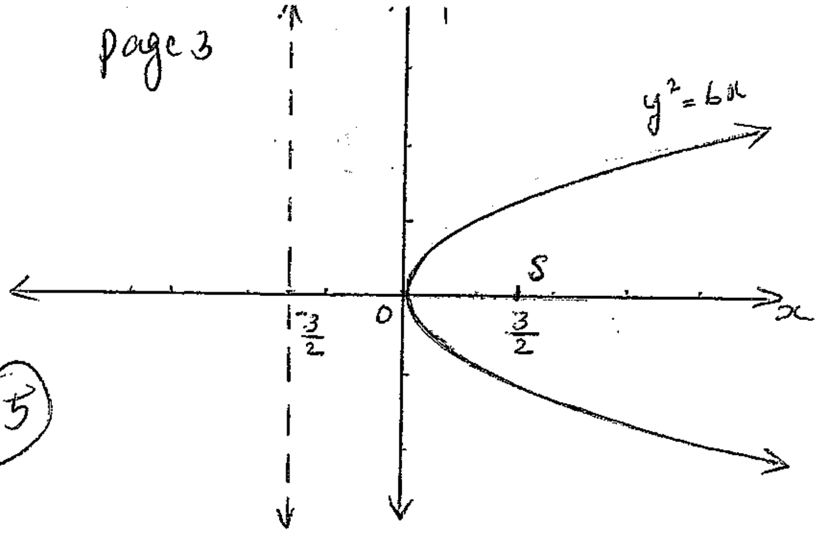
page 3

$y^2 = 4ax$

$4a = 6$; $a = \frac{3}{2}$

Focus $= \left(\frac{3}{2}, 0\right)$

directrix : $x = \frac{-3}{2}$ (5)



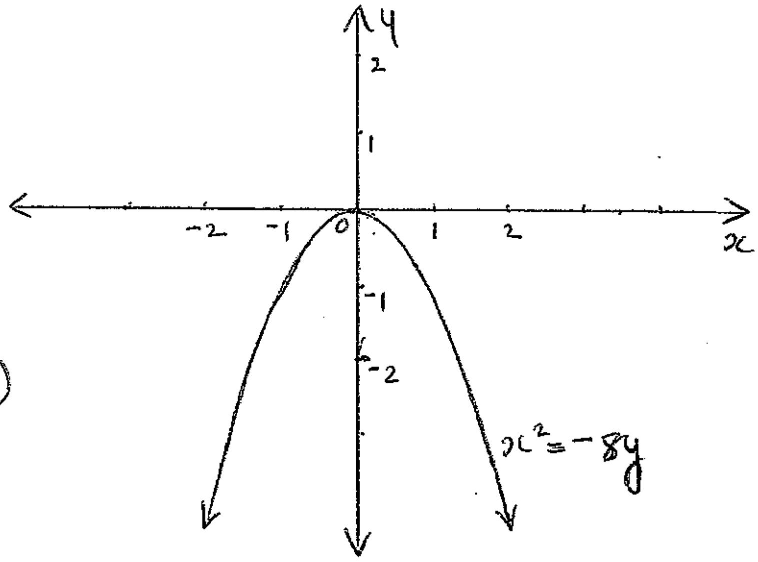
(β) $x^2 = -8y$

$x^2 = -4ay$

$4a = 8$; $a = 2$

Focus $(0, -2)$

directrix : $y = 2$ (5)



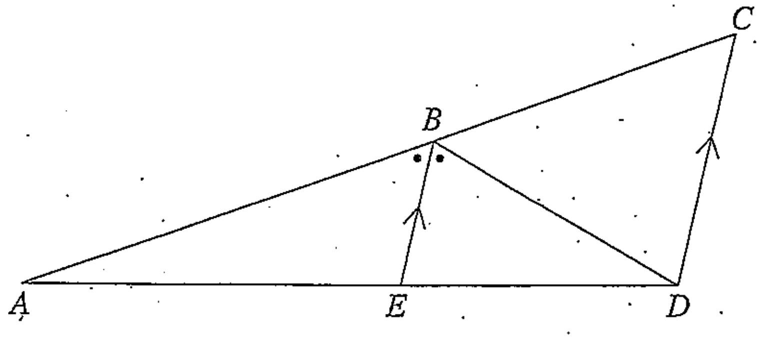
Question 3 (16 marks)

(a) (i) Angle sum $= (n-2) \times 180$

$= (9-2) \times 180 = 1260°$ (1)

(ii) One interior angle $= \frac{1260}{9} = 140°$ (1)

(b)



(i) $\angle EBD = \angle BDC$ (alternate angles equal, $BE \parallel CD$)

(2)

(ii) $\angle ABE = \angle BCD$ (Corresponding angles equal, $BE \parallel CD$)

$\angle EBD = \angle BDC$ (from (i))

$\angle EBD = \angle ABE$ (given)

$\therefore \angle BCD = \angle BDC$

$\therefore \triangle BCD$ is isosceles (two equal angles)

(2)

(iii) $\frac{AE}{ED} = \frac{AB}{BC}$ (equal ratios on transversals by parallel lines)

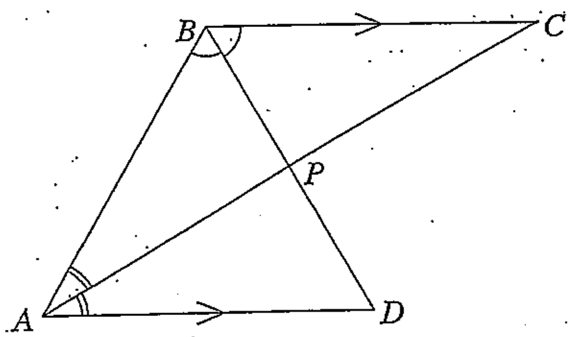
$BC = BD$ (isosceles $\triangle BCD$)

$\therefore \frac{AE}{ED} = \frac{AB}{BD}$

ie $\underline{AE : ED = AB : BD}$

(2)

(c)



(i) $\angle BAC = \angle CAD$ (AC bisects $\angle BAD$)

$\angle CAD = \angle BCA$ (alternate angles equal, $BC \parallel AD$)

$\therefore \angle BAC = \angle BCA$.

(2)

(ii) Consider $\triangle ABP$ and $\triangle CBP$

$\angle BAC = \angle BCA$ (from (i))

$\angle ABP = \angle CBP$ (BP bisects $\angle ABC$)

BP is common

$\therefore \underline{\triangle ABP \equiv \triangle CBP \text{ (AAS)}}$

(3)

(iii) $AP = CP$ (matching sides of congruent triangles ABP and CBP)

$\angle APB = \angle CPB$ (matching angles of congruent triangles ABP and CBP)

$\angle APB + \angle CPB = 180^\circ$ (APC is a straight angle, 180°)

$\therefore \angle APB = \angle CPB = 90^\circ$

In ΔABP and ΔADP

$\angle BAP = \angle DAP$ (AP bisects $\angle BAD$)

$\angle APB = \angle APD$ ($BP \perp AC$)

AP is common

3

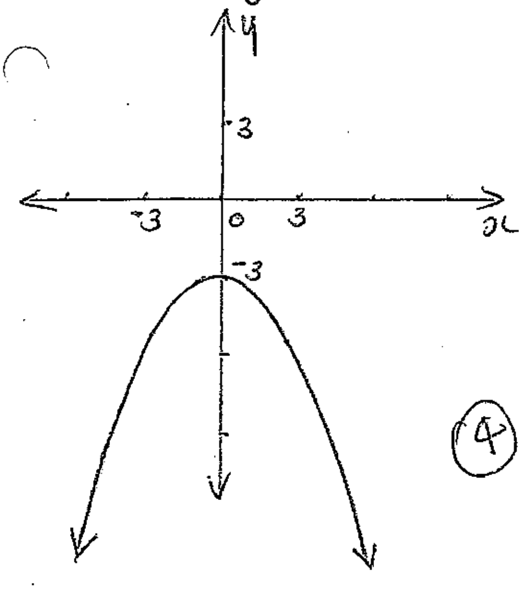
$\therefore \Delta ABP \equiv \Delta ADP$ (AAS)

$BP = PD$ (matching sides of congruent triangles ABP and ADP)

ABCD is a rhombus (diagonals bisect each other at right angles)

<u>Question 4</u> (30 marks)

(a) (α) $y = -x^2 - 3$



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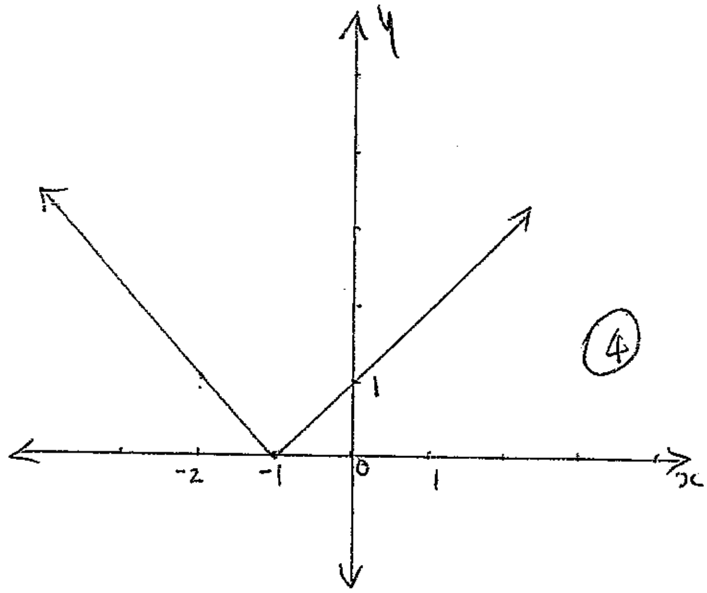
D : all real x

R : $y \le -3$, y real

(β) $y = |x+1|$

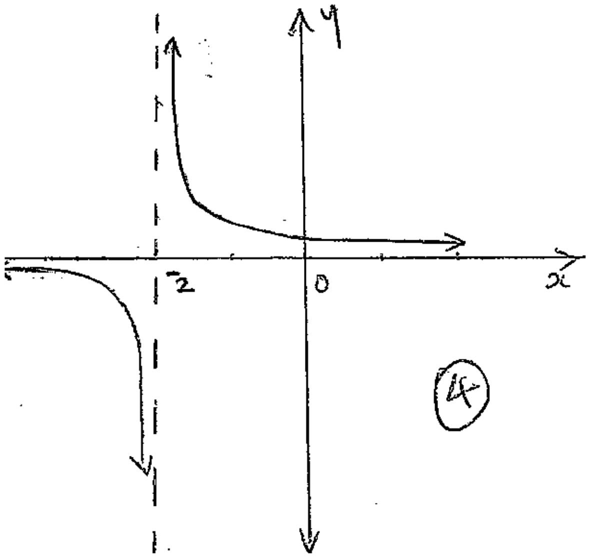


4

D : all real x

R : $y \ge 0$, y real

(r) $y = \frac{1}{x+2}$ page 6

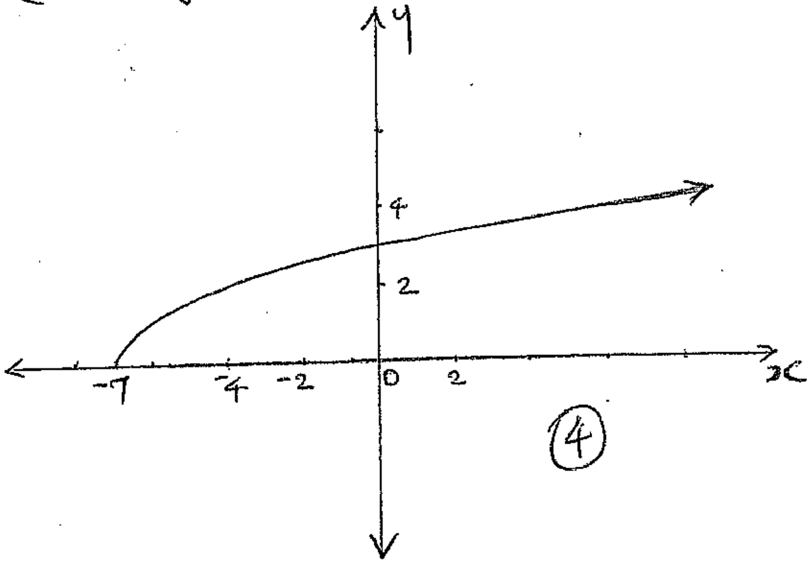


④

D: all real x, $x \neq -2$

R: all real y, $y \neq 0$

(δ) $y = \sqrt{7+x}$



④

D: $x \geq -7$

R: $y \geq 0$

(b) $y = \frac{1}{\sqrt{64-x^2}}$

$\sqrt{64-x^2}$ exists when

$64 - x^2 \geq 0$

$64 \geq x^2$

$x^2 \leq 64$ ②

$-8 \leq x \leq 8$

D: $-8 < x < 8$

(c) $f(x) = x^4 + 2x$

$f(-x) = (-x)^4 + 2(-x)$

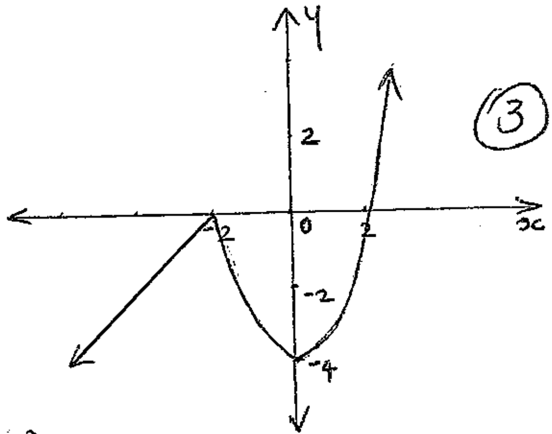
$= x^4 - 2x$

$f(-x) \neq f(x)$ ③

and $f(-x) \neq -f(x)$

$\therefore f(x)$ is neither even nor odd function.

(d)

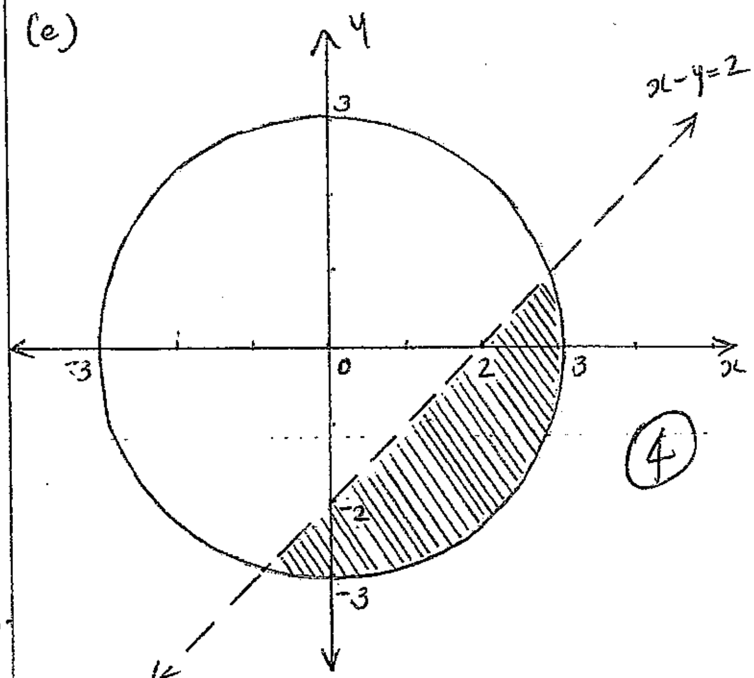


③

$g(2) = 0$ ②

$g(-3) = -3 + 2 = -1$

(e)



④

Question 5 (23 marks)

(a) (i) $y = 4x^2 - 7x + 3 + 5x^{-2}$

$y' = 4 \times 2x - 7 + 5 \times -2x^{-3}$

$= \underline{\underline{8x - 7 - 10x^{-3}}}$ (2)

(ii) $y = (2x+5)(x^2-1)^4$

$y' = (2x+5)\,4(x^2-1)^3 \times 2x + (x^2-1)^4 \times 2$ (3)

$= \underline{\underline{8x(2x+5)(x^2-1)^3 + 2(x^2-1)^4}}$

(iii) $y = \dfrac{5x}{x+1}$

$y' = \dfrac{(x+1) \times 5 - 5x \times 1}{(x+1)^2}$

$= \dfrac{5x+5-5x}{(x+1)^2}$ (3)

$= \underline{\underline{\dfrac{5}{(x+1)^2}}}$

(iv) $y = \sqrt{2x-7}$

$y' = \dfrac{1}{2\sqrt{2x-7}} \times 2$

$= \underline{\underline{\dfrac{1}{\sqrt{2x-7}}}}$ (3)

(b) (i) $\lim\limits_{x \to 4} \dfrac{x-4}{x^2-16} = \lim\limits_{x \to 4} \dfrac{x-4}{(x+4)(x-4)}$

$= \lim\limits_{x \to 4} \dfrac{1}{x+4}$

$= \dfrac{1}{8}$ (2)

(ii) $\lim\limits_{x \to \infty} \dfrac{4x^2+1}{3x^2}$

$= \lim\limits_{x \to \infty} \dfrac{4 + \frac{1}{x^2}}{3}$ (2)

$= \dfrac{4}{3} \quad \left(\because \lim\limits_{x \to \infty} \dfrac{1}{x} = 0\right)$

(c) $3y^2 - 8y + 3 = 0$

(i) $\alpha + \beta = \dfrac{-(-8)}{3} = \underline{\underline{\dfrac{8}{3}}}$ (1)

(ii) $\alpha\beta = \dfrac{3}{3} = \underline{\underline{1}}$ (1)

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\dfrac{8}{3}\right)^2 - 2 = \dfrac{64}{9} - 2 = \underline{\underline{5\tfrac{1}{9}}}$ (2)

page 8

(d) $k(x+1)(x-8) - (x+4)$

$= k(x^2 - 7x - 8) - x - 4$

$= kx^2 - 7kx - 8k - x - 4$

$= kx^2 - x(7k+1) - 4 - 8k$ — (1)

(1) is negative definite

$\Longrightarrow k < 0$ and $\Delta < 0$

$\Delta = (7k+1)^2 - 4k(-4-8k)$

$= 49k^2 + 14k + 1 + 16k + 32k^2$

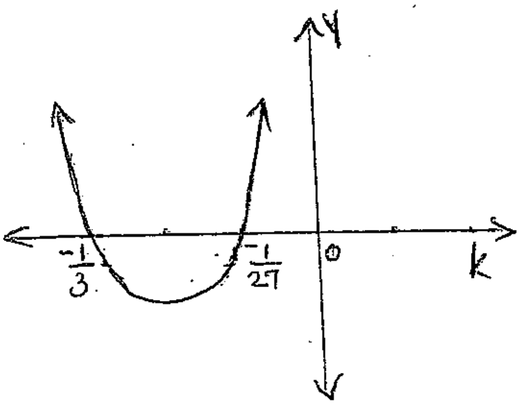
$= 81k^2 + 30k + 1$

X intercepts of Δ are given by

$k = \dfrac{-30 \pm \sqrt{900 - 4\times 81 \times 1}}{2 \times 81}$

$= \dfrac{-30 \pm \sqrt{900 - 324}}{162}$ (4)

$= \dfrac{-1}{27}$ or $\dfrac{-1}{3}$



From the graph $\Delta < 0$

when $\dfrac{-1}{3} < k < \dfrac{-1}{27}$

Question 6 (12 marks)

(a) $d = \dfrac{|5\times -2 - 12 \times 3 + 20|}{\sqrt{5^2 + 12^2}}$

$= \left|\dfrac{-10 - 36 + 20}{13}\right|$

$= \dfrac{26}{13} = 2$ (2)

b(i) $3x + 6y - 7 = 0$

$6y = -3x + 7$

$y = -\dfrac{3x}{6} + \dfrac{7}{6}$

gradient $= \dfrac{-1}{2}$

gradient of $l = 2$

Equation of l

$y - 2 = 2(x+1)$ (3)

$y - 2 = 2x + 2$

$y = 2x + 4$

(ii) $y = 2x + 4$

$y = 0 \Longrightarrow 2x + 4 = 0$

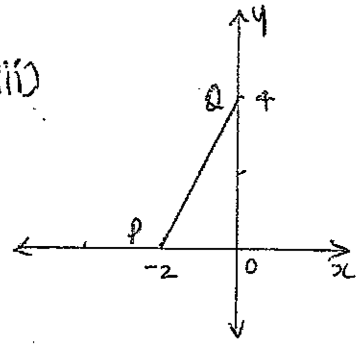
$2x = -4 \quad x = \dfrac{-4}{2} = -2$

$P(-2, 0)$

$x = 0 \Longrightarrow y = 4$ (2)

$Q = (0, 4)$

(iii)



Area of ΔOPQ (2)

$= \frac{1}{2} \times 2 \times 4$

$= 4 \text{ unit}^2$

(C) $x - 5y = 17$ — ①

$3x - 2y = 12$ — ②

① × 3

$3x - 15y = 51$ — ③

$3x - 2y = 12$ — ②

③ - ② $\quad -13y = 39$

$y = -3$

$x = 17 + 5y$

$= 17 - 15 = 2$

Point of intersection of ① and ② is $(2, -3)$

substitute $(2, -3)$ in

$5x + y - 7 = 0$

LHS $= 5 \times 2 + -3 - 7$

$= 10 - 3 - 7 = 0$

RHS $= 0$ (3)

LHS = RHS.

The point $(2, -3)$ is on the line $5x + y - 7 = 0$. Therefore the given lines are concurrent.

Question 7 (16 marks)

(a) (i) $\cos\theta = \frac{-1}{\sqrt{2}}$

acute $\angle\theta = 45^\circ$

$\theta = 180 - 45, 180 + 45$ (2)

$= 135^\circ, 225^\circ$

(ii) $\tan 2\theta = \sqrt{3}, \quad 0 \le \theta \le 360^\circ$

Let $u = 2\theta \qquad 0 \le 2\theta \le 720^\circ$

$\tan u = \sqrt{3}, \quad 0 \le u \le 720^\circ$

$u = 60^\circ, 240^\circ, 420^\circ, 600^\circ$ (3)

$\theta = 30^\circ, 120^\circ, 210^\circ, 300^\circ$

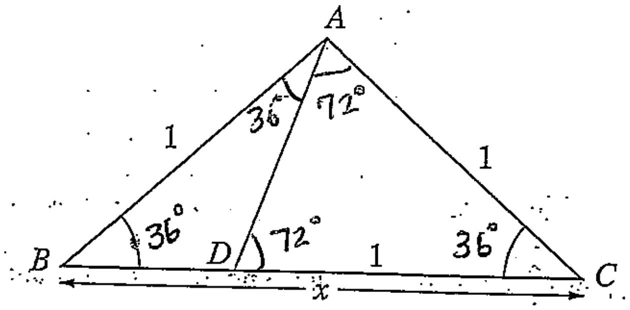
(b) LHS $= (\sin\theta + \cos\theta)^2 - (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)$

$= 1 + 2\sin\theta\cos\theta - 1 + 2\sin\theta\cos\theta$ (3)

$= 4\sin\theta\cos\theta =$ RHS.

(c)



(i) $\angle ADC = \frac{180-36}{2} = 72°$ ($\because \Delta ADC$ is isosceles)

$\angle ADC = \angle DBA + \angle BAD$ (exterior angle theorem)

ie $72 = 36 + \angle BAD$

$\therefore \angle BAD = 36°$

In ΔDBA and ΔABC (3)

$\angle DBA = \angle ACB = 36°$

$\angle BAD = \angle ABC = 36°$

$\therefore \Delta DBA \parallel\!| \Delta ABC$ (Equi angular)

(ii) Since $\Delta DBA \parallel\!| \Delta ABC$ we have

$\frac{AB}{BC} = \frac{BD}{AB} = \frac{DA}{AC}$ (2)

$\frac{1}{x} = \frac{x-1}{1}$

$1 = x^2 - x$ $\quad \therefore x^2 - x - 1 = 0$

(iii) In ΔABC,

$$\cos = \frac{x^2+1^2-1^2}{2 \cdot x \cdot 1}$$

$$\cos 36^\circ = \frac{x^2}{2x} = \frac{x}{2}$$

From (ii) we have $x^2-x-1=0$

$$x = \frac{-(-1) \pm \sqrt{1-4\times 1 \times -1}}{2}$$

$$= \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore \cos 36 = \frac{1 \pm \sqrt{5}}{2} \times \frac{1}{2}$$ 3

$$= \frac{1 \pm \sqrt{5}}{4}$$

Since $\cos 36$ is positive we have

$$\cos 36^\circ = \frac{1+\sqrt{5}}{4}$$