Gosford High School

Year 11

2007
Preliminary
Higher School Certificate

Mathematics

Assessment Task 4

Time Allowed – 2.5 hours

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

Question 1 (12 marks)

(a) Factorise $x^3 + 27$ (1)

(b) Write down the exact value of $\cos(210)°$ (2)

(c) Solve $2^x = \frac{1}{\sqrt{2}}$ (1)

(d) Solve $|2x-7| \le 5$ (2)

(e) Solve $\frac{3x}{5} - 4 = \frac{x}{2}$ (2)

(f) In 2007 Council rates increased by $7\frac{1}{2}\%$. The new rate for a property is \$1735.
What was the old rate for this property. Give your answer correct to the nearest dollar. (2)

(g) The speed of light is 299 725 km/h. Write this number correct to 3 significant figures. (1)

(h) Find the value of $\sqrt[3]{456.7 \div 3\pi}$ correct to 1 decimal place. (1)

Question 2 (12 marks)

(a) Solve the equations $3x - 2y = 7$ and $y = 3 - 2x$ simultaneously (2)

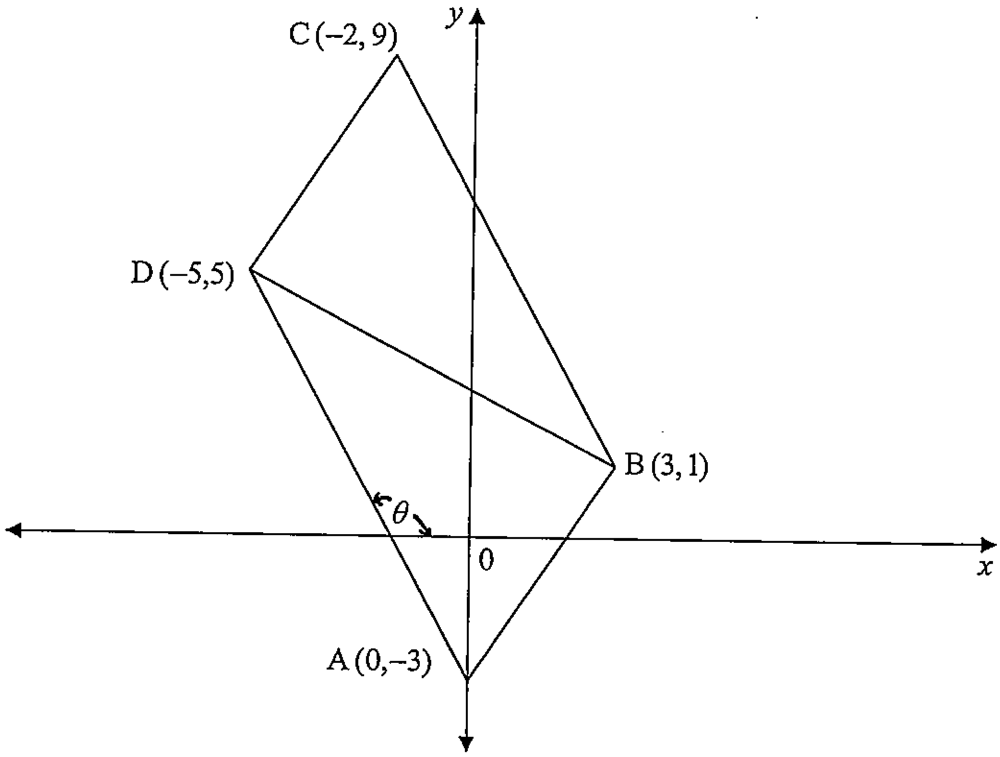
(b) Solve $6x^2 - 7x - 20 = 0$ (2)

(c) Find rational numbers a and m if $\frac{\sqrt{2}}{3\sqrt{2} - 2\sqrt{3}} = a + m\sqrt{6}$ (3)

(d) Simplify $\frac{x+6}{x^2-4} - \frac{2}{x-2}$ (3)

(e) Solve the equation $2x^2 = x + 5$ giving your answers in simplest surd form (2)

Question 3 **(12 marks)**



(a) Find the gradient of AD (1)

(b) Find θ correct to the nearest degree (1)

(c) Find the coordinates of M the midpoint of BD (1)

(d) Prove that ABCD is a parallelogram (2)

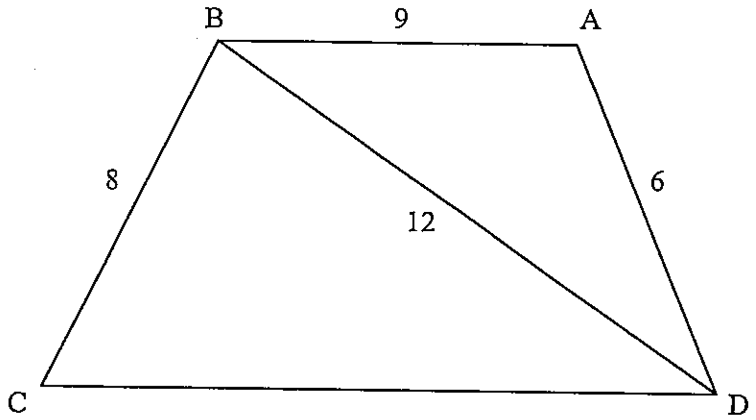
(e) Show that the equation of the line AB is $4x-3y-9=0$ (3)

(f) Find the perpendicular distance from the point D to the line AB. (2)

(g) Find the area of the quadrilateral ABCD (2)

Question 4 **(12 marks)**

(a) In the diagram $\angle DAB = \angle CBD$

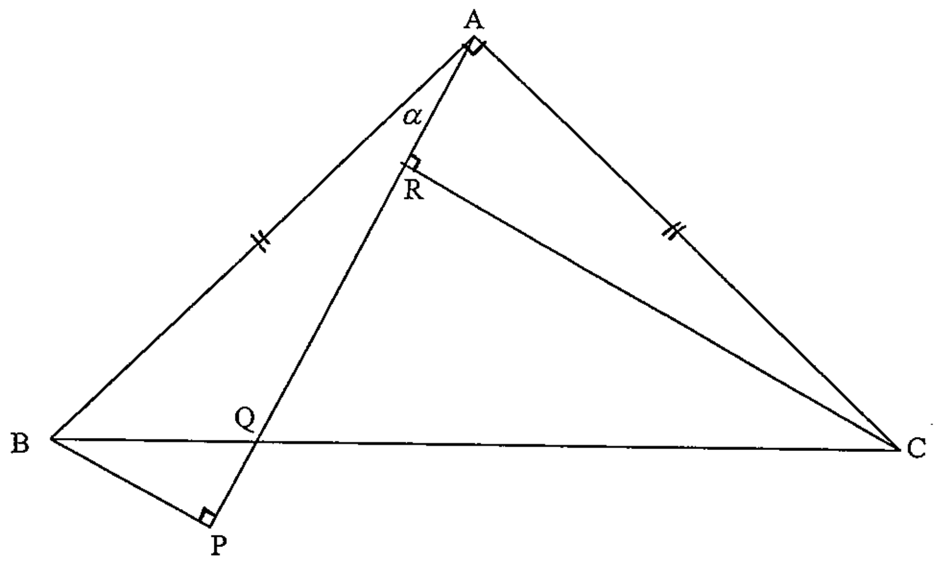


(i) Prove triangles ABD and BDC are similar. (3)

(ii) Find the length of CD (1)

(iii) Prove that AB and CD are parallel (1)

(b)



In the diagram above $AB = AC$, $\angle BAC = \angle BPA = \angle CRA = 90°$; $\angle BAP = \alpha$

Prove that

(i) $\angle ACR = \alpha$ (1)

(ii) triangles ABP and CAR are congruent (2)

(iii) triangles BPQ and CRQ are similar (2)

(iv) $\dfrac{PQ}{QR} = \dfrac{RA}{AP}$ (2)

Question 5 **(12 marks)**

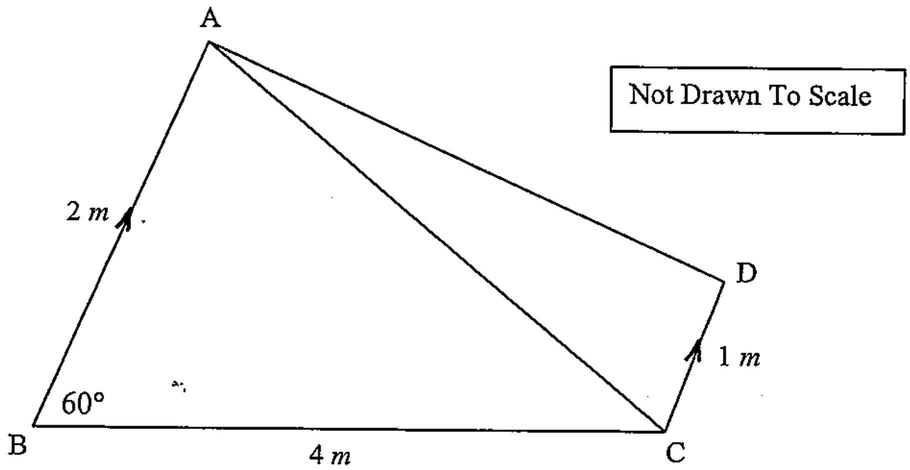
(a) Shade the region satisfying the following inequations simultaneously $\begin{cases} y \le 0 \\ y \le 1 - x^2 \\ y > \left|\dfrac{x}{2}\right| \end{cases}$ (3)

(b) In the diagram below ABCD is a quadrilateral in which AB is parallel DC, $\angle ABC = 60°$, AB = 2 metres, BC = 4 metres and DC = 1 metre.

(i) Find the exact area of triangle ABC (2)

(ii) Find the exact length of AC (2)

(iii) Find the exact length of AD (3)



(c) Evaluate $\lim_{x \to 3}\left[\dfrac{x^3 - 3x^2 + x - 3}{x - 3}\right]$ (2)

Question 6 **(12 marks)**

(a) Draw neat labelled sketches of each of the following functions:

(i) $y=(x-2)^3$ (2)

(ii) $y=1+2^x$ (2)

(iii) $y=2x(4-x)$ (2)

(b) State the domain and range of the function $y=\sqrt{9+x^2}$ (2)

(c) (i) Find the value of $G(-2)+G(3)$ given that $G(x)=\begin{cases} x+5 & \text{for } x<0 \\ \sqrt{25-x^2} & \text{for } x\geq 0 \end{cases}$ (1)

(ii) Is the function continuous at $x=0$? (1)

(d) Write a quadratic equation with roots $3+\sqrt{5}$ and $3-\sqrt{5}$ (2)

Question 7 **(12 marks)**

(a) Find the derivatives of

(i) $7x^3-2x^2+x-8$ (1)

(ii) $y=\sqrt{4-x^3}$ (2)

(b) Show that if $y=\dfrac{7x}{x^2+1}$ then $\dfrac{dy}{dx}=\dfrac{7(1-x^2)}{(x^2+1)^2}$ (2)

(c) Find $f'(-1)$ if $f(x)=2x(3+2x)^5$ (2)

(d) Find the equation of the normal to the curve $y=x^3-x+5$ at the point $(2,11)$ on the curve. (2)

(e) Find the coordinates of the point on the curve $y=3x^2-2x+1$ where the tangent is parallel to the line $4x-y-1=0$ (3)

Question 8 **(12 marks)**

(a) Simplify $\frac{1-\cos^2 x}{\cos(90°-x)}$ (2)

(b) If $\tan\theta = \frac{-5}{12}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$ (3)

(c) Solve $\cos^2\theta - 1 = 0$ for $0^\circ \le \theta \le 360^\circ$ (3)

(d) (i) State the range of the function $y = \sin x$ (1)

(ii) Hence state the minimum value of the expression $7 + 6\sin x$ (1)

(e) Solve $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0^\circ \le \theta \le 360^\circ$ (2)

Question 9 **(12 marks)**

(a) If α and β are the roots of the equation $2x^2 + 4x - 3 = 0$, what is the value of :

(i) $\alpha + \beta$ (1)

(ii) $\alpha\beta$ (1)

(iii) $\alpha^2 + \beta^2$ (2)

(b) Prove that $x^2 + (k-3)x - k$ has real roots for all values of k. (2)

(c) $a(x-1)(x-2) + b(x-1) + c \equiv 2x^2 - 3x + 5$. Find a, b and c (3)

(d) The quadratic equation $2x^2 - (m+2)x + m = 0$ has one root which is twice the other. Find the values of *m*. (3)

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QUESTION 1

a) $(x+3)(x^2-3x+9)$

b) $\cos 210° = -\frac{\sqrt{3}}{2}$

c) $2^x = 2^{-\frac{1}{2}}$

$\therefore x = -\frac{1}{2}$

d) $-5 \le 2x - 7 \le 5$

$2 \le 2x \le 12$

$1 \le x \le 6$

e) $\frac{3x}{5} - 4 = \frac{x}{2}$

$6x - 40 = 5x$

$x = 40$

f) $107\frac{1}{2}\%$ of Old Rate = \$1735

$\therefore$ Old Rate $= \frac{\$1735}{1.075}$

= \$1614 (to nearest \$)

g) 300000 km/h

h) 3.6

QUESTION 2

a) $3x - 2(3 - 2x) = 7$

$3x - 6 + 4x = 7$

$7x = 13$

$x = \frac{13}{7}$

$x = 1\frac{6}{7}$

$y = 3 - 2\left(\frac{13}{7}\right)$

b) $(3x+4)(2x-5) = 0$

$x = -\frac{4}{3}, \frac{5}{2}$

c) $\frac{\sqrt{2}}{3\sqrt{2} - 2\sqrt{3}} \times \frac{(3\sqrt{2} + 2\sqrt{3})}{(3\sqrt{2} + 2\sqrt{3})}$

$= \frac{6 + 2\sqrt{6}}{18 - 12}$

$= \frac{6 + 2\sqrt{6}}{6}$

$= 1 + \frac{1}{3}\sqrt{6}$

$\therefore a = 1, m = \frac{1}{3}$

d) $\frac{x+6}{(x+2)(x-2)} - \frac{2}{x-2}$

$= \frac{x + 6 - 2(x+2)}{(x+2)(x-2)}$

$= \frac{x + 6 - 2x - 4}{(x+2)(x-2)}$

$= \frac{2 - x}{(x+2)(x-2)}$

$= \frac{-1}{x+2}$

e) $2x^2 - x - 5 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(2)(-5)}}{4}$

$x = \frac{1 \pm \sqrt{41}}{4}$

QUESTION 3

a) $m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3 - 5}{0 + 5}$

$= \frac{-8}{5}$

b) $\tan\theta = \frac{-8}{5}$

$\theta = \tan^{-1}\left(-\frac{8}{5}\right)$

$\theta = 122°$

c) Midpoint of BD $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-5+3}{2}, \frac{5+1}{2}\right)$

$= \left(\frac{-2}{2}, \frac{6}{2}\right)$

$= (-1, 3)$

d) i Midpoint of AC $= \left(\frac{-2+0}{2}, \frac{9-3}{2}\right)$

$= \left(\frac{-2}{2}, \frac{6}{2}\right)$

$= (-1, 3)$

$\therefore$ ABCD is a parallelogram Since diagonals of ABCD bisect each other

e) $m_{AB} = \frac{1+3}{3}$

$= \frac{4}{3}$

Equation of AB is

$y = \frac{4}{3}x - 3$

$3y = 4x - 9$

$4x - 3y - 9 = 0$

f) $d = \left|\frac{\ldots}{\sqrt{a^2 + b^2}}\right|$ $b = -3$ $y_1 = \ldots$ $c = -9$

$= \left|\frac{-20 - 15 - 9}{\sqrt{16 + 9}}\right|$

$= \frac{44}{5}$ units

g) $d_{AE} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - 0)^2 + (1 + 3)^2}$

$= \sqrt{9 + 16}$

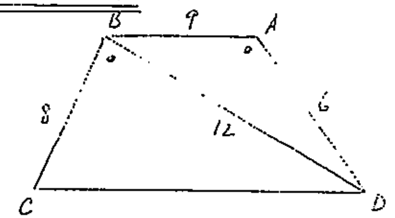
$= 5$ units

Area ABCD $= 5 \times \frac{44}{5}$

= 44 sq units

QUESTION 4

a)



i) Proof

In Δ's ABD & BDC

$B\hat{A}D = C\hat{B}D$ (given)

$\frac{AB}{AD} = \frac{9}{6} = \frac{3}{2}$

$\frac{BD}{BC} = \frac{12}{8} = \frac{3}{2} = \frac{AB}{AD}$

$\therefore \Delta ABD \;|||\; \Delta BDC$ (one pair of sides in proportion and an equal included angle)

(ii) $\frac{CD}{BD} = \frac{BC}{AD}$

$\frac{CD}{12} = \frac{8}{6}$

$CD = 16$

(iii) $A\hat{B}D = B\hat{D}C$ (corresp. ∠'s of similar Δ's in (i))

∴ $AB \parallel CD$ (alternate ∠'s are equal)

b) $R\hat{A}C = 90° - \alpha$ (adj. compl. ∠'s)

$A\hat{C}R = 180° - [(90° - \alpha) + 90°]$

(∠ sum of Δ ACR is 180°)

$A\hat{C}R = \alpha$

(ii) Proof

$B\hat{A}P = A\hat{C}R = \alpha$ (proven in (i))

$B\hat{P}A = A\hat{R}C$ (given right ∠'s)

$AB = AC$ (given)

∴ $\triangle BAP \equiv \triangle ACR$ (AAS)

(iii) $Q\hat{R}A = 180° - C\hat{R}A$

$= 180° - 90°$

$= 90°$

(adj. suppl. ∠'s)

∴ $Q\hat{R}A = B\hat{P}Q$

$= 90°$

$B\hat{Q}P = C\hat{Q}R$ (vert. opp. ∠'s)

∴ $\triangle BPQ \parallel\!| \triangle CRQ$

iv) $\frac{PQ}{RQ} = \frac{BP}{RC}$

but $BP = AR$ and

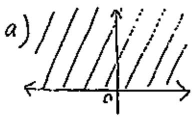
$RC = AP$

(corresp. ∠'s of congruent Δ's)

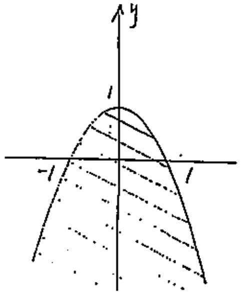
∴ $\frac{PQ}{QR} = \frac{RA}{AP}$

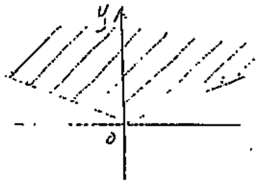
QUESTION 5

a)



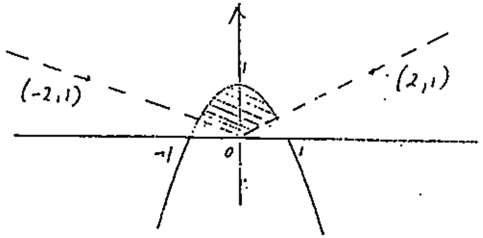
$y \geq 0$





$y > \left|\frac{x}{2}\right|$

Solution



b) (i) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 2 \times 4 \times \sin 60°$

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ sq units

(ii) $AC^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 60°$

$= 20 - 8$

(iii) Let $A\hat{C}B = \alpha$

∴ $\frac{\sin \alpha}{2} = \frac{\sin 60°}{2\sqrt{3}}$

$\sin \alpha = \frac{2 \times \frac{\sqrt{3}}{2}}{2\sqrt{3}}$

$= \frac{1}{2}$

$\alpha = 30°$

∴ $A\hat{C}D = 180° - (30 + 60°)$

(co-int. ∠'s, $AB \parallel DC$)

$A\hat{C}D = 90°$

∴ $AD^2 = AC^2 + DC^2$

$= (2\sqrt{3})^2 + 1^2$

$= 12 + 1$

$= 13$

∴ $AD = \sqrt{13}$ metres

c) $\lim_{x \to 3} \left[\frac{x^2(x-3) + (x-3)}{(x-3)} \right]$

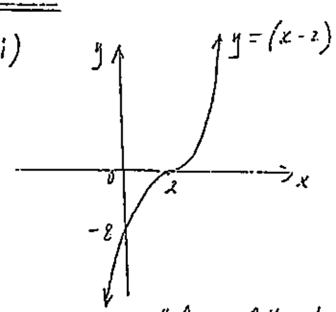
$= \lim_{x \to 3} \left[\frac{(x-3)(x^2+1)}{x-3} \right]$

$= \lim_{x \to 3} [x^2 + 1]$ for $x \neq 3$.

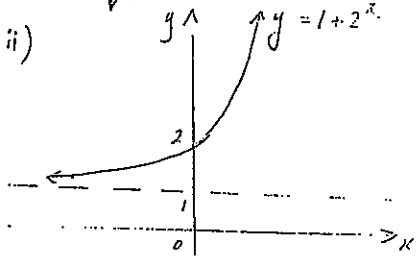
$= 10$

Question 6

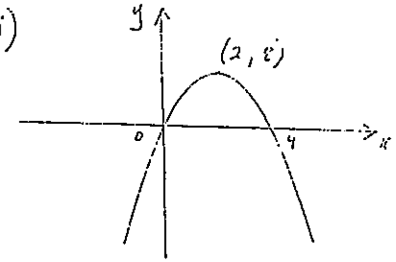
a) i)



ii)



iii)



b) Domain x is any Real number

Range $y \geq 3$

c) i) $G(-2) + G(3) = 3 + 4$

$= 7$

ii) Yes

d) $x^2 - (3+\sqrt{5}+3-\sqrt{5})x + (3+\sqrt{5})(3-\sqrt{5}) = 0$

$x^2 - 6x + 4 = 0$

QUESTION 7

a) i) $\frac{dy}{dx} = 21x^2 - 4x + 1$

ii) $y = (4 - x^3)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(4 - x^3)^{-\frac{1}{2}} \times (-3x^2)$

$= \frac{-3x^2}{2\sqrt{4 - x^3}}$

b) $\frac{dy}{dx} = \frac{(x^2+1) \times 7 - 7x \times 2x}{(x^2+1)^2}$

$= \frac{7x^2 + 7 - 14x^2}{(x^2+1)^2}$

$= \frac{7 - 7x^2}{(x^2+1)^2}$

$= \frac{7(1 - x^2)}{(x^2+1)^2}$

c) $f'(x) = (3+2x)^5 \times 2 + 2x \times 5(3+2x)^4 \times 2$

$= 2(3+2x)^5 + 20x(3+2x)^4$

$f'(-1) = 2 \times 1 + (-20) \times 1.$

$= 2 - 20$

$= -18$

d) $y = x^3 - x + 5$

$\frac{dy}{dx} = 3x^2 - 1$

$= 11$ at $x = 2$

Gradient of Normal is -1

Equation of Normal is

$y - 11 = \frac{-1}{11}(x - 2)$

$11y - 121 = -x + 2$

$x + 11y - 123 = 0$

e) $y = 3x^2 - 2x + 1$

$\frac{dy}{dx} = 6x - 2$

$\therefore 6x - 2 = 4$

$6x = 6$

$x = 1$

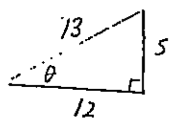
$\therefore$ Pt is $(1, 2)$

QUESTION 8

a) $\frac{1 - \cos^2 x}{\cos(90^\circ - x)} = \frac{\sin^2 x}{\sin x}$

$= \sin x$

b) θ lies in 3rd Quadrant.



$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{1}{-\frac{5}{13}}$

$= -\frac{13}{5}$

c) $\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\therefore \theta = 0, 180^\circ, 360^\circ$

d) i) $-1 \leq y \leq 1$

ii) Minimum Value $= 7 + 6(-1)$

$= 1$

e) $\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = 150^\circ, 330^\circ$

QUESTION 9

a) (i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{-4}{2}$

$= -2$

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-3}{2}$

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2 \times \left(\frac{-3}{2}\right)$

$= 4 + 3$

$= 7$

b) For Real Roots

$b^2 - 4ac \geq 0$

$b^2 - 4ac = (k-3)^2 - 4(1)(-k)$

$= k^2 - 6k + 9 + 4k$

$= k^2 - 2k + 9$

$= k^2 - 2k + 1 + 8$

$\therefore b^2 - 4ac \geq 0$ f...

Since $(k-1)^2$ and 8 are both positive

c) True for $x = 1$

$\therefore c = 2 - 3 + 5$

$c = 4$

True for $x = 2$

$\therefore b + c = 8 - 6 + 5$

$b + 4 = 7$

$b = 3$

True for $x = 0$

$\therefore 2a - b + c = 5$

$2a - 3 + 4 = 5$

$2a = 4$

$a = 2$

d) Let α and 2α be the roots

$\alpha + 2\alpha = \frac{m+2}{2}$

$3\alpha = \frac{m+2}{2}$

$\alpha = \frac{m+2}{6}$

$2\alpha \times \alpha = \frac{m}{2}$

$2\alpha^2 = \frac{m}{2}$

$\alpha^2 = \frac{m}{4}$

$\therefore \left(\frac{m+2}{6}\right)^2 = \frac{m}{4}$

$\frac{m^2 + 4m + 4}{36} = \frac{m}{4}$

$m^2 + 4m + 4 = 9m$

$m^2 - 5m + 4 = 0$

$(m - 4)(m - 1) = 0$