Question 1 (10 marks)

(a) Simplify the ratio $1\cdot6 : 0\cdot12$ (1)

(b) Write $0\cdot00618$ in scientific notation. (1)

(c) Rueben scored 43 out of 56 in a Calculus test.
Write his result as a percentage correct the nearest percent. (1)

(d) Convert 270 millimetres to metres. (1)

(e) Find $\sqrt[5]{710000}$ correct to 2 significant figures. (2)

(f) If x is an integer, state the set of values satisfying $-2 \le x+1 < 3$ (2)

(g) Write $\dfrac{\sqrt{2}}{2-\sqrt{2}}$ in simplest form and with a rational denominator. (2)

Question 2 (12 marks)

(a) Expand and simplify $100-30(20-t)$ (1)

(b) Expand $(x-3y)^2$ (1)

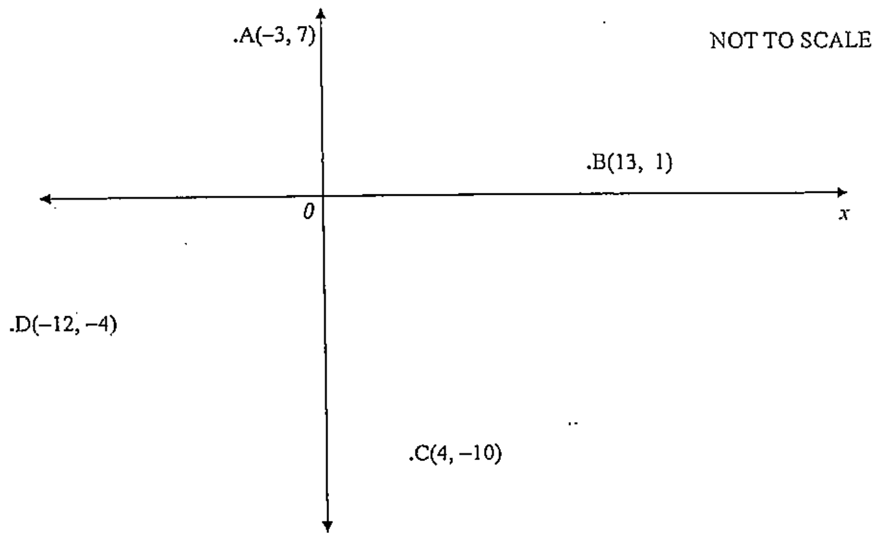
(c) Factorise $64-8a^3$ (2)

(d) Solve $\dfrac{6-x}{2} \le x$ (2)

(e) Solve $\left(\dfrac{1}{27}\right)^{x-2} = \left(\sqrt{3}\right)^{2x}$ (3)

(f) Solve $3x-4 = |2x-11|$ (3)

Question 3 (12 Marks)



$A(-3,7)$, $B(13,1)$, $C(4,-10)$, and $D(-12,-4)$ form the vertices of a quadrilateral $ABCD$.

(a) Find the length of the diagonal BD. (2)

(b) Show that AC and BD bisect each other. (2)

(c) Show that AC is **NOT** perpendicular to BD. (3)

(d) Explain why $ABCD$ is a parallelogram. (1)

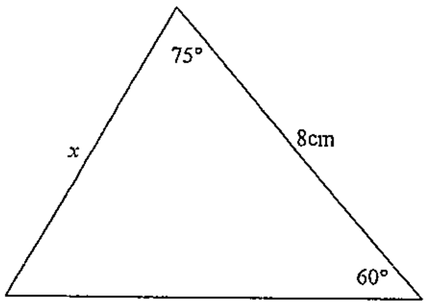
(e) Show that the equation of BD is $x-5y-8=0$ (2)

(f) Find the perpendicular distance from A to BD. (2)

Question 4 (12 marks)

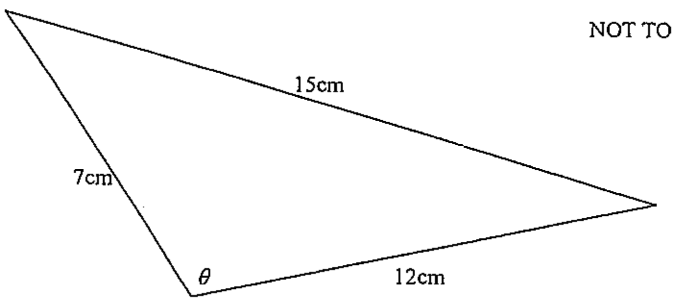
(a) Find the exact value of x in the triangle below. (3)

NOT TO SCALE



(b) Find the value of $\cos\theta$ in the diagram, writing your answer as a simplified fraction. (2)

NOT TO SCALE



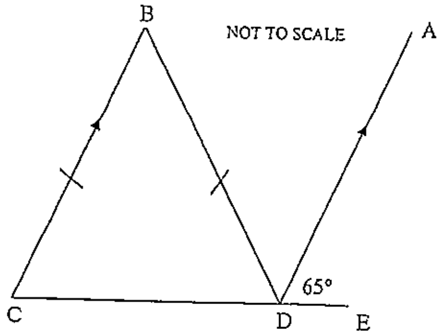
(c) Given $\cos\alpha = \frac{4}{7}$ and $\sin\alpha < 0$ find the exact value $\tan\alpha$ (2)

(d) Solve $\sin x = \sqrt{3}\cos x$ for $0 \le x \le 360°$ (3)

(e) Prove $(\operatorname{cosec}^2 x - 1)(1 - \cos^2 x) = \cos^2 x$ (2)

Question 5 (18 marks)

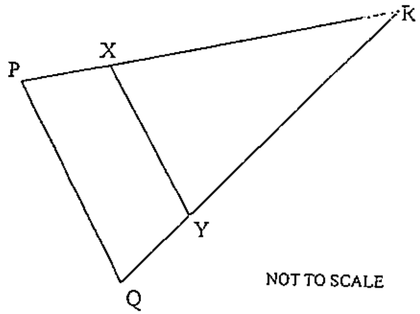
(a)



In the diagram $\angle ADE = 65°$, $BC = BD$ and BC is parallel to AD.

Find, giving reasons, the size of $\angle CBD$. (

(b)



In the diagram $PR = 36$cm, $PX = 12$cm, $QR = 30$cm and $QY = 10$cm.

(i) Prove $\Delta PRQ\ |||\ \Delta XRY$. (3

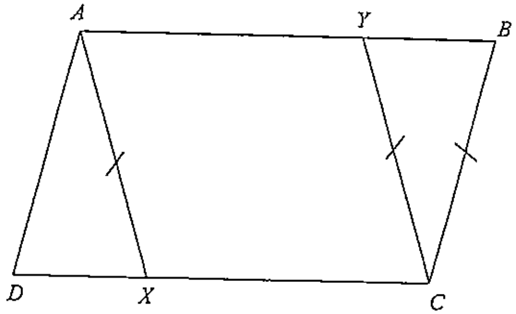
(ii) Hence, or otherwise, state why $\angle RPQ = \angle RXY$. (1

(iii) State why PQ is parallel to XY. (1

(iv) If $XY = 16$cm, find PQ. (2)

(c) Find the number of sides of a polygon if the sum of the interior angles is 3780°. (2)

(d) $ABCD$ is a parallelogram. The point X lies on CD, the point Y lies on AB, and $AX = CY = BC$, as shown in the diagram.



NOT TO SCALE

(i) Explain why $\angle ADX = \angle CBY$ (1)

(ii) Show that $AD = AX$ (2)

(iii) Show that triangles ADX and CBY are congruent. (3)

Question 6 (18 marks)

(a) Neatly sketch of each of the following curves on separate sets of axes. Label all critical points.

(i) $y=\sqrt{9-x^2}$ (3)

(ii) $y=1+2^{-x}$ (3)

(iii) $y=(1-x)^3$ (3)

(b) State the domain and range of $y=\dfrac{5}{2-x}$ (2)

(c)

$$\text{If } f(x)=\begin{cases} 6, & x<-1 \\ x^2+6, & -1\le x<2 \\ 3-x, & x\ge 2 \end{cases}$$

Find the value of $f(-1)+f(-12)-f(5)$ (2)

(d) Show that $g(x)=\dfrac{3x}{3+x^2}$ is an odd function (2)

(e) Draw a sketch of the region satisfying each of the following inequalities simultaneously (3)

$$\begin{cases} x\ge 0 \\ x-y<2 \\ xy\le 2 \end{cases}$$

Question 7 (18 marks)

(a) For the parabola $y=2x^2+4x+3$,

(i) find the equation of the axis of symmetry. (1)

(ii) find the vertex. (2)

(iii) sketch the parabola, labelling the vertex, y intercept and axis of symmetry. (3)

(iv) state the minimum value of $2x^2+4x+3$ (1)

(b) Solve $12m-4m^2>0$ (2)

(c) Find where the curve $y=2x^2+5x-12$ cuts the x axis. (2)

(d) Solve the equation $3x^2=2x+2$ writing your answers in simplest exact form. (3)

(e) Find the point(s) of intersection of the line $y=6-3x$ and the curve $xy=3$ (3)

(f) The quadratic equation $x^2-2x+k=0$ has roots $x=1\pm\sqrt{2}$. Find k. (1)

2008 — YR 11 PRELIMINARY H.S.C FINAL ASSESSMENT

SOLUTIONS

Question 1

a) $1.6 : 0.12 = 160 : 12$
$= 40 : 3$

b) $0.00618 = 6.18 \times 10^{-3}$

c) $\frac{43}{56} \times 100\% = 76.78\%$
$= 77\%$

d) $270\text{ mm} = 0.27\text{ m}$

e) $\sqrt[5]{710\,000} = (710\,000)^{\frac{1}{5}}$
$= 14.799\ldots$
$= 15$ to 2 sign. figs

f) $-2 \leq x+1 < 3$
$-3 \leq x < 2$
$\therefore x = \{-3, -2, -1, 0, 1\}$

g) $\frac{\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}}{(2-\sqrt{2})} \times \frac{(2+\sqrt{2})}{(2+\sqrt{2})}$
$= \frac{2\sqrt{2}+2}{4-2}$
$= \frac{2(\sqrt{2}+1)}{2}$
$= \sqrt{2}+1$

Question 2

a) $100 - 30(20-t) = 100 - 600 + 30t$
$= 30t - 500$

b) $(x-3y)^2 = x^2 - 6xy + 9y^2$

c) $64 - 8a^3 = 8(8-a^3)$
$= 8(2-a)(4+2a+a^2)$

d) $\frac{6-x}{2} \leq x$
$6 - x \leq 2x$
$6 \leq 3x$
$2 \leq x$
$x \geq 2$

e) $\left(\frac{1}{27}\right)^{x-2} = (\sqrt{3})^{2x}$
$\left(\frac{1}{3^3}\right)^{x-2} = \left(3^{\frac{1}{2}}\right)^{2x}$
$(3^{-3})^{x-2} = 3^x$
$3^{6-3x} = 3^x$
$\therefore 6-3x = x$
$6 = 4x$
$x = 1\frac{1}{2}$

f) $3x-4 = 2x-11$ & $-(3x-4) = 2x-11$
$x = -7$ $\quad -3x+4 = 2x-11$
Which does not satisfy $\quad 15 = 5x$
$3 = x$
Which satisfies
$\therefore x = 3$ only

Question 3

a) $BD = \sqrt{(3+12)^2+(1+4)^2}$
$= \sqrt{25^2+5^2}$
$= \sqrt{625+25}$
$= \sqrt{650}$

b) Midpoint of AC $= \left(\frac{-3+4}{2}, \frac{7-10}{2}\right)$
$= \left(\frac{1}{2}, -1\frac{1}{2}\right)$

Midpoint of BD $= \left(\frac{13-12}{2}, \frac{1-4}{2}\right)$
$= \left(\frac{1}{2}, -1\frac{1}{2}\right)$
$=$ Midpoint of AC

$\therefore$ AC & BD bisect each other

c) $M_{AC} = \frac{-10-7}{4+3}$ $\quad M_{BD} = \frac{-4-1}{-12-13}$
$= \frac{-17}{7}$ $\quad = \frac{-5}{-25}$
$= \frac{1}{5}$

$M_{AC} \times M_{BD} = \frac{-17}{7} \times \frac{1}{5}$
$= \frac{-17}{35}$
$\neq -1$

$\therefore$ AC is NOT perpendicular to BD

d) ABCD is a parallelogram since the diagonals AC & BD bisect each other

e) Equation of BD is
$y-(-4) = \frac{1}{5}(x-(-12))$
$5(y+4) = x+12$
$5y+20 = x+12$
$x - 5y - 8 = 0$

f) $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$
$= \left|\frac{1(-3)+(-5)(7)+(-8)}{\sqrt{1^2+(-5)^2}}\right|$
$= \left|\frac{-3-35-8}{\sqrt{1+25}}\right|$
$= \left|\frac{-46}{\sqrt{26}}\right|$
$= \frac{46}{\sqrt{26}}$

Question 4

a) Third angle in triangle is 45°

$$\frac{x}{\sin 60^\circ} = \frac{8}{\sin 45^\circ}$$

$$x = \frac{8\sin 60^\circ}{\sin 45^\circ}$$

$$= \frac{8 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$$

$$= 4\sqrt{3} \times \frac{\sqrt{2}}{1}$$

$$= 4\sqrt{6}$$

b) $$\cos\theta = \frac{7^2 + 12^2 - 15^2}{2 \times 7 \times 12}$$

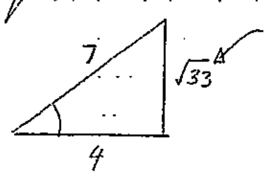
$$= \frac{49 + 144 - 225}{168}$$

$$= \frac{-32}{168}$$

$$= \frac{-4}{21}$$

c) $\cos\alpha = \frac{4}{7}$ & $\sin\alpha < 0$

$\therefore \alpha$ lies in the 4th quadrant.



$$\therefore \tan\alpha = -\frac{\sqrt{33}}{4}$$

d) $\sin x = \sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \sqrt{3}$ ... See note below

$\tan x = \sqrt{3}$

$x = 60^\circ, 240^\circ$

Note & Check if $\cos x = 0$

$x = 90^\circ, 270^\circ$

But these two values do not satisfy the given equation.

e) L.H.S. $= (\operatorname{cosec}^2 x - 1)(1 - \cos^2 x)$

$= \cot^2 x \times \sin^2 x$

$= \frac{\cos^2 x}{\sin^2 x} \times \sin^2 x$

$= \cos^2 x$

$=$ R.H.S.

Question 5

a) $B\hat{C}D = A\hat{D}E$ (corresp. $\angle$'s, $AD \parallel BC$)

$= 65^\circ$

$B\hat{D}C = B\hat{C}D$ (equal $\angle$'s opposite equal sides of isosceles Δ)

$= 65^\circ$

$C\hat{B}D = 180^\circ - (B\hat{C}D + B\hat{D}C)$ (Angle sum of ΔBCD)

$= 180^\circ - (65^\circ + 65^\circ)$

$= 50^\circ$

b)(i) Proof: $\hat{R}$ is common to both Δ's

$$\frac{PR}{RQ} = \frac{36}{30} = \frac{6}{5}$$

$$\frac{RX}{RY} = \frac{24}{20} = \frac{6}{5}$$

$\therefore \Delta PRQ \ |||\ \Delta XRY$

(one pair of corresponding sides are in proportion and an equal included angle)

(ii) $R\hat{P}Q = R\hat{X}Y$

(corresp. $\angle$'s of similar Δ's PRQ and XRY)

(iii) $PQ \parallel XY$ since corresponding angles RPQ and RXY are equal).

(iv) $\frac{PQ}{XY} = \frac{PR}{XR}$

(corresp. sides of similar Δ's are in proportion)

$\frac{PQ}{16} = \frac{3}{2}$

$PQ = \frac{3}{2} \times 16$

$PQ = 24$ cm

c) Sum of Interior $\angle$'s $= (n-2) \times 180^\circ$

$3780^\circ = (n-2) \times 180^\circ$

$21 = n - 2$

$n = 23$

$\therefore$ Polygon has 23 sides.

d) i) $A\hat{D}X = C\hat{B}Y$

(opposite $\angle$'s of a parallelogram are equal)

(ii) $AD = BC$

(opposite sides of a parallelogram are equal)

$BC = CY = AX$ (given)

$AD = AX$ (equal to equals)

(iii) Proof $A\hat{X}D = A\hat{D}X$

(equal $\angle$'s opp. equal sides of isos. Δ)

$C\hat{Y}D = C\hat{B}Y$

(equal $\angle$'s opp. equal sides of isos. Δ)

$\therefore A\hat{X}D = B\hat{Y}C$

(equal to equal $\angle$'s)

$A\hat{D}X = C\hat{B}Y$

(proven in (i))

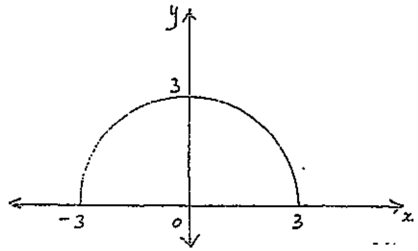
$AD = BC$

(equal sides of parallelogram)

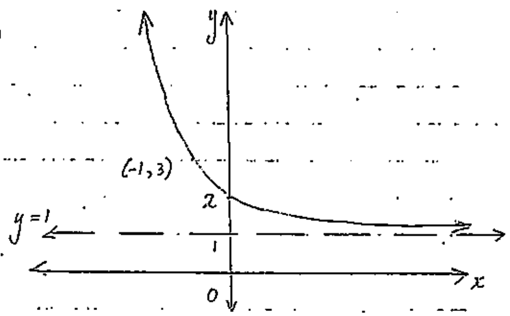
$\Delta ADX \equiv \Delta CBY$ (AAS)

QUESTION 6

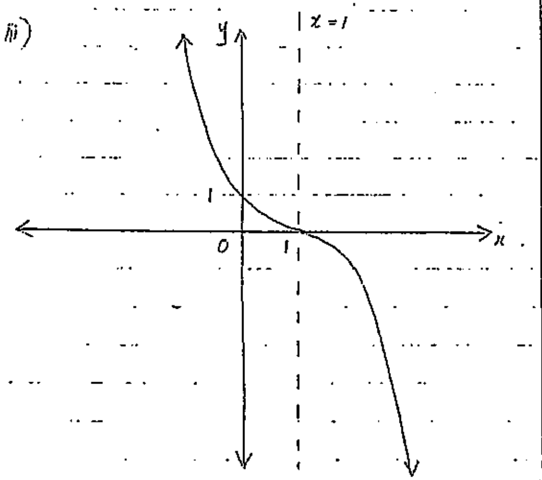
(i)



ii)



iii)



b) Domain : All real x but $x \neq 2$
Range : All real y but $y \neq 0$

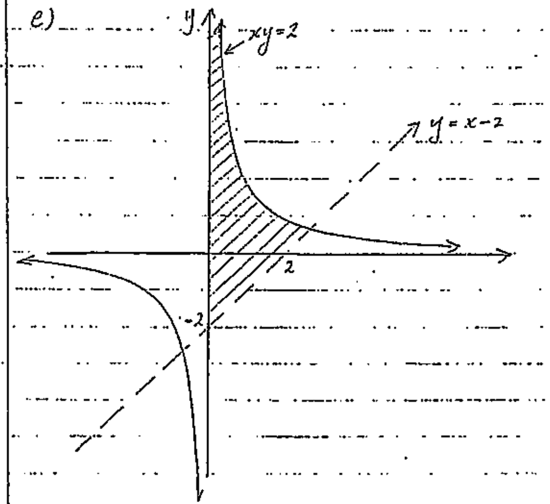
c) $f(-1) = (-1)^2 + 6 = 7$
$f(-12) = 6$
$f(5) = 3 - 5 = -2$

$\therefore f(-1) + f(-12) - f(5) = 7 + 6 - (-2)$
$= 15$

d) $g(-x) = \dfrac{3(-x)}{3 + (-x)^2}$
$= \dfrac{-3x}{3 + x^2}$
$= -g(x)$

$\therefore g(x)$ is ODD

e)



For $x - y < 2$ Test $(0,0)$
$0 < 2$ True

For $xy \leq 2$ Test $(0,0)$
$0 \leq 2$ True

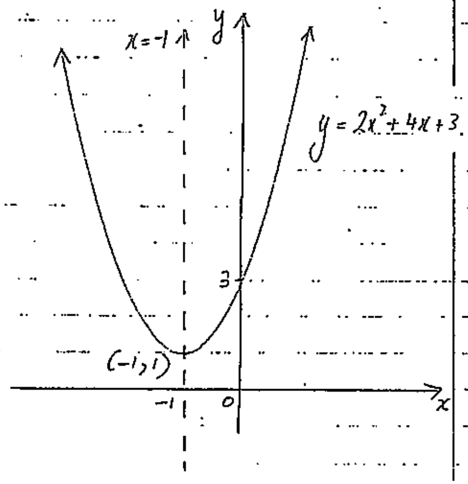
QUESTION 7

a) $y = 2x^2 + 4x + 3$

i) $x = \dfrac{-b}{2a}$
$x = \dfrac{-4}{4}$
$x = -1$ is Axis of Symmetry

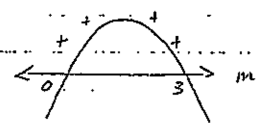
ii) when $x = -1$,
$y = 2(-1)^2 + 4(-1) + 3$
$y = 1$
$\therefore$ Vertex is $(-1, 1)$

iii)



iv) Minimum value of $2x^2 + 4x + 3$ is '1'.

b) $4m(3 - m) > 0$



$\therefore 0 < m < 3$

c) Parabola cuts the x axis when $y = 0$

$\therefore 2x^2 + 5x - 12 = 0$
$(2x - 3)(x + 4) = 0$
$\therefore x = 1\frac{1}{2}, -4$

Parabola cuts the x axis at $(1\frac{1}{2}, 0)$ & $(-4, 0)$

d) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{4 - 4(3)(-2)}}{2 \times 3}$

$x = \dfrac{2 \pm \sqrt{28}}{6}$

$x = \dfrac{2 \pm 2\sqrt{7}}{6}$

$x = \dfrac{1 \pm \sqrt{7}}{3}$

e) At pt. of intersection
$x(6 - 3x) = 3$
$6x - 3x^2 = 3$
$3x^2 - 6x + 3 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$x = 1$, $y = 3$

f) $k = (1 + \sqrt{2})(1 - \sqrt{2})$
$= 1 - 2$
$= -1$