Gosford High School

Year 11

2009
Preliminary
Higher School Certificate

Mathematics

Assessment Task 4

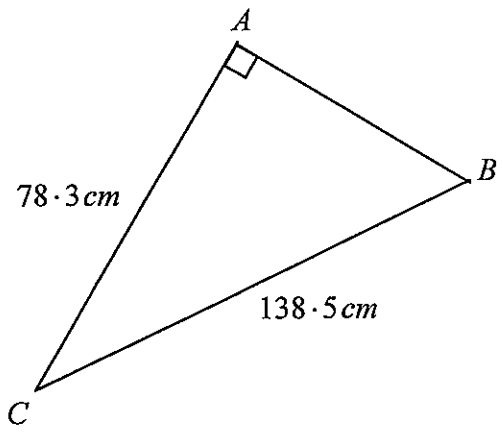
Time Allowed – 2 hours

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1 **(13 marks)**

(a)



Find the length of the interval AB correct to 3 significant figures. (2)

(b) A bicycle is bought for \$840 and sold 3 years later for \$315.
Express the loss as a percentage of the cost price. (1)

(c) Expand and simplify $(x-3)^2+5x$ (2)

(d) Solve $\frac{3x}{2}+4=x$ (1)

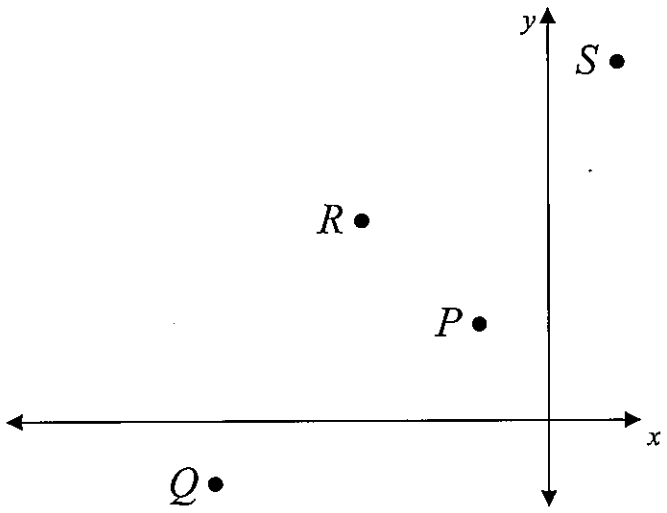
(e) Simplify $\frac{3x^2-5x-2}{9x^2-1}$ (2)

(f) Find the value of x and y if $x+\sqrt{y}=\frac{3}{2\sqrt{3}-3}$ (2)

(g) Solve $|2x|=5x+9$ (3)

QUESTION 2 (13 marks)

(a) A quadrilateral has vertices $P(-1\frac{1}{2}, 2)$, $Q(-6, -1)$, $R(-3\frac{1}{2}, 4)$ and $S(1, 7)$.



(i) Find the midpoint of PR (1)

(ii) Show that PQRS is a parallelogram (2)

(iii) Find the length of QS (1)

(iv) Find the gradient of QS (1)

(v) Find the equation of QS in general form (2)

(vi) Find the perpendicular distance from P to QS (2)

(vii) Find the area of the parallelogram PQRS (2)

(b) Find the point of intersection of the lines $x + y + 3 = 0$ and $3x - 4y + 2 = 0$ (2)

QUESTION 3 (24 marks)

(a) Sketch $y = 2^{-x}$ neatly on a number plane. (2)

(b) Sketch $y = x^2 + 2x - 3$ neatly, showing the coordinates of the vertex and the intercepts with the coordinate axes. (3)

(c) Shade the region satisfying $y < |x-1|$ and $x \geq 0$ (3)

(d) Sketch the curve $y = \sqrt{4 - x^2}$ (2)

(e) Given $F(x) = \sqrt{2-x}$

(i) Find $F(-3)$ (1)

(ii) Find x if $F(x) = 0$ (1)

(iii) State the domain and range of the function $y = F(x)$ (2)

(iv) Sketch the curve $y = F(x)$ (2)

(v) Find the point of intersection of the curve $y = F(x)$ and the line $x + y = 0$ (3)

(f) Consider the function $g(x) = \dfrac{\cos x}{x}$

(i) Show that the function is an ODD function (2)

(ii) For what value of x is the function $y = \dfrac{\cos x}{x}$ undefined (1)

(iii) Find where the curve $y = \dfrac{\cos x}{x}$ crosses the x axis, if $0 \leq x \leq 360°$ (2)

QUESTION 4 (13 marks)

(a) Write down the exact value of $\sec(135)°$ (1)

(b) A triangle has sides 7cm, 4cm and 5cm.
Find the size of the largest angle in the triangle correct to the nearest minute. (2)

(c) Simplfy $\dfrac{\sqrt{1-\sin^2 A}}{\cot A}$ (2)

(d) If $\operatorname{cosec}\theta = -2$ and $\cos\theta > 0$, find the exact value of $\tan\theta$. (2)

(e) Solve $2\cos^2 x + \cos x = 0$ for $0 \le x \le 360°$ (3)

(f) A fishing boat anchored at a point A observes a lighthouse L on a bearing of 050°N.
The boat sails due North until it reaches and anchors at a buoy B.
Charts tell the fisherman that the lighthouse L is 30 km from buoy B and has a bearing of 125°N.

(i) Draw a neat diagram, showing clearly on your diagram all the relevant given information. (1)

(ii) Find the distance from point A to the buoy B. (2)

QUESTION 5 (24 marks)

(a) Find the solutions to equation $2x^2 - 4x - 1 = 0$ in simplest surd form (3)

(b) Solve $x(3-x) \leq 0$ (1)

(c) Find the minimum value of the expression $x^2 + 4x + 3$ (2)

(d) If α and β are the roots of the equation $x^2 - 2x + 5 = 0$, find

(i) $\alpha + \beta$ (1)

(ii) $\alpha\beta$ (1)

(iii) $(\alpha - 2)(\beta - 2)$ (2)

(iv) $\alpha^2 + \beta^2$ (2)

(e) Solve $(x^2 - 2x)^2 - (x^2 - 2x) - 6 = 0$ (3)

(f) Find A , B *and* C if $x^2 - 3x + 6 \equiv A(x-2)^2 + Bx + C$ (3)

(g) Find the range of values of k for which the equation $2x^2 - 3kx + 2 = 0$ has no real roots (2)

(h) Find the values of m if the equation $x^2 - 3mx + (m+3) = 0$ has

(i) one root is the reciprocal of the other (1)

(ii) roots that are equal but opposite in sign (1)

(iii) one root is double the other (3)

QUESTION 6 (13 marks)

(a) Find $\frac{d}{dx}\left[(2x-1)^2\right]$ (2)

(b) Find $\frac{dy}{dx}$ if $y = 4x^3 - \frac{3}{x} + 5$ (2)

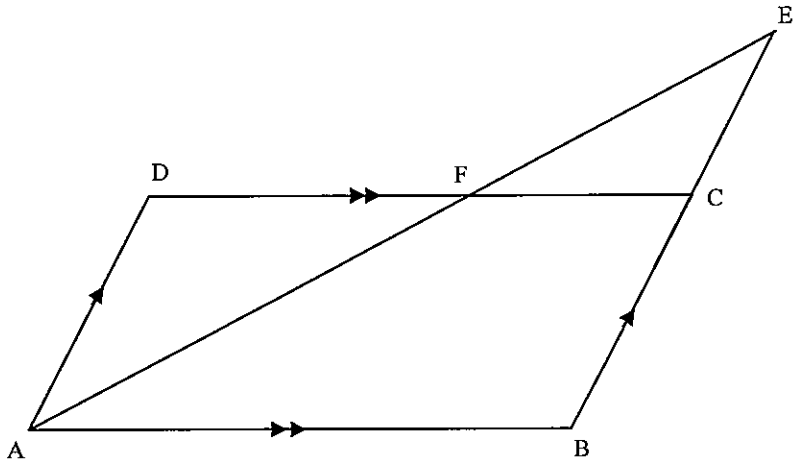
(c) Find $f'(4)$ if $f(x) = 2\sqrt{x}$ (2)

(d) Use first principles to find the gradient of the tangent to the curve $y = x^2 - 3x$ (2)

(e) In the diagram below ABCD is a parallelogram.
F lies on DC such that AF produced meets BC produced at E

(i) Prove $\Delta ADF \;|||\; \Delta EBA$ (3)

(ii) Find DF if FC = 6cm, EC = 5cm and CB = 8cm (2)



Solutions to 2009 Preliminary HSC Maths Task 4

Q1 a) $c^2 = 138.5^2 - 78.3^2$

$= 13051.36$

$c \doteq 114$ cm (3 sig figs)

b) Loss $= \$525$

$\%$ Loss $= \frac{525}{840} \times 100\%$

$= 62.5\%$

c) $x^2 - 6x + 9 + 5x = x^2 - x + 9$

d) $3x + 8 = 2x$

$x = -8$

e) $\frac{(3x+1)(x-2)}{(3x+1)(3x-1)} = \frac{x-2}{3x-1}$

$3x \quad +1$

$x \quad -2$

f) $\frac{3}{2\sqrt{3}-3} \times \frac{2\sqrt{3}+3}{2\sqrt{3}+3} = \frac{6\sqrt{3}+9}{12-9}$

$= \frac{3(2\sqrt{3}+3)}{3} = 2\sqrt{3}+3$

g) $|2x| = 5x + 9$

$2x = 5x + 9$ or $-2x = 5x + 9$

$-9 = 3x$ $\quad -9 = 7x$

$-3 = x$ $\quad -\frac{9}{7} = x$

Check $|-6| \stackrel{?}{=} -15 + 9$ $\quad \left|-\frac{18}{7}\right| \stackrel{?}{=} -\frac{45}{7} + 9$

No $\quad$ Yes

$\therefore x \doteq -\frac{9}{7}$ only

Q2

i) $\left(-\frac{5}{2}, \frac{6}{2}\right) = (-2\frac{1}{2}, 3)$

ii) Midpoint of QS: $\left(\frac{-6+1}{2}, \frac{-1+7}{2}\right)$

$= (-2\frac{1}{2}, 3)$

Equal midpoints ∴ Diagonals bisect each other. ∴ PQRS is a parallelogram.

iii) $d^2 = 7^2 + 8^2$

$d = \sqrt{113}$

iv) $m = \frac{7+1}{1+6} = \frac{8}{7}$

v) $y - 7 = \frac{8}{7}(x - 1)$

$7y - 49 = 8x - 8$

$0 = 8x - 7y + 41$

vi) $d = \left|\frac{8\times\frac{-3}{2} - 7\times 2 + 41}{\sqrt{8^2+7^2}}\right|$ $\left(-1\frac{1}{2}, 2\right)$

$d = \left|\frac{15}{\sqrt{113}}\right| = \frac{15}{\sqrt{113}}$

vii) $A = bh = \sqrt{113} \times \frac{15}{\sqrt{113}}$

$= 15 \text{ u}^2$

b) $x + y = -3$ — ①

$3x - 4y = -2$ — ②

① ×4 $4x + 4y = -12$ — ③

② + ③ $7x = -14$

$x = -2$

Sub into ①

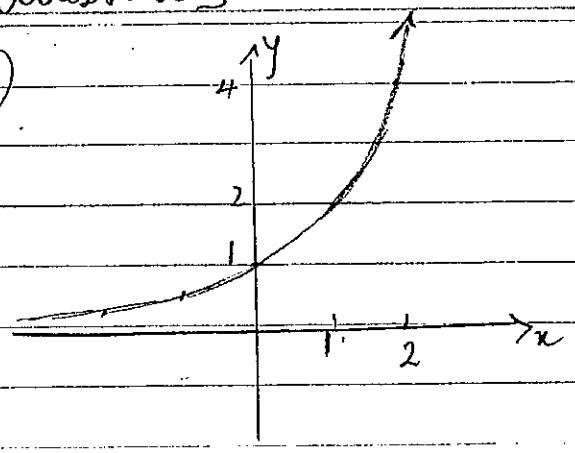
$-2 + y = -3$

$y = -1$

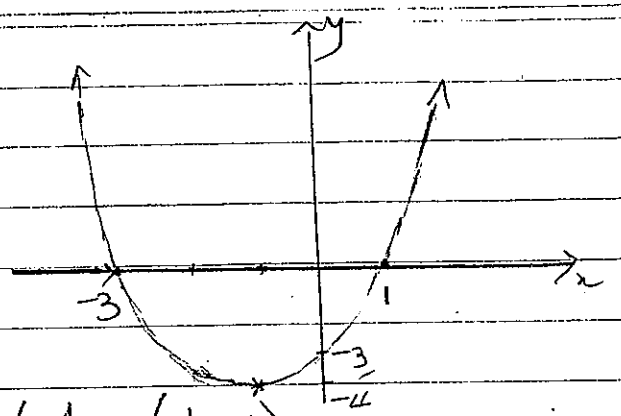
$(-2, -1)$

Question 3

a)

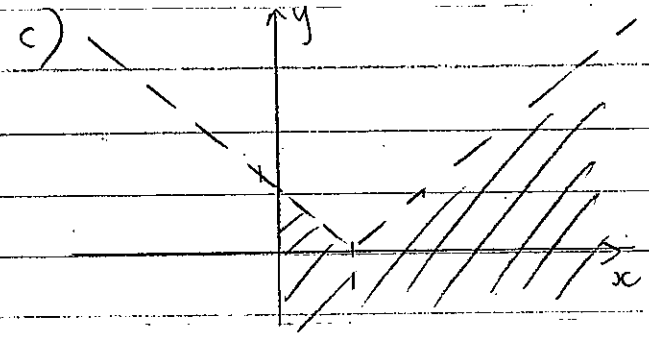


b)

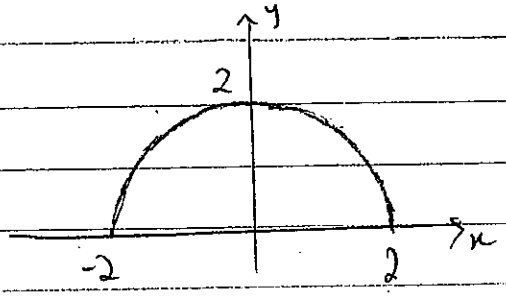


Vertex $(-1, -4)$

c)



d)



e) i) $F(-3) = \sqrt{5}$

ii) $0 = \sqrt{2-x}$

$0 = 2 - x$

$x = 2$

iii) D: $x \leq 2$

R: $y \geq 0$

v) $-x = \sqrt{2-x}$

$x^2 = 2 - x$

$x^2 + x - 2 = 0$

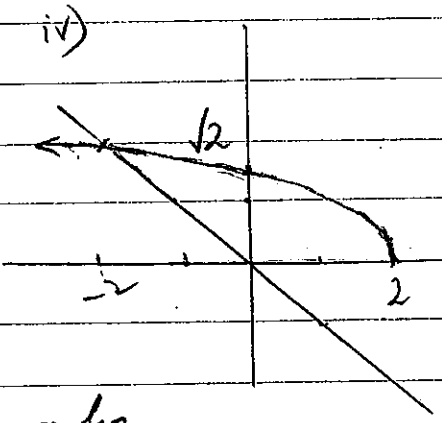
$(x+2)(x-1) = 0$

$x = -2$ or 1

Only $x = -2$ works

Point of intersection is $(-2, 2)$

iv)



f) $g(x) = \frac{\cos x}{x}$

$g(-x) = \frac{\cos(-x)}{-x}$

$= \frac{\cos x}{-x}$

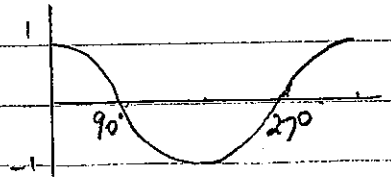
$g(-x) = -g(x)$

$\therefore$ odd.

ii) $x = 0$

iii) $\frac{\cos x}{x} = 0$

$\cos x = 0$

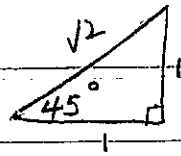


$x = 90°$ or $270°$

Question 4

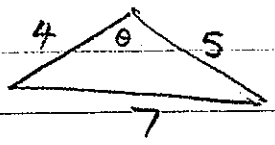
a) $\sec 135^\circ = -\sec 45^\circ$

$= -\sqrt{2}$



b) $\cos\theta = \dfrac{4^2+5^2-7^2}{2\times4\times5} = -\dfrac{1}{5}$

$\angle\theta = 101^\circ\ 32'$



c) $\sqrt{\cos^2 A} \times \dfrac{\sin A}{\cos A} = |\cos A| \times \dfrac{\sin A}{\cos A}$

$= \sin A$ if $\cos A > 0$ ie Quads ① + ④

$= -\sin A$ if $\cos A < 0$ ie quads ② + ③

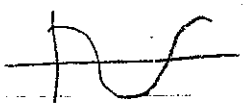
d) $\sin\theta = -\dfrac{1}{2}$ θ lies in 4th quad

$\theta = 330^\circ$ $\therefore \tan\theta = -\dfrac{1}{\sqrt{3}}$

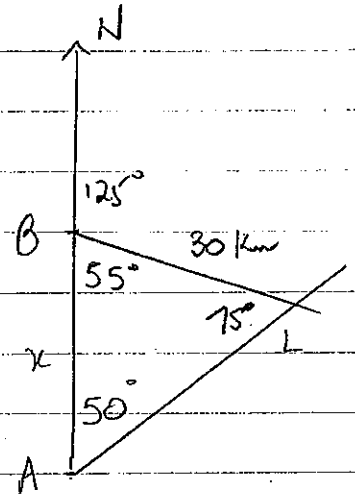
e) $\cos x(2\cos x + 1) = 0$

$\cos x = -\dfrac{1}{2}$ or $\cos x = 0$

$x = 120^\circ$ or 240° or 90° or 270°



f)



ii) $\dfrac{x}{\sin 75^\circ} = \dfrac{30}{\sin 50^\circ}$

$x = \dfrac{30\sin 75^\circ}{\sin 50^\circ}$

$x = 37.827798$

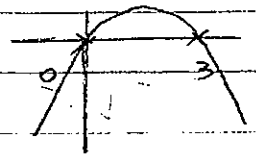
$x \doteq 37.83$ km

Question 5

a) $x = \frac{4 \pm \sqrt{16+8}}{8}$

$x = \frac{4 \pm 2\sqrt{6}}{8} = \frac{2 \pm \sqrt{3}}{4}$

b)



$x \leq 0$ or $x \geq 3$

c) $x^2+4x+4-1$

$(x+2)^2 - 1$

Minimum value is -1

d) i) $\alpha+\beta = 2$

ii) $\alpha\beta = 5$

iii) $\alpha\beta - 2(\alpha+\beta) + 4$

$= 5 - 4 + 4$

$= 5$

iv) $(\alpha+\beta)^2 - 2\alpha\beta$

$= 4 - 10$

$= -6$

e) $u = x^2 - 2x$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3$ or -2

$x^2 - 2x - 3 = 0$ or $x^2 - 2x + 2 = 0$

$(x-3)(x+1) = 0$ $\quad \Delta = 4 - 8$

$x = 3$ or -1 $\quad = -4$

no solutions

$\therefore x = 3$ or -1

f) Equate coeffs of x^2 $\quad A = 1$

Sub $x = 0$ $\quad 4A + C = 6$

$C = 2$

Sub $x = 2$ $\quad 4 - 6 + 6 = 2B + 2$

$2B = 2 \Rightarrow B = 1$

$A = 1, \quad B = 1, \quad C = 2$

g) $2x^2 - 3kx + 2 = 0$

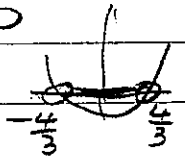
$\Delta < 0$

$9k^2 - 4 \times 2 \times 2 < 0$

$9k^2 - 16 < 0$

$(3k-4)(3k+4) < 0$

$-\frac{4}{3} < k < \frac{4}{3}$



h) i) $\alpha\beta = 1$

$m + 3 = 1$

$m = -2$

ii) $b = 0$

$-3m = 0$

$m = 0$

iii) let roots be α & 2α

$\therefore 3\alpha = 3m$

$\alpha = m$

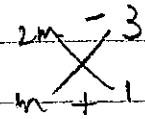
$2\alpha^2 = m + 3$

$2m^2 = m + 3$

$2m^2 - m - 3 = 0$

$(2m-3)(m+1) = 0$

$m = \frac{3}{2}$ or -1



Question 6

a) $y' = 2(2x-1)^1 \cdot 2 = 4(2x-1)$ or $8x-4$

b) $y = 4x^3 - 3x^{-1} + 5$

$y' = 12x^2 + 3x^{-2} \quad = 12x^2 + \frac{3}{x^2}$

c) $f(x) = 2x^{\frac{1}{2}} \qquad f'(x) = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$f'(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$

d) $y + \Delta y = (x+\Delta x)^2 - 3(x+\Delta x)$

$y + \Delta y = x^2 + 2x\Delta x + (\Delta x)^2 - 3x - 3\Delta x$

$\Delta y = 2x\Delta x + (\Delta x)^2 - 3\Delta x$

$\frac{\Delta y}{\Delta x} = 2x + \Delta x - 3$

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x - 3$

e) i) In Δ's ADF, EBA

1. $\angle DAF = \angle FEC$ (alternate $\angle$'s $DA \parallel EB$)

2. $\angle ADF = \angle ABE$ (opposite angles of parallelogram are equal)

$\therefore \Delta ADF /// \Delta EBA$ (2 angles test)

ii) $\frac{DF}{AB} = \frac{AD}{EB}$ (corresponding sides in similar Δ's)

$\frac{x}{x+6} = \frac{8}{13}$

$13x = 8x + 48$

$5x = 48$

$x = \frac{48}{5}$

$\therefore DF = \frac{48}{5}$ cm.