YEAR 11 Mathematics
Preliminary Course
Yearly Examination 2011

1. There are 6 questions.
2. Each question is worth 12 marks.
3. Answer each question in a **NEW** booklet
4. You can use more than one booklet to answer a question if required.
5. Calculators may be used

TOTAL /72

Total Marks – 72
Attempt Questions 1 to 6
All questions are of equal value

Marks

Question 1. (12 marks)

(a) Evaluate $\dfrac{\sqrt[3]{8.24\times10^8}}{21\times5}$ correct to four significant figures. **2**

(b) Express $\dfrac{2\sqrt{3}-2}{1+\sqrt{3}}$ in the form $a+b\sqrt{3}$. **2**

(c) Find the values of x for which $|2x-1|>3$. **2**

(d) Express $\dfrac{x-1}{5}-\dfrac{2x-3}{9}$ as a fraction in its simplest form. **2**

(e) Factorise x^3-8y^3. **2**

(f) A country has a 15% rate of GST. A golf club sells for \$184 which includes GST. Find the amount of GST paid for the golf club. **2**

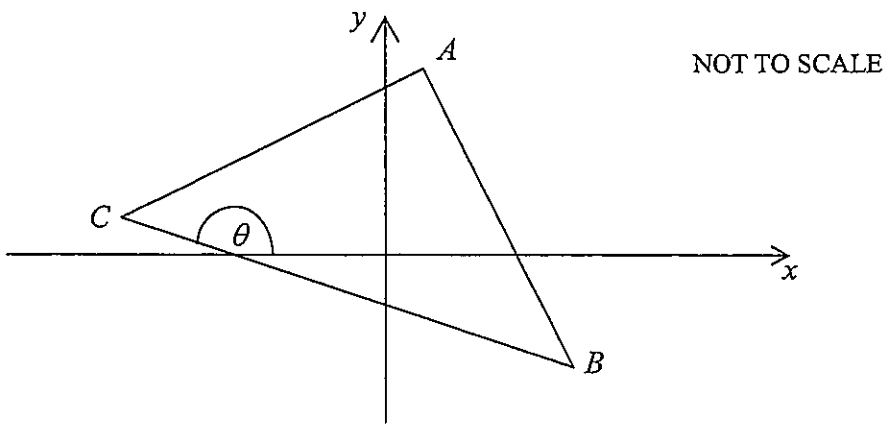
Marks

Question 2. (12 marks)

(a) Simplify $\dfrac{x^2+2x-3}{x^2+3x}$ **2**

(b) In the diagram, A, B and C are the points $(1, 5)$, $(5, -3)$, $(-7, 1)$ respectively. The angle between the line BC and the x-axis is θ.

The gradient of the line BC is $-\dfrac{1}{3}$.



Copy this diagram into your writing booklet showing all relevant information.

(i) Calculate the size of θ correct to the nearest degree. **1**

(ii) Show the line BC has equation $x+3y+4=0$. **1**

(iii) Find the perpendicular distance of the point A from the line BC. Leave answer in exact form. **2**

(iv) Find the exact length of the interval BC. **1**

(v) Find the exact area of ΔABC. **1**

(vi) Find the coordinates of D, the midpoint of BC. **1**

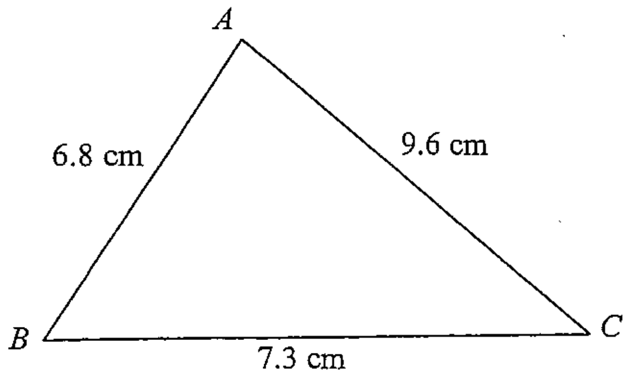
(vii) Find the gradient of the AD. **1**

(viii) By considering the answers to the above questions determine the type of triangle ΔABC represents. Give reasons for your answer. **2**

Marks

Question 3. (12 marks)

(a)



NOT TO SCALE

The diagram shows triangle ABC.

(i) Find the size of the largest angle correct to the nearest minute. **3**

(ii) Hence determine the area of the triangle. Correct to 2 decimal places. **1**

(b) Solve $2\cos 2\theta = -1$ for $0° \le \theta \le 180°$ **3**

(c) Sketch the curve $y = \sin x$ for $-180° \le x \le 180°$ **2**

(d) Simplify $\sin(90° - \alpha)\sec(180° + \alpha)$ **3**

Question 4 (12 marks) **Marks**

(a) Show algebraically that $f(x) = \dfrac{x^3}{x^4 - x^2}$ is an **odd** function. **2**

(b) State the domain and range of $y = x^2 + x - 6$ **2**

(c) If $f(x) = \begin{cases} 0 & if \;\; x \leq 0 \\ -1 & if \;\; 0 < x < 2 \\ x & if \;\; x \geq 2 \end{cases}$

Evaluate $f(-1) + f(1) + f(5)$ **2**

(d) (i) Sketch the function $y = 3(x - 1)$ in the domain $-4 \leq x \leq 4$, indicating both the x and y intercepts. **2**

(ii) On the same set of axes sketch $y = 3 + 2x - x^2$, showing intercepts **2**

(iii) Deduce the ***x*-values** of the points of intersection of the two curves **1**

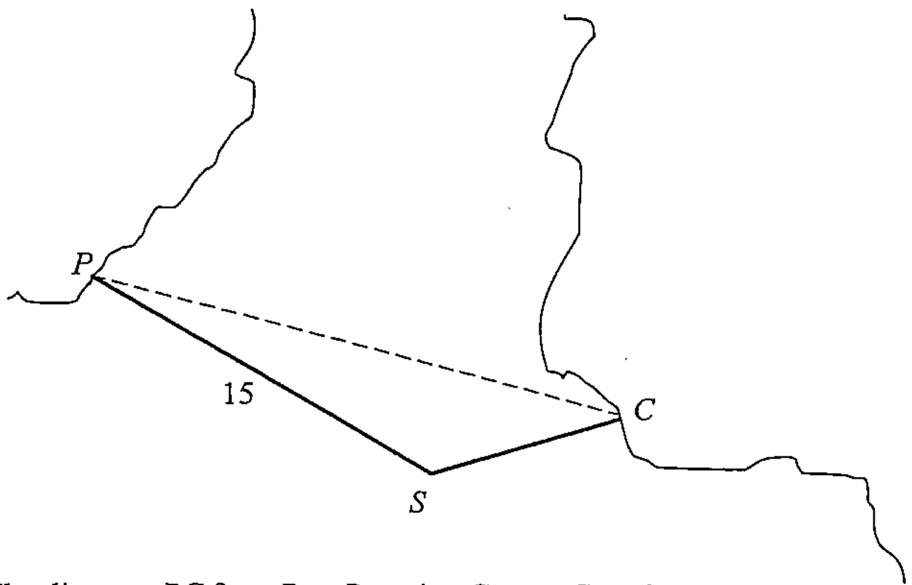
(iv) Hence, or otherwise, solve $3(x - 1) < 3 + 2x - x^2$ **1**

Marks

Question 5. (12 marks)

(a) Solve $x^2 + 12 \geq 7x$. **2**

(b) Vanessa sets sail on her yacht from Port Pourri (P) on a bearing of $137°$.
She sails for 15 nautical miles before a storm forces her to change direction and sail on a bearing of $078°$ to Craggy Cove (C).
The diagram shows her course and the point S indicates where she changed direction due to the storm.



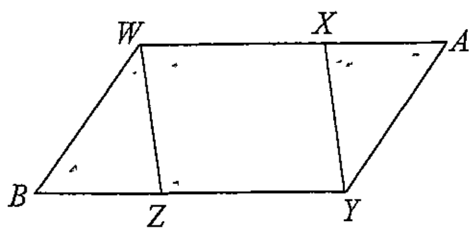
The distance PC from Port Pourri to Craggy Cove is 17.4 nautical miles.
Copy the diagram into your answer booklet showing all relevant information.

(i) Show that $\angle PSC = 121°$. **1**

(ii) Find $\angle PCS$ correct to the nearest degree. **2**

(iii) Hence find the bearing of Port Pourri (P) from Craggy Cove (C). **2**

(c)



NOT TO SCALE

$WXYZ$ is a parallelogram. WX is produced to A and YZ is produced to B such that $AX = BZ$.
Copy the diagram into your writing booklet.

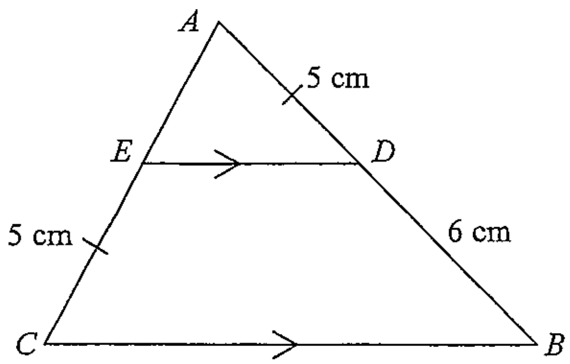
(i) Show, with reasons, that $\Delta AYX \equiv \Delta BWZ$. **3**

(ii) Hence, prove that $BW \parallel YA$. **2**

Question 6. (12 marks) **Marks**

(a) Find all real values of x for which $4^x - 5(2)^x + 4 = 0$. **3**

(b)



The diagram shows triangle ABC, where $ED \parallel CB$, $AD = EC = 5$ cm and $BD = 6$ cm.

(i) Prove that $\Delta ABC \parallel\!| \Delta ADE$. **2**

(ii) Find the exact length of AC. **1**

(c) For what values of k will the expression $kx^2 + 2x + k$ always be negative? **3**

(d) Show that $\dfrac{1}{\cot x + \operatorname{cosec} x} = \operatorname{cosec} x - \cot x$. **3**

End of paper

Solutions to Yr11 Mathematics Yearly 2011

Question 1

(a) 8.929 (2)

(b) $\frac{2\sqrt{3}-2}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{2\sqrt{3}-6-2+2\sqrt{3}}{-2}$

$= \frac{4\sqrt{3}-8}{-2}$

$= 4-2\sqrt{3}$ (2)

$a = 4$

$b = -2$

(c) $(2x-1) > 3$ OR $-2x+1 > 3$

$2x > 4$ $\quad -2x > 2$

$x > 2$ OR $x < -1$ (2)

(d) $\frac{9(x-1) - 5(2x-3)}{45} = \frac{9x-9-10x+15}{45}$

$= \frac{-x+6}{45}$ (2)

(e) $x^3 - (2y)^3 = (x-2y)(x^2+2xy+4y^2)$ (2)

(f). $115\% = 184$

$1\% = 184 \div 115$

$100\% = (184 \div 115) \times 100$ (2)

$\$160 \quad \therefore \text{GST} = \$24$

Q2/

(a) $\frac{\cancel{(x+3)}(x-1)}{x\cancel{(x+3)}} = \frac{x-1}{x}$ (2)

(b) (i) $\tan\theta = -\frac{1}{3}$

$\theta = 180° - 18°$

$= 162°$ (1)

(ii) $y+3 = -\frac{1}{3}(x-5)$

$3y+9 = -x+5$

$x+3y+4=0$ (1)

(iii) $d = \left|\frac{(1)(1)+(3)(5)+4}{\sqrt{1+9}}\right|$

$= \frac{20}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$ (2)

$= 2\sqrt{10}$

(iv) $d = \sqrt{(5+7)^2+(-3-1)^2}$

$= \sqrt{144+16}$

$= \sqrt{160}$

$= 4\sqrt{10}$ (1)

(v) $A = \frac{1}{2} \times 2\sqrt{10} \times 4\sqrt{10}$

$= 40u^2$ (1)

(vi) $\left(\frac{5-7}{2}, \frac{-3+1}{2}\right) = (-1,-1)$ (1)

(vii) $m = \frac{-1-5}{-1-1} = 3$ (1)

(viii) AD $\perp$ BC

D is midpoint $\therefore$ Δ ABC is isosceles

NB With further investigation it is a right isosceles Δ. (2)

Q3/ (i) $\cos B = \dfrac{(7.3)^2 + (6.8)^2 - (9.6)^2}{2 \times 7.3 \times 6.8}$

$B = 85^\circ 45'$ (3)

(ii) $A = \frac{1}{2} \times 7.3 \times 6.8 \times \sin 85^\circ 45'$

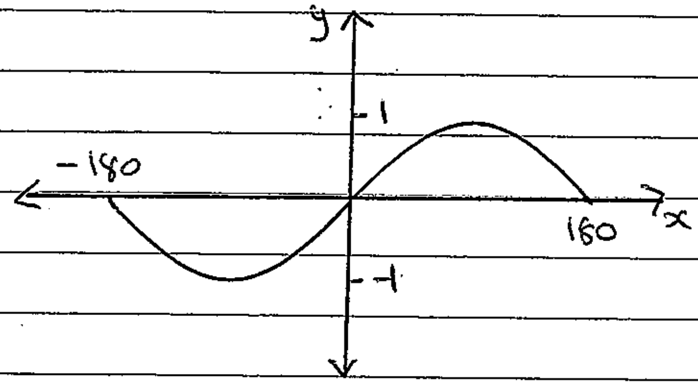
$= 24.75 \text{ cm}^2$ (1)

(b) $\cos 2\theta = -\frac{1}{2}$ $\quad 0^\circ \leq 2\theta \leq 360^\circ$

$2\theta = 120^\circ, 240^\circ$

$\theta = 60^\circ, 120^\circ$ (3)

(c)



(2)

(d) $\sin(90-\alpha)\sec(180+\alpha) = \cos\alpha \times \dfrac{1}{\cos(180+\alpha)}$

$= \cos\alpha \times \dfrac{1}{-\cos\alpha}$ (3)

$= -1$

Q4/ (a) $f(x) = \dfrac{x^3}{x^4 - x^2}$ $\quad f(-x) = \dfrac{(-x)^3}{(-x)^4 - (-x)^2}$

$= \dfrac{-(x)^3}{x^4 - x^2}$ (2)

$\therefore f(x) = -f(-x)$

$\therefore$ odd

(b) Domain is all real x

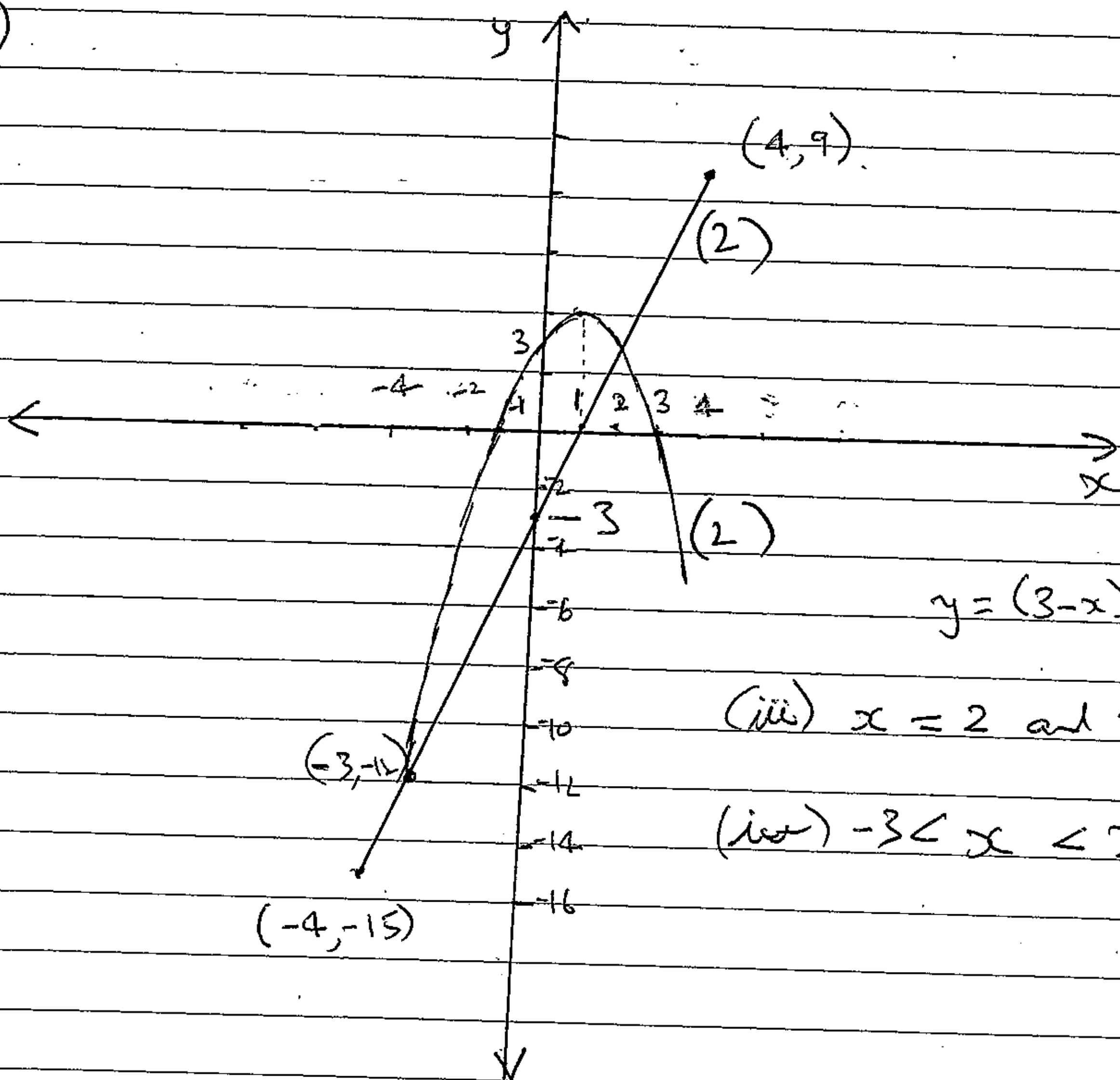
A.O.S $x = -\frac{1}{2}$ $\quad y = (-\frac{1}{2})^2 - \frac{1}{2} - 6$

$y = -6\frac{1}{4}$ (2)

$\therefore$ rage is $y \geq -6\frac{1}{4}$

(c) $(0) + (-1) + (5) = 4$ (2)

(d)



$y = (3-x)(1+x)$

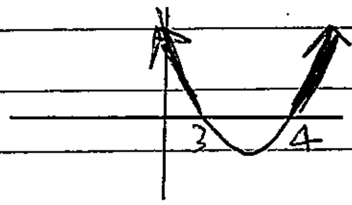
(iii) $x = 2$ and $x = -3$ (1)

(iv) $-3 < x < 2$ (1)

Q5/

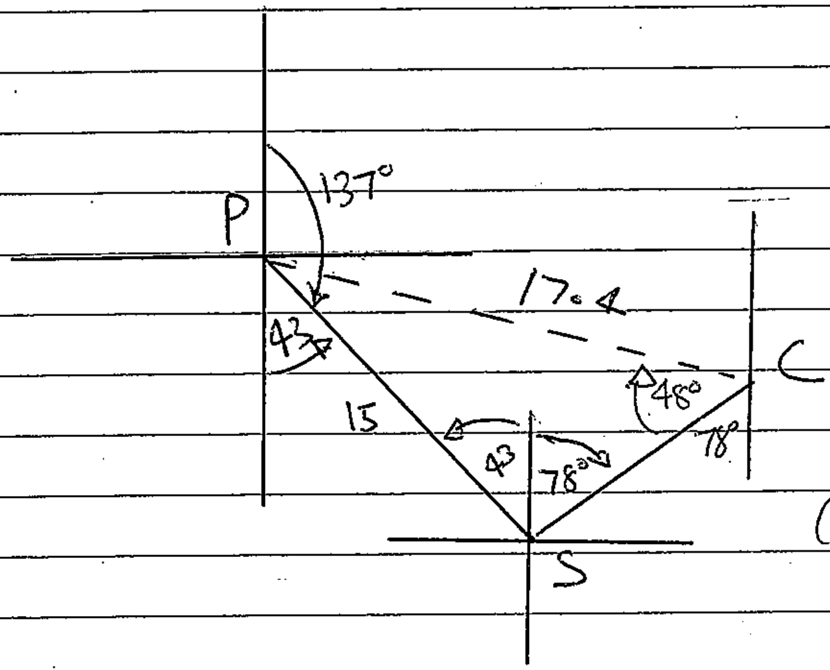
(a) $x^2 - 7x + 12 \geq 0$

$(x-4)(x-3) \geq 0$ (2)



$x \geq 4$ OR $x \leq 3$

(b)



(i) See diagram (1)

(ii) $\dfrac{\sin C}{15} = \dfrac{\sin 121°}{17.4}$

$C = \sin^{-1}\left[\dfrac{15 \sin 121°}{17.4}\right]$ (2)

$\angle PCS = 48°$

(iii) $180 + 78 + 48 = 306°$ (2)

(c) (i) $XY = WZ$ (opposite sides of parallelogram $WXYZ$)

$\angle WXY = \angle WZB = \alpha$ (opposite $\angle$'s " " " ")

$AX = BZ$ (given)

$\angle AXY = \angle WZB = 180 - \alpha$ (adjacent supplementary $\angle$'s)

$\therefore \triangle WZB \equiv \triangle AXY$ (SAS) (3)

(ii)

$\angle XAY = \angle WBZ = \theta$ (corresponding $\angle$'s of $\equiv \triangle$'s)

Produce BY to T

$\angle XAY = \angle AYT = \theta$ (alternate $\angle$'s $WA \parallel BT$)

$\therefore \angle AYT = \angle WBZ = \theta$

and these angles are in corresponding positions

$\therefore BW \parallel YA$

Q6/ (a) $2^{2x} - 5(2^{x}) + 4 = 0$

Let $u = 2^{x}$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4$ OR $\cancel{x}u = 1$ (3)

$2^{x} = 4$ OR $2^{x} = 1$

$x = 2$ $x = 0$

(b) $\angle A$ is common

$\angle AED = \angle ACB$ (Corres $\angle$'s $ED \parallel CB$)

(i) $\therefore \triangle ABC \,|||\, \triangle ADE$ (equiangular) (2)

(ii) $\frac{AE}{AC} = \frac{AD}{AB}$

$\frac{x}{x+5} = \frac{5}{11}$

$11x = 5x + 25$ (1)

$6x = 25$

$x = \frac{25}{6}$

$AC = x + 5$

$= 9\frac{1}{6}$

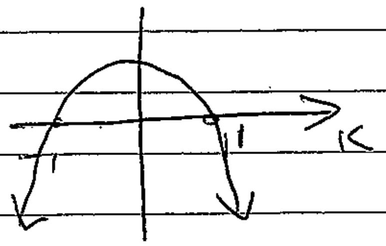
(c) $K < 0$ & $\Delta < 0$ for negative definite

$$4 - 4 \times K \times K < 0$$

$$4 - 4K^2 < 0$$

$$1 - K^2 < 0$$

$$(1-K)(1+K) < 0$$



$K > 1$ OR $K < -1$ (3)

But $K < 0$ also

$\therefore$ $K < -1$ for negative definite

(d) $$LHS = \frac{1}{\cot x + \operatorname{cosec} x} \times \frac{\cot x - \operatorname{cosec} x}{\cot x - \operatorname{cosec} x}$$

$$= \frac{\cot x - \operatorname{cosec} x}{\cot^2 x - \operatorname{cosec}^2 x}$$

$$= \frac{\cot x - \operatorname{cosec} x}{-1}$$ (3)

$$= \operatorname{cosec} x - \cot x$$

$$= RHS$$