Student Name: ....

Student Number: ....

GOSFORD HIGH SCHOOL

Year 11 Preliminary 2012

Yearly Examination (Task 3)

MATHEMATICS

Time: 2 Hours *(plus 5 minutes reading time)*

General Instructions:

- Board approved calculators may be used.
- Write using black or blue pen.
- All necessary working should be shown in Questions 11 – 16
- Write your student number **AND** name on this examination paper

Total Marks – 70

Section 1 - **10 Marks**

- Attempt Questions 1 – 10 on the separate answer sheet. (No working is to be shown on this sheet – use the question sheet.)
- Allow about 15 minutes for this section.

Section 2 - **60 Marks**

- Attempt all Questions 11 – 16
- Start each question in a new booklet.
- Allow about 1 hour 45 minutes for this section

This paper **MUST NOT** be removed from the examination room.

SECTION 1: (10 Marks)

- Attempt all questions.
- Allow about 15 minutes for this section.
- Use the multiple choice answer sheet (shade in carefully).
- Any working out can be completed on this question sheet. (**Do not** do working on the answer sheet.)

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1. What is the value of $\frac{6.4+8.7}{5.9-2.3}$ correct to 2 significant figures?

   A. 4.19

   B. 4.2

   C. 5.57

   D. 5.6

2. $\frac{1+\sqrt{3}}{2-\sqrt{3}} =$

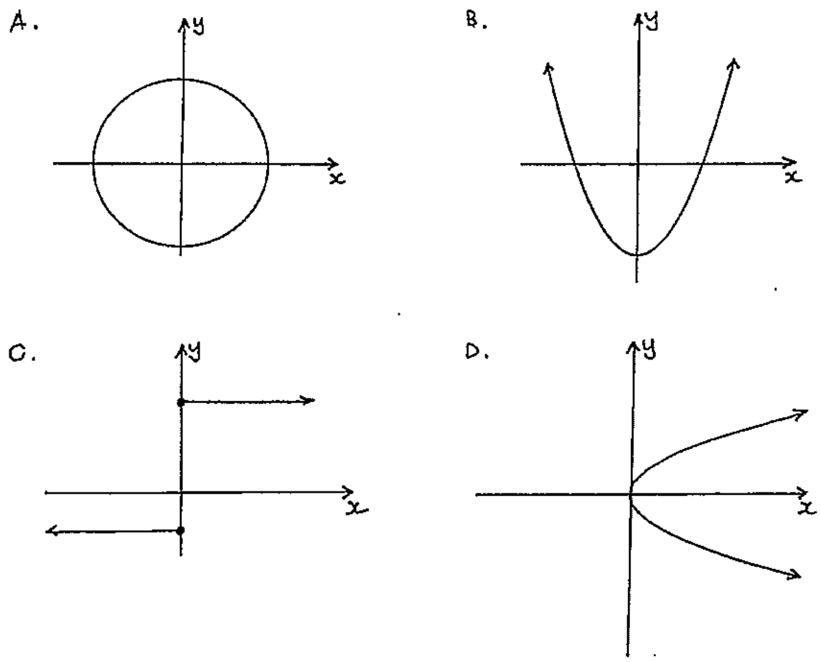
   A. $5+3\sqrt{3}$

   B. $\frac{5+3\sqrt{3}}{7}$

   C. $\sqrt{3}-1$

   D. $\frac{\sqrt{3}-1}{7}$

3. Which of the following graphs represents a function?



4. What is the value(s) of θ, in the domain $0° \leq \theta \leq 360°$, when $\cos\theta = \frac{1}{2}$

A. $60°$

B. $60°\ and\ 120°$

C. $60°\ and\ 240°$

D. $60°\ and\ 300°$

5. Expand and simplify $3(2x-1)-(x-4)$

A. $5x-7$

B. $5x-5$

C. $5x+1$

D. $5x+3$

6. Which of the following is true for the function

$$f(x) = \frac{3}{x^2} - 2\ ?$$

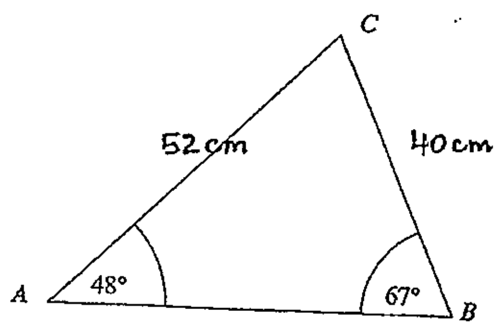
A. Even function

B. Odd function

C. Neither odd nor even function

D. Zero function

7. What is the correct expression for the area of triangle ABC?



Not to scale

A. $\frac{1}{2} \times 52 \times 40 \times cos\ 67°$

B. $\frac{1}{2} \times 52 \times 40 \times sin\ 67°$

C. $\frac{1}{2} \times 52 \times 40 \times cos\ 65°$

D. $\frac{1}{2} \times 52 \times 40 \times sin\ 65°$

8. The domain and range for the function $y = 7 - x^2$ is:

A. $x \leq 7$, all real y

B. $x \leq \sqrt{7}$, all real y

C. All real x, $y \leq 7$

D. All real x, $y \leq \sqrt{7}$

9. The values of k for which the expression

$$-\frac{x^2}{4} - x - k$$

is a negative definite is:

A. $k < 1$

B. $k > 1$

C. $k < 4$

D. $k > 4$

10. For the quadratic equation $x^2 + ax + c = 0$, the roots $\propto$ *and* β are reciprocals of each other. Which statement **must** be true?

A. $\propto \beta = \frac{c}{a}$

B. $\propto \beta = -\frac{c}{a}$

C. $c = 1$

D. $\frac{c}{a} = 1$

SECTION II

60 marks
Attempt Questions 11 – 16
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question

Question 11 (10 marks). Use a SEPARATE booklet.

Marks

a. Express $2.3\dot{6}\dot{4}$ as a rational number in simplest form. 2

b. Solve:

i. $\frac{5}{x+3} = \frac{3}{x}$ 2

ii. $|2x+1| = 3x+2$ 2

c. Solve the following equations simultaneously: 2

$$y = 2x + 4 \; and \; y = x^2 + 1$$

d. Simplify fully: 2

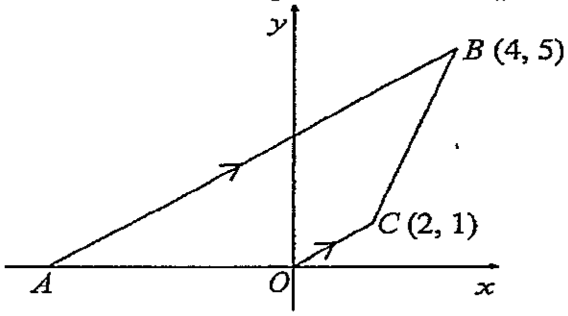
$$\frac{x^2 + y^2 + 2xy}{x^2 - y^2}$$

End of Question 11

Question 12 (10 marks). Use a SEPARATE booklet.

Marks

In the diagram below $ABCO$ is a trapezium with $AB \parallel OC$.



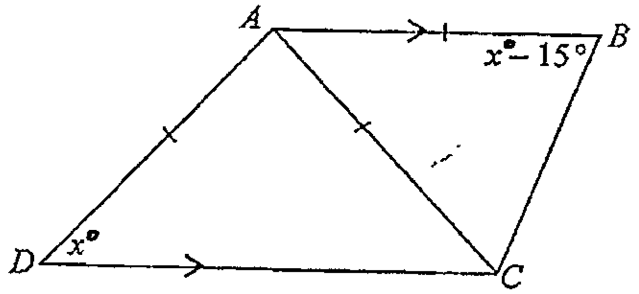
(i)	Find the coordinates of the midpoint of BC.	1
(ii)	Calculate the exact length of OC.	1
(iii)	Find the gradient of OC.	1
(iv)	Find the size of $\angle AOC$, correct to the nearest degree.	1
(v)	Show that the equation of the line AB is $x-2y+6=0$.	1
(vi)	Find the coordinates of A.	1
(vii)	Show that the perpendicular distance from C to the line AB is $\frac{6\sqrt{5}}{5}$.	2
(viii)	Hence, or otherwise, calculate the area of trapezium $ABCO$.	2

End of Question 12

Question 13 (10 marks). Use a SEPARATE booklet.

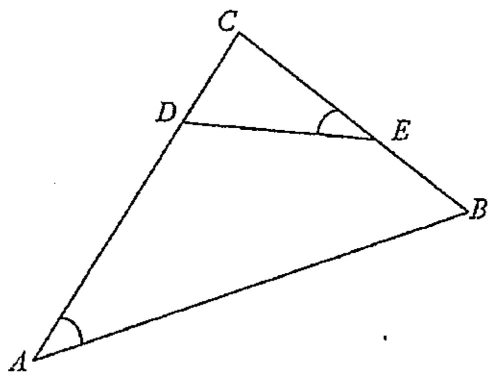
Marks

a. In quadrilateral ABCD, AB = AC = AD and AB||DC as shown $\angle ADC = x^\circ$ and $\angle ABC = x^\circ - 15^\circ$ 2



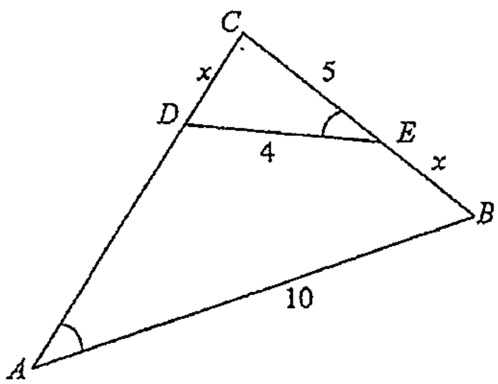
Find the value of x giving reasons.

b. In the diagram ABC is a triangle.
The points D and E lie on AC and BC respectively so that $\angle BAC = \angle DEC$



Copy or trace the diagram into your writing booklet.

i. Prove that $\Delta ABC ||| \Delta EDC$ 2



ii. Hence, if EC = 5 cm, DE = 4 cm, AB = 10 cm and DC = BE = x cm, as shown in the diagram above, find the value of x. 2

Give reasons for your answer.

Question 13 (Continued)

c. c. A function is defined as:

$$f(x) = \begin{cases} x - 5 & for \ \ x \geq 0 \\ -2 & for \ \ -1 < x < 0 \\ 2 + x^2 & for \ \ x \leq -1 \end{cases}$$

i. Evaluate $f(2) + f(-1)$ 1

ii. On a one third page number plane neatly sketch the function $y = f(x)$ clearly indicating all important features. 2

iii. Write an expression for $f(a^2)$ 1

End of Question 13

Question 14 (10 marks). Use a SEPARATE writing booklet.

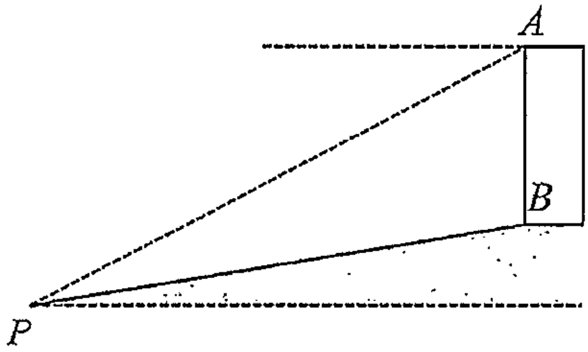
Marks

a. Prove 2

$$\frac{1}{cosec^2\theta} + \frac{1}{sec^2\theta} + \frac{1}{cot^2\theta} = sec^2$$

b. Given that $\tan\theta = \frac{4}{5}$ and $\sin\theta < 0$, find the exact value of $\cos\theta$ 2

c. A point P is at the foot of the hill which is inclined at 5° to the horizontal. The base B, of a tower AB, is situated 150 metres up the incline of the hill from P. From the top of the tower the angle of depression of the point P is 32°.



i. Copy the diagram and mark on it the size of ∠APB and of ∠PAB. 1

ii. Find the height of the tower, correct to the nearest metre. 2

d. Two ships leave a port P at the same time. Ship A travels at 12 km/h on a bearing of 213°. Ship B travels in a south easterly direction at a speed of 8 km/h.

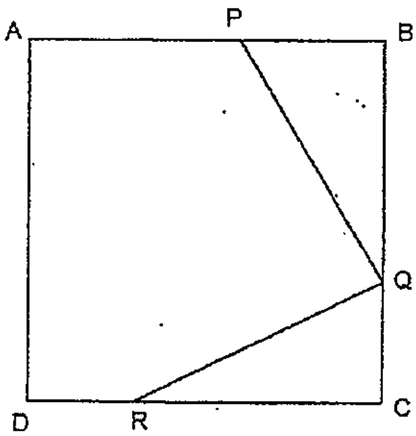
i. Draw a diagram to illustrate the relative positions of the two ships after 3 hours, correct to the nearest km. 1

ii. Calculate the distance between the ships after 3 hours, correct to the nearest km. 2

Question 15 (10 marks). Use a SEPARATE writing booklet.

Marks

a. In the diagram below, ABCD is a square. P, Q and R are points on sides AB, BC and CD respectively such that PB = QC = RD.



Copy the diagram onto your worksheet. 2

Prove that $\Delta BPQ \equiv \Delta CQR$

b. Solve $x^2 - 5x - 24 \leq 0$ 2

c. If $\propto$ and β are the roots of the equation $3x^2 + 5x - 4 = 0$, find

i. $\propto + \beta$ and $\propto \beta$ 1

ii. $\frac{3}{\propto} + \frac{3}{\beta}$ 1

iii. $2a^2 + 2\beta^2$ 2

d. Given the equation $(k+1)x^2 + (k+2)x + k = 0$ find the value of k if the equation has equal roots. 2

End of question 15

Question 16 (10 marks). Use a SEPARATE writing booklet.

Marks

a. Solve for θ, where $-180° \leq \theta \leq 180°$ 2

$$\sqrt{3}\, tan\theta + 1 = 0$$

b. Find the values of a, b and c if 3

$$\frac{x^2+1}{x^2+3x+2} \equiv \frac{a}{x+2} + \frac{bx+c}{x+1}$$

c. Solve: 3

$$\frac{1}{x^4} - \frac{3}{x^2} - 4 = 0$$

d. If $f(x) = x^2 - 3x + 5$, 2

Find an expression, in its simplest form for

$$\frac{f(x+h) - f(x)}{h}$$

End of Examination

SECTION 1.

1) 4.194444... (calc)

$= 4.2$ **B**

2) $\frac{1+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}+2\sqrt{3}+3}{4-3}$

$= 5+3\sqrt{3}$ **A**

3) **B**

4) $\cos\theta = \frac{1}{2}$

$\therefore \theta = 60^\circ, 300^\circ$ **D**

5) $3(2x-1)-(x-4)$

$= 6x-3-x+4$

$= 5x+1$ **C**

6) Since $f(x) = f(-x)$

$\therefore$ even function **A**

7) Using the sin rule:

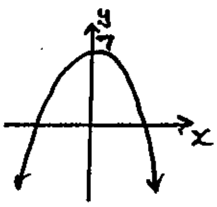
$\frac{YZ}{\sin Y\hat{X}Z} = \frac{22}{\sin 55^\circ}$

$\therefore YZ = \frac{22\sin 60^\circ}{\sin 55^\circ}$ **B**

8) From the graph:

D: all real x

R: $y \leq 7$ **C**



9) to be a negative definite

$a<0$ and $\Delta<0$

$\Delta = b^2-4ac$

$= 1-4(-\frac{1}{4})(-k)$

$= 1-k$

$\therefore 1-k<0$ **B**

$1<k$

10) If reciprocals then product of roots = 1 **C**

$\therefore c = 1$

SECTION II

Q11) a. let $x = 0.3\dot{6}\dot{4}$

$\therefore 100x = 36.46464...$

$x = 0.36464...$

$99x = 36.1$

$\therefore 990x = 361$

$x = \frac{361}{990}$

$\therefore$ answer $= 2\frac{361}{990}$

b. (i) $\frac{5}{x+3} = \frac{3}{x}$

$5x = 3x+9$

$2x = 9$

$x = \frac{9}{2}$

(ii) $|2x+1| = 3x+2$

$2x+1 = 3x+2$ OR $2x+1 = -(3x+2)$

$-1 = x$

$2x+1 = -3x-2$

$5x = -3$

$x = -\frac{3}{5}$

check solutions :

$x = -1$ ✗ does not work

$x = \frac{-3}{5}$ ✓ works

$\therefore x = -\frac{3}{5}$ is the only solution

c. $y = 2x+4$ — ①

$y = x^2+1$ — ②

Sub ① in ② :

$2x+4 = x^2+1$

$0 = x^2-2x-3$

$0 = (x-3)(x+1)$

$\therefore x = 3, -1$

$y = 10, 2$

d. $\frac{x^2+y^2+2xy}{x^2-y^2}$

$= \frac{(x+y)^2}{(x-y)(x+y)}$

$= \frac{x+y}{x-y}$

Q12 (i) Midpt BC

$= \left(\frac{4+2}{2}, \frac{5+1}{2}\right)$

$= (3,3)$

(ii) $OC^2 = 2^2+1^2$

$= 5$

$\therefore OC = \sqrt{5}$ units

(iii) $m_{OC} = \frac{1-0}{2-0} = \frac{1}{2}$

(iv) Let $C\hat{O}X = \theta$

$\therefore \tan\theta = \frac{1}{2}$

$\therefore \theta = 26°34'$

$\therefore \angle AOC = 180° - 26°34'$

$= 153°26'$

$= 153°$ (nearest deg)

(v) $m_{AB} = \frac{1}{2}$

$\therefore$ eqn $\Rightarrow y-5 = \frac{1}{2}(x-4)$

$2y-10 = x-4$

$x-2y+6 = 0$

as req.

(vi) Sub $y=0$

$\therefore x = -6$

$\therefore A = (-6, 0)$

(vii) $\perp d = \frac{|(1)(2)+(-2)(1)+6|}{\sqrt{1^2+(-2)^2}}$

$= \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$ units

as req.

(viii) $A = \frac{h}{2}(a+b)$

Now $d_{AB} = \sqrt{(4+6)^2+(5-0)^2}$

$= \sqrt{10^2+5^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$

$\therefore A = \frac{6\sqrt{5}}{10}(\sqrt{5}+5\sqrt{5})$

$= \frac{(6\sqrt{5})^2}{10}$

$= 18 \text{ units}^2$

13. a. $\angle ACD = x^\circ$ (equal base $\angle$'s in isosceles Δ)

$\angle CAB = x^\circ$ (alternate $\angle$'s in parallel lines are equal)

Also $\angle ACB = x^\circ - 15^\circ$ (equal base $\angle$'s in isosceles Δ)

$\therefore x^\circ + x^\circ - 15^\circ + x^\circ - 15^\circ = 180^\circ$ ($\angle$ sum of $\Delta = 180^\circ$)

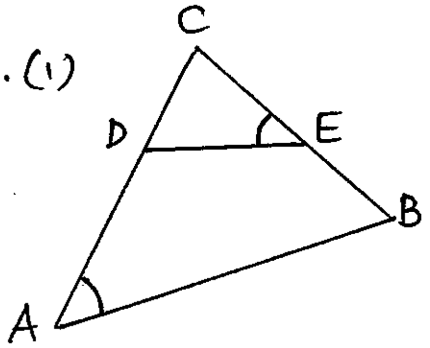
$3x^\circ - 30^\circ = 180^\circ$

$3x^\circ = 210^\circ$

$x^\circ = 70^\circ$

$\therefore x = 70$

b. (i)



$\angle BAC = \angle DEC$ (given)

$\angle BCA = \angle DCE$ (common)

$\therefore \Delta ABC \parallel\!| \Delta EDC$ (equiangular)

(ii) Since similar, corresponding sides are in proportion

i.e. $\frac{CD}{CB} = \frac{DE}{AB}$

$\frac{x}{5+x} = \frac{4}{10}$

$10x = 20 + 4x$

$6x = 20$

$x = \frac{10}{3}$

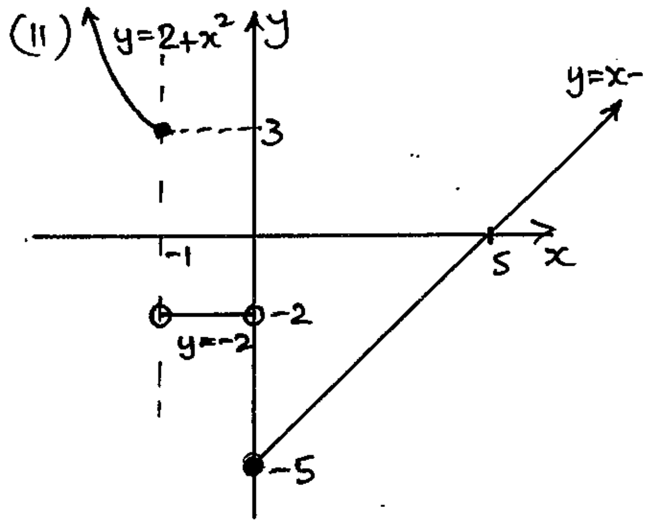
c. (i) $f(2) = 2 - 5 = -3$

$f(-1) = 2 + (-1)^2 = 3$

$\therefore f(2) + f(-1) = -3 + 3$

$= 0$

(ii)



(iii) $f(a^2) = a^2 - 5$

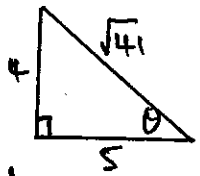
14 a. $LHS = \sin^2\theta + \cos^2\theta + \tan^2\theta$

$= 1 + \tan^2\theta$

$= \sec^2\theta$

$=$ RHS as req.

b. $\tan\theta = \frac{4}{5} \Rightarrow$

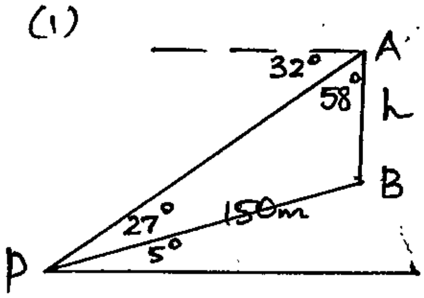


and $\sin\theta < 0$

$\therefore$ θ is in 3rd Quad.

$\therefore \cos\theta = -\frac{5}{\sqrt{41}}$

c. (i)

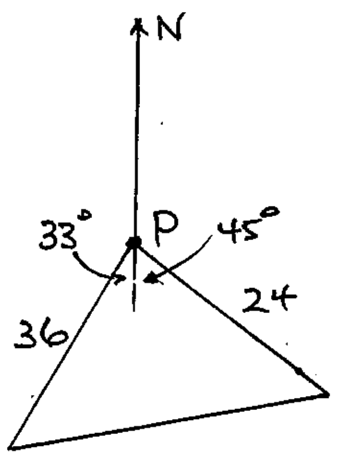


(ii) Using the sin rule

$$\frac{h}{\sin 27^\circ} = \frac{150}{\sin 58^\circ}$$

$$h = \frac{150 \sin 27^\circ}{\sin 58^\circ}$$

d. (i)



(ii) $d^2 = 36^2 + 24^2 - 2 \times 36 \times 24 \times \cos 78^\circ$

$= 1512.728598$ (calc)

$\therefore d = 38.8938\ldots$ (calc)

$\therefore$ distance is 39 kms.

Q15 a. $\angle PBQ = \angle RCQ$ ($90^\circ \angle$ in a square)

$PB = QC$ (given)

$BQ = RC$ (equal lengths subtracted from equal sides of a square)

$\therefore \Delta BPQ \equiv \Delta CQR$ (SAS)

b. Consider $x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$

$x = 8, -3$

Test in $x^2 - 5x - 24 \le 0$

$\therefore -3 \le x \le 8$

c. (i) $\alpha\beta = \frac{c}{a} = -\frac{4}{3}$

$\alpha + \beta = -\frac{b}{a} = -\frac{5}{3}$

(iii) $2\alpha^2 + 2\beta^2 = 2(\alpha^2 + \beta^2)$

$= 2[(\alpha+\beta)^2 - 2\alpha\beta]$

$= 2\left(\left(-\frac{5}{3}\right)^2 - 2\left(-\frac{4}{3}\right)\right)$

$= 2\left(\frac{25}{9} + \frac{8}{3}\right)$

$= \frac{98}{9}$

(ii) $\frac{3}{\alpha}+\frac{3}{\beta} = \frac{3\alpha+3\beta}{\alpha\beta}$

$= \frac{3(\alpha+\beta)}{\alpha\beta}$

$= \frac{3\left(-\frac{5}{3}\right)}{-\frac{4}{3}}$

$= \frac{15}{4}$

d. Equal roots when $\Delta = 0$

i.e $(k+2)^2 - 4(k+1)k = 0$

$k^2+4k+4 - 4k^2-4k = 0$

$-3k^2+4 = 0$

$4 = 3k^2$

$\frac{4}{3} = k^2$

$\pm\frac{2}{\sqrt{3}} = k$

Q16 a. $\sqrt{3}\tan\theta + 1 = 0$

$\sqrt{3}\tan\theta = -1$

$\tan\theta = -\frac{1}{\sqrt{3}}$

for $0^\circ \le \theta \le 360^\circ$

$\theta = 150^\circ, 330^\circ$

So in $-180^\circ \le \theta \le 180^\circ$

$\theta = -30^\circ, 150^\circ$

b. $\frac{x^2+1}{x^2+3x+2} \equiv \frac{a}{x+2} + \frac{bx+c}{x+1}$

$\therefore x^2+1 \equiv a(x+1) + (bx+c)(x+2)$

$= ax+a+bx^2+2bx+cx+2$[illegible]

$= bx^2+(a+2b+c)x+(a+2$[illegible]

$\therefore b = 1$ — ①

$a+2b+c = 0$ — ②

$a+2c = 1$ — ③

② $\Rightarrow$ $a+c = -2$ — ④

③ $\Rightarrow$ $a+2c = 1$ — ⑤

④ − ⑤ $-c = -3$

$c = 3$

$\therefore a = -5$

$\therefore a = -5,\ b = 1,\ c = 3$

c. let $A = \frac{1}{x^2}$

$\therefore A^2 - 3A - 4 = 0$

$(A-4)(A+1) = 0$

$\therefore A = 4, -1$

$\therefore \frac{1}{x^2} = 4$ or ~~$\frac{1}{x^2} = -1$~~ no real solution

$\therefore x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$ are the only solutions.

d. $f(x) = x^2 - 3x + 5$

$$f(x+h) = (x+h)^2 - 3(x+h) + 5$$
$$= x^2 + 2xh + h^2 - 3x - 3h + 5$$

$$\therefore \frac{f(x+h) - f(x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= 2x + h - 3$$