Hornsby Girls' High – 2006

Mathematics Preliminary Examination

Question 1 (14 marks)	
a) Evaluate the following. Express your answer in Scientific Notation correct to 3 significant figures. $\frac{-0.1}{\sqrt{(0.1)^2 + 8}}$	2
b) State the exact value of Cos 210°	2
c) Show that the roots of the quadratic equation $3x^2 - 8x + 10 = 0$ are not real.	2
d) Find $\lim_{x \to 3} \frac{x^2 - 9}{2x - 6}$	2
e) Simplify: $\tan \theta \propto \sin (90^\circ - \theta)$	2
 f) A ship sails due south for 10km and due west for 3km. Calculate its bearing from its starting point. 	2
g) Solve simultaneously: $2x + 5y = 16$ 10x - 3y = -4	2

Question 2 (16 marks) START A NEW PAGE

a) Differentiate the following

i) $y = x^4 - 3x^3 + 12x$

ii) y = (3x+5)(8-2x) 2

$$y = \frac{x}{x^2 + 4}$$

b) For the parabola $(x-4)^2 = 12(y+3)$ find: i) the vertex 1 ii) the focus 1 the equation of its directrix iii) 1 c) For the quadratic $2x^2 - 6x + 1 = 0$ find: i) $\alpha + \beta$ l αβ ii) 1 iii) $\alpha^2 + \beta^2$ 2 d) Solve for θ : $0 \le \theta \le 360^{\circ}$ 2 $2\sin\theta - \sqrt{3} = 0$ e) Solve for x: $4^x - 9(2^x) + 8 = 0$ 2 Question 3 (15 marks) START A NEW PAGE a) If f''(x) = 6x - 8, f'(2) = 0& f(-1) = 3. 2 Find an expression for f(x). b) Find the equation of a parabola with focus (5.-3) 2 and directrix x = 9. c) For the equation $y = \frac{1}{3}x^3 - x^2 - 8x - 6$ find: i) the y-intercept 1 the stationary points and determine their nature ii) 2 any inflection points iii) 2 Sketch the curve in the domain $-4 \le x \le 6$. 3 State the minimum value of the function in the above domain. 1 d) Find the equation of the normal to the curve $y = \frac{1}{x^2}$ at the point where x = 2. Express your 2

answer in general form.

Question 4 (14 marks) START A NEW PAGE

a) What is the range of
$$y = 2\sqrt{9 - x^2}$$
?

b) For what values of *a* is the line
$$y = ax$$
 a tangent 2
to the hyperbola $y = \frac{3}{x-2}$?

c) Differentiate:
$$y = (\sqrt{x} + 1)^{100}$$
 2

d) Consider the function $f(x) = 2x^2 + x$

i) Find
$$f'(x)$$
 using the First Principles Method 2

ii) Show that
$$a(1 + f'(a)) = 2f(a)$$
. 2

e) Show that the line
$$5x-12y+26=0$$
 is a tangent 2
to the circle $x^2 + y^2 = 4$.

f) Find the domain over which the curve $f(x) = x^3 - 7x^2 + 1$ 2 is concave downwards.

Question 5 (13 marks) START A NEW PAGE

a) What is the domain of
$$y = \sqrt{x^2 - 3x - 4}$$
?

b) sketch a curve which is monotonic increasing and concave up for all values of x.

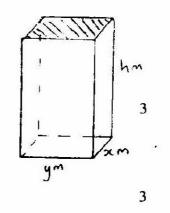
c) Prove:
$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta - \sin^2 \theta$$
 3

- d) A box with a rectangular base and height h metres is open at the top and has a volume of $32m^3$. The dimensions of the base of the box are x & x.
 - i) Show that the surface area (A) of the box is

$$A = xy + 64\left(\frac{1}{x} + \frac{1}{y}\right)$$

ii) Considering that for the surface area above y is constant and x is the variable, show that the minimum surface area of the

box is
$$A = \left(16\sqrt{y} + \frac{64}{y}\right)m^2$$
.



END OF PAPER

SOLUTIONS - YEAR II YEARLY EXAM &	2006
- 2	Q2. (unt.)
$\frac{11}{2}a - 3.53 \times 10^{-1}$	
$\int \cos 2\omega = -\cos 3\omega$	$\frac{1}{10} y = \frac{x}{x^2 + 4} v$
$=-\frac{J3'}{2}$	
	y' = (x'+4)(1) - x(2x)
$3\pi^{2} - 8\pi + 10 = 0$	$(x^2+4)^2$
$\Delta = b^2 - 4ac$	$-\frac{y'^2}{(x^2+4)^2} + \frac{4-x^2}{(x^2+4)^2}$
= 64 - 4(3)(10)	
= - 56 < 0 unreal roots.	b) $(x-4)^2 = 12(y+3)$
$\frac{1}{2} \frac{1}{2} \frac{1}$	$(y - 4)^{2} = 4(3)(y + 3)$
x+3 2n-6	i) -: Verkx (4,-3)
= (1 (x - 3)(x + 3))	ii) focus is (4,0)
n+3 2(2-3)	ii) directrix : y=-6
= .3	c) $2x^2 - 6x + 1 = 0$
) ta 8 x 2: (90-8)	$i) \propto +\beta = 3$
<u>5-9 x Coss</u>	ii) $\alpha \beta = \frac{1}{2}$
دورجه	iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2 - \beta$
= S.: 8	$= 3^2 - 2(\frac{1}{2})$
$\frac{1}{10} + n \theta = \frac{3}{10}$	= 8
.: 9 = 16°42'	0-12 9.20 11
bearing is 196° 42'T	Sin D = J3 180-60 + 60
· bearing is 196° 42' Τ σr S16° 42' ω	$S \stackrel{\frown}{i} \stackrel{O}{=} \frac{J3!}{2} \stackrel{180-634}{=} 4 \stackrel{60}{=} 60$
516°42'W	
5 S 16°42'W 1) 2x+5y=16 -0 x 5	9= 60°, 120°
$5 = \frac{5}{6} + \frac{2}{42} + \frac{5}{42} = \frac{16}{2} + \frac{5}{2} = \frac{16}{2} + \frac{5}{2} = -\frac{16}{2} + \frac{10}{2} = -4 - \frac{10}{2}$	$\frac{1}{2} \frac{9}{9} = 60^{\circ}, 120^{\circ}$ e) $4^{\pi} - 9(2^{\pi}) + 8 = 0$
$5 = 5 = 16 - 0 \times 5$ $10 \times -3y = -4 - 0$ $10 \times -3y = -4 - 0$ $10 \times +25y = 80 - 3$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\times} \ - \ 9(2^{\times}) \ + \ 8 = 0 \\ e + \ m = \ 2^{\times} \end{array}$
$5 = \frac{5}{42'} + $	$\frac{\partial f^{x}}{\partial f^{x}} = \frac{60^{\circ}}{120^{\circ}}$ e) $\frac{4^{x}}{-9(2^{x})} + 8 = 0$ let $m = 2^{x}$ $\therefore m^{2} - 9m + 8 = 0$
$5 = 16^{\circ} 42' W$ $3 = 16 - 0 \times 5$ $10x - 3y = -4 - 2$ $10x + 25y = 80 - 3$ $-28y = -84$ $28y = -84$	$\frac{1}{2} \frac{9 = 60^{\circ}, 120^{\circ}}{120^{\circ}}$ e) $\frac{4^{\pi} - 9(2^{\pi}) + 8 = 0}{120^{\circ}}$ let $m = 2^{\pi}$ $\frac{1}{2^{\pi}} \frac{m^{2} - 9m}{10^{\pi}} + 8 = 0$ $(m - 8)(m - 1) = 0$
$5 6^{\circ} 42' W$ $3 2\pi + 5y = 16 - 0 \times 5$ $10\pi - 3y = -4 - 2$ $10\pi + 25y = 80 - 3$ $-28y = -84$ $3 + 3 - 28 + 3 - 3$ $2\pi + 15 = 16$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\varkappa} \ - \ 9(2^{\varkappa}) \ + \ 8 = 0 \\ et \ m = \ 2^{\varkappa} \\ \vdots \ m^{2} \ - \ 9m \ + \ 8 = 0 \\ (m - \ 8)(m - 1) = 0 \\ \vdots \ m = \ 8, \ m = 1 \end{array}$
$5 6^{\circ} 42' W$ $3 2\pi + 5y = 16 - 0 \times 5$ $10\pi - 3y = -4 - 2$ $10\pi + 25y = 80 - 3$ $-28y = -84$ $3 + 3 - 28 + 3 - 3$ $2\pi + 15 = 16$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\varkappa} \ - \ 9(2^{\varkappa}) \ + \ 8 = 0 \\ et \ m = \ 2^{\varkappa} \\ \vdots \ m^{2} \ - \ 9m \ + \ 8 = 0 \\ (m - \ 8)(m - 1) = 0 \\ \vdots \ m = \ 8, \ m = 1 \end{array}$
$\sum_{x = 1}^{3} \frac{5}{6} \frac{42'}{42'} = \frac{16}{2} \frac{-0}{5} \times 5$ $\frac{10x - 3y}{10x - 3y} = -4 - 2$ $\frac{10x - 3y}{10x + 25y} = \frac{80}{-28} - 2$ $\frac{-28y}{-28y} = -84$ $\frac{-28y}{-28} = -84$ $\frac{-3}{2x + 15} = \frac{16}{-3}$ $\frac{-3x}{2x + 15} = \frac{16}{2}$ $\frac{-3x}{2x + 15} = \frac{16}{2}$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e + \ m = 2^{\pi} \\ \vdots \ m^{2} - 9m + 8 = 0 \\ (m - 8)(m - 1) = 0 \\ \vdots \ M = 8, \ M = 1 \\ b = 1 \\ b = 1 \\ b = 1 \\ \vdots \ 2^{\pi} = 8 \\ \vdots \ 2^{\pi} = 1 \end{array}$
$r = \frac{5}{6} \frac{42}{42} \frac{10}{10} \times \frac{5}{10} \times \frac{5}{10}$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\varkappa} \ - \ 9(2^{\varkappa}) \ + \ 8 = 0 \\ e \ M = \ 2^{\varkappa} \\ \vdots \ m^{2} \ - \ 9m \ + \ 8 = 0 \\ (m - \ 8)(m - 1) = 0 \\ \vdots \ m = \ 8, \ m = 1 \\ b \ m = \ 2^{\varkappa} \\ \vdots \ 2^{\varkappa} = \ 8, \ 2^{\varkappa} = 1 \\ \vdots \ \chi = \ 3, \ \chi = 0. \end{array}$
$\int 2\pi + 5y = 16 - 0 \times 5$ $\frac{10x - 3y}{10x + 25y} = -4 - 2$ $\frac{10x - 3y}{10x + 25y} = 80 - 3$ $-28y = -84$ $\therefore y = 3.$ $2x + 15 = 16$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}, y = 3.$ $\frac{12}{2} \text{ (a)} \text{ (b)} y = x^{4} - 3x^{3} + 12x$ $y' = 4x^{3} - 9x^{2} + 12$	$\begin{array}{c} \vdots \ \vartheta = \ 60^{\circ}, \ 120^{\circ} \\ e) \ 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ et \ m = 2^{\pi} \\ \vdots \ m^{2} - 9m + 8 = 0 \\ (m - 8)(m - 1) = 0 \\ \vdots \ m = 8, \ m = 1 \\ b = 1 \\ b = 1 \\ b = 1 \\ \vdots \ m = 2^{\pi} \\ \vdots \ 2^{\pi} = 8 \\ z^{\pi} = 1 \\ \vdots \ n = 3 \\ x = 0. \\ \hline Q3. \ f''(\pi) = 6\pi - 8 \\ f'(2) = 0 \ f(-1) = 3 \\ \end{array}$
$y' = 4x^{3} - 9x^{2} + 12$ $y = 16 - 0 \times 5$ $10x - 3y = -4 - 2$ $-28y = -84$	$ \begin{array}{c} \vdots & \vartheta = 60^{\circ}, 120^{\circ} \\ e) & 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e & 164 \\ e & 12^{\pi} \\ \vdots & m^{2} - 9m + 8 = 0 \\ (m - 8)(m - 1) = 0 \\ \vdots & m = 8, m = 1 \\ b & 16m = 2^{\pi} \\ \vdots & 2^{\pi} = 8, 2^{\pi} = 1 \\ \vdots & 2^{\pi} = 1, 2^{\pi} = 1 \\ \vdots & 2^{\pi} = 1, 2^{\pi} = 1, 2^{\pi} = 1 \\ \vdots & 2^{\pi} = 1, 2^{\pi} =$
$\int 316^{\circ} 42' W$ $\int 2\pi + 5y = 16 - 0 \times 5$ $\frac{10\pi - 3y = -4 - 2}{10\pi + 25y = 80}$ $\int -28y = -84$ $\therefore y = 3$ $2\pi + 15 = 16$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $\therefore y = 3$ $\frac{12}{2} \text{ a) i) y = \pi^{4} - 3x^{3} + 12x$ $y' = 4\pi^{3} - 9\pi^{2} + 12$ $y = (3\pi + 5)(8 - 2\pi)$ $y' = (8 - 2\pi)(3) + (3\pi + 5)(-2)$	$ \begin{array}{c} \vdots & \vartheta = 60^{\circ}, 120^{\circ} \\ e) & 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e & 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e & 16^{\pi} - 9m + 8 = 0 \\ (m - 8)(m - 1) = 0 \\ \vdots & m = 8, m = 1 \\ b & 16^{\pi} - 8, m = 1 \\ b & 16^{\pi} - 8, m = 1 \\ \vdots & m = 2^{\pi} \\ \vdots & 2^{\pi} = 8, 2^{\pi} = 1 \\ \vdots & 2^{\pi} = 1, 2^{\pi} = 1 \\ \vdots & 2^{\pi} = 1, 2^{\pi}$
$y' = 4x^{2} - 9x^{2} + 12x^{2} + 1$	$ \begin{array}{c} \vdots & \vartheta = 60^{\circ}, 120^{\circ} \\ e) & 4^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e & 16^{\pi} - 9(2^{\pi}) + 8 = 0 \\ e & 16^{\pi} - 9m + 8 = 0 \\ (m - 8)(m - 1) = 0 \\ \vdots & m = 8, m = 1 \\ b & 16^{\pi} - 8, m = 1 \\ b & 16^{\pi} - 8, m = 1 \\ \vdots & 16^{\pi} - 8, m = 1, m = 1 \\ \vdots & 16^{\pi} - 8, m = 1, $
$y = \frac{516^{\circ} 42' W}{1}$ $y = \frac{5}{10x - 3y} = -4\frac{3}{2}$ $10x - 3y = -4\frac{3}{2}$ $-\frac{3}{2}x + 25 - \frac{3}{2}x - 3$	$ \frac{\partial f^{n}}{\partial f'} = 60^{\circ}, 120^{\circ} $ e) $4^{n} - 9(2^{n}) + 8 = 0$ $ c+m=2^{n}$ $\therefore m^{2} - 9m + 8 = 0$ (m-8)(m-1) = 0 $\therefore m=8, m=1$ $b_{n}t = 2^{n}$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 3, x = 0.$ $\underline{Q3}. f''(x) = 6x - 8, f'(2) = 0, f(-1) = 3$ $f'(x) = 3x^{2} - 8x + C,$ $0 = 3(2)^{2} - 8(2) + C,$ $\therefore c_{1} = 4$ $\therefore f'(x) = 3x^{2} - 8x + 4$
$y = \frac{5}{6} + \frac{2}{42} + \frac{5}{42} + \frac{5}{4$	$ \frac{\partial f^{n}}{\partial t} = 60^{\circ}, 120^{\circ} $ e) $4^{n} - 9(2^{n}) + 8 = 0$ $ e+ m = 2^{n}$ $\therefore m^{2} - 9m + 8 = 0$ (m - 8)(m - 1) = 0 $\therefore m = 8, m = 1$ $b_{n} + m = 2^{n}$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 1, 2^{n} = 4, 2^{n} = $
$y = \frac{5}{6} + \frac{2}{42} + \frac{5}{42} + \frac{5}{4$	$ \frac{\partial f^{n}}{\partial t} = 60^{\circ}, 120^{\circ} $ e) $4^{n} - 9(2^{n}) + 8 = 0$ $ e+ m = 2^{n}$ $\therefore m^{2} - 9m + 8 = 0$ (m - 8)(m - 1) = 0 $\therefore m = 8, m = 1$ $b_{n} + m = 2^{n}$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 1, 2^{n} = 4, 2^{n} = $
$y = \frac{5}{6} + \frac{2}{42} + \frac{5}{42} + \frac{5}{4$	$ \frac{\partial f^{n}}{\partial t} = 60^{\circ}, 120^{\circ} $ e) $4^{n} - 9(2^{n}) + 8 = 0$ $ e+ m = 2^{n}$ $\therefore m^{2} - 9m + 8 = 0$ (m - 8)(m - 1) = 0 $\therefore m = 8, m = 1$ $b_{n} + m = 2^{n}$ $\therefore 2^{n} = 8, 2^{n} = 1$ $\therefore 2^{n} = 1$

23 (cont).) S(5,-3), directrix is x= 9. x=9 -3 S vertex is (7,-3) a = 2. $(y+3)^2 = -8(x-7)$ $) y = \frac{1}{3} \chi^{3} - \chi^{2} - 8 \chi$ -6 y-intercept)) is -6 y = 2 - 22 - 8 = 0 for stat pts. (n-4)(n+2)=0, x=-2 = 4 y=-32 3 , y= 33 . Stat pt3 are $(4, -32\frac{2}{3}) \neq (-2, 3\frac{1}{3})$ y"= 2x-2 at $(4, -32\frac{2}{3})$ + $(-2, 3\frac{1}{3})$ at x=4, y"=670 Minimum N=-2 4" at i) For pts of inflexion y'=0 2x - 2 = 00 y=-chaye in a (-2,3\$) $\frac{x-1}{y=-14\frac{2}{3}}$ -210 · (1,-1+=) · a pt of) 1 5 z 3 - 2 --3 :2: - 10 - 15 - 20 - 25 -30 (4,-323) -35

14 y= 257-x e) Centre (0,0) r=2 $d = \frac{5(0) - 12(0) + 26}{5^2 + 12^2}$ 2 $d = \frac{26}{13} = 2.$ Ronge : all real y where Osys6.) $\alpha x = \frac{3}{x-z}$ since the perp. dist = the radius o circle the live must be a t ax(x-2)=3 $f(x) = x^3 - 7x^2 + 1$ $f'(x) = 3x^2 - 14x$ $an^2 - 2an - 3 = 0$ For year to be a tangent to the f" (2) = 62 - 14 <0 for concave d hyperpola there must only be 6x <14 . 'x < + one solution ie A=o. 62-4ac =0 Q5 a) x2-2x-420 $4a^2 - 4(a)(-3) = 0$ (x-4)(x+1)20 $4a^2 + 12a = 0$. . x=4 , x=-1 4a(a+3)=0) y= (Jx1+1)" y'= 100 (Jx+1) x 1/2 x 2 . Domain is : Sx & TR where x6-1, x3 6) 50 (52 +1) $1)if(x) = 2x^2 + x$ $f(x+h) = 2(x+h)^2 + (x+h)$ f'(n)= 1im 2(x+L)2+(x+L)-2x2-x h-70 = 1- 2xt + 4xh + 2h2 + 2h + h - 2h2 - xt h+0 = 1i K(4n+2h+1) c) LHS= 1-tan 2 4+0 1+ ta 20 ... - f'(x) = 4x + 1= 1 - tan20) Show that a(1+f'(a))=2f(a)Seco LHS = A(1 + 4A + 1)= Cos2 0 - Si2 0 x Cos $=2a + 4a^{2}$ Cos2 0 $= 2(2a^2+9)$ = Cos 2 9 - Si 2 9 = 2f(a)= RH3 - . LHS = RHS . . LHS = RHS QED!

25 (cont.)i) SA = 2xh + 2yh + xy V = xyh V = 32 : 32 = xyh 32 h = $\frac{SA = 2x \left(\frac{32}{xy}\right) + 2y \left(\frac{32}{xy}\right) + xy}{xy}$ $\frac{SA = 64 + 4}{9}$ <u>64</u> 2 $\frac{1}{x} + \frac{1}{y} QED!$ $\frac{1}{x^{-1}} + \frac{1}{y} \qquad (3)$ $\frac{2}{(keeping y constant)}$ $SA = \chi y + 64$) SA'= y -64 x -2 = y - 64 = 0 for stat. pts $\frac{64}{\chi^2} = y$ $\chi = \frac{8}{\sqrt{4}}$ (since x>0 Test for a スレデシュ 10 $\frac{F}{SA''} = \frac{128}{x^3}$DR 0.24 when $\chi = \frac{\theta}{\sqrt{y'}}$ x>= A'>0 Minima Surface Aven SA = 8 . y $(\frac{4}{8} + \frac{1}{4})$ = 859 <u>64</u> 4 SA = (16 Jy 64) m2 is the minimum