Solutions

YEAR 11 PRELIMINARY EXAMINATION 2000

MATHEMATICS

2 UNIT

Time Allowed – 85 minutes

All questions may be attempted

All questions are of equal value

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

The answers to all questions are to be returned in separate bundles clearly labelled Section A, Section B, etc. Each bundle must show your Candidate's Number.

SECTION A: **START A NEW PAGE**

(i) Evaluate $6x^{-2}$ when $x = 3$.

(ii) Simplify $\log_2 40 - \log_2 5$.

(iii) Solve $4x^2 - 12x = 0$.

(iv) A circle has area 500 m^2. Find its circumference correct to the nearest meter.

(v) Given that $\frac{8}{3-\sqrt{5}}$ can be written in the form $p + q\sqrt{5}$, find the values of p and q.

SECTION B: **START A NEW PAGE**

(i) Differentiate with respect to x:

(a) $5x^2 + 4x\sqrt{x}$.

(b) $\cos 4x$.

(c) $x^2 e^{3x}$

(ii) (a) Express the equation of the parabola $y = \frac{1}{8}x^2 - x - 3$ in the form $(x - x_o)^2 = 4a(y - y_o)$, where a, x_o and y_o are constants.

(b) Hence write down the coordinates of the vertex and focus of the parabola $y = \frac{1}{8}x^2 - x - 3$.

SECTION C: **START A NEW PAGE**

(i) A chord PQ of length 10cm is drawn in a circle of radius 8cm. If the chord PQ subtends an angle α at the centre, find

(a) the value of $\cos\alpha$.

(b) the area of the minor segment cut off by the chord PQ. (Give your answer to the nearest integer)

(ii) (a) Find the equation of the normal to the curve $y = \frac{x}{x-2}$ at the origin.

(b) Given that the normal intersects the curve again, find the coordinates of this point.

SECTION D: **START A NEW PAGE**

(i) An arithmetic sequence has first two terms 7 and 10.

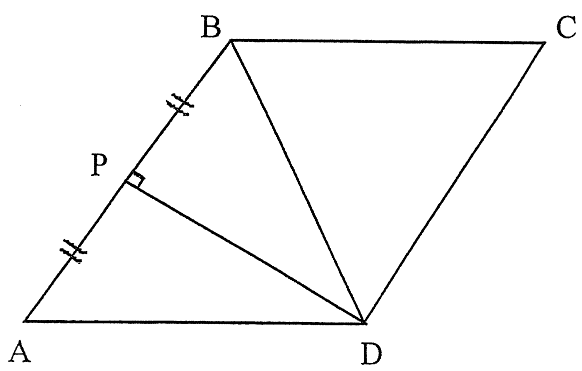
(a) Find an expression for the sum (S_n) of the first n terms of this sequence.

(b) Find the number of terms needed to make the sum of the first n terms equal to 527.

(ii) ABCD is a rhombus and P lies on AB so that DP is the perpendicular bisector of AB.

(a) Prove that ΔAPD and ΔBPD are congruent.

(b) Find the size of $\angle BAD$.



SECTION E: START A NEW PAGE

(i) Find all the values of β for which $2\cos\beta + \sqrt{3} = 0$ for $0 \le \beta \le 2\pi$.

(ii) Given the function $f(x) = 2x\sqrt{x+6}$:

(a) Write down the domain of $y = f(x)$.

(b) Find the coordinates of all stationary points on $y = f(x)$ and determine their nature.

(c) Discuss the gradient of $y = f(x)$ when $x = -6$.

(d) Sketch $y = f(x)$ for $x \le 3$.

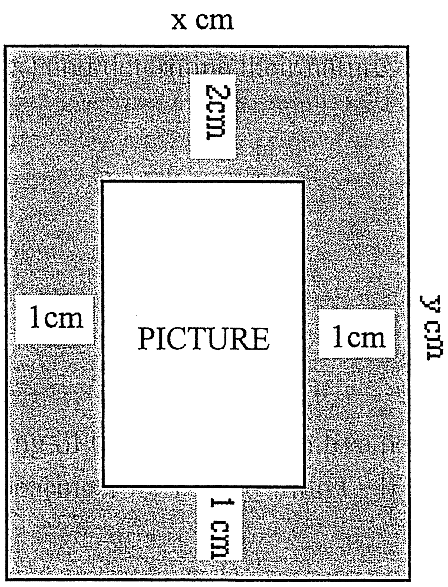
SECTION F: START A NEW PAGE

(i) An ultralight plane is flown from an airport A on a bearing of 030^{o}T for 150km to a position B. From position B the ultralight is then flown on a new course bearing 135^{o}T to position C. If the bearing of position C from the airport is 075^{o}T,

(a) Draw a diagram to illustrate the above information.

(b) Find the exact distance from B to C.

(ii) A rectangular sheet of blue cardboard has area $600cm^2$. A rectangular picture is to be pasted onto the cardboard sheet so that there is a blue border of cardboard surrounding the picture. The border above the picture is 2cm wide and the other three borders are 1cm wide. (see diagram)



(a) Show that the area ($A\ cm^2$) of the picture is given by

$$A = 606 - 3x - \frac{1200}{x}$$

where x cm is the width of the cardboard.

(b) Find the dimensions of the picture that has the greatest area.

THIS IS THE END OF THE EXAMINATION PAPER

JRAHS YR11 PRELIMINARY 2UNIT 2000

SECTION A

(i) $6/x^2 = 6/9$

$= 2/3$

(ii) $\log_2(40/5) = \log_2 8$

$= 3$

(iii) $4x(x-3) = 0$

$x = 0, 3$

(iv) $A = \pi r^2$

$r = \sqrt{\frac{A}{\pi}} \quad (r>0)$

$(\doteqdot 12.61567)$

$C = 2\pi r$

$= 2\pi . \sqrt{\frac{A}{\pi}}$

$= 79.27$

$= 79$ m (to nearest m)

(v) $\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{8(3+\sqrt{5})}{9-5}$

$= 2(3+\sqrt{5})$

$= 6 + 2\sqrt{5}$

$p = 6 \quad q = 2$

SECTION B

(i)(a) $y = 5x^2 + 4x^{1\frac{1}{2}}$

$y' = 10x + 6\sqrt{x}$

(b) $y' = -4\sin 4x$

(c) $y' = (2x)(e^{3x}) + (x^2)(3e^{3x})$

$= 2xe^{3x} + 3x^2e^{3x}$

(ii)(a) $8y = x^2 - 8x - 24$

$x^2 - 8x + 16 = 8y + 40$

$(x-4)^2 = 8(y+5)$

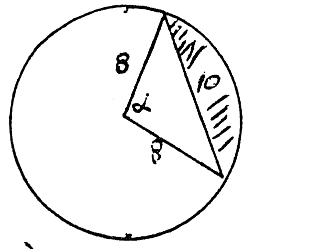
(b) focal length $= 2$

vertex $(4, -5)$ focus $(4, -3)$

SECTION C

(i)(a) $\cos\alpha = \frac{8^2 + 8^2 - 10^2}{2(8)(8)}$

$= \frac{7}{32}$



(b) $A = \frac{1}{2}r^2(\alpha - \sin\alpha) \qquad \alpha = \cos^{-1}\left(\frac{7}{32}\right)$

$= \frac{1}{2}(8)^2(1.35026 - \sin 1.35026) \qquad \doteqdot 1.35026$

$= 11.9834$

$= 12$ cm^2 (to nearest integer)

(ii)(a) $\frac{dy}{dx} = \frac{(x-2)(1) - (1)(x)}{(x-2)^2}$

$= \frac{-2}{(x-2)^2}$

at $x = 0$, $y' = -2/4$

$= -1/2$

$\therefore$ slope of normal $= 2$

when $x = 0$, $y = 0$

normal is $y - 0 = 2(x - 0)$

$y = 2x$

(b) $y = 2x$ & $y = \frac{x}{x-2}$

meet when $2x = \frac{x}{x-2}$

$2x^2 - 4x = x$

$2x^2 - 5x = 0$

$x(2x - 5) = 0$

$\therefore x = 0, 2\frac{1}{2}$

$x = 2\frac{1}{2}$, $y = 5$

$\therefore$ pt is $(2\frac{1}{2}, 5)$

SECTION D

(i)(a) $a = 7 \quad d = 3$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$= \frac{n}{2}(14 + 3(n-1))$$
$$= \frac{n}{2}(3n + 11)$$

(b) $\frac{n}{2}(3n+11) = 527$

$$3n^2 + 11n = 1054.$$
$$3n^2 + 11n - 1054 = 0$$
$$n = \frac{-11 \pm \sqrt{12769}}{6}$$
$$= \frac{-11 \pm 113}{6}$$
$$= 17, -20\tfrac{2}{3}$$

but $n > 0 \quad \therefore n = 17.$

$\therefore$ no. terms $= 17$

(ii)(a) In $\triangle APD$ & $\triangle BPD$

$AP = BP$ (DP is bisector of AB)

$A\hat{P}D = B\hat{P}D$ (both 90°, $AB \perp PD$)

$PD = PD$ (common)

$\therefore \triangle APD \equiv \triangle BPD$ (SAS)

(b) let $\hat{A} = \theta$

$\therefore A\hat{B}D = \theta$ (corresponding angles in congruent Δ's)

$C\hat{B}D = \theta$ (diagonals of rhombus bisect opposite angles)

$AD \parallel BC$ (opposite sides of rhombus are parallel)

$3\theta = 180$ ($AD \parallel BC$ cointerior angles are supplementary)

$\theta = 60$

$\therefore B\hat{A}D = 60°$

SECTION E

(i) $\cos\beta = -\sqrt{3}/2$

$$\beta = \pi - \pi/6, \ \pi + \pi/6$$
$$= \frac{5\pi}{6}, \frac{7\pi}{6}$$

(ii)(a) $x + 6 \geq 0$

$x \geq -6$

(b) $y = 2x(x+6)^{1/2}$

$$y' = 2(x+6)^{\frac{1}{2}} + (2x)\tfrac{1}{2}(x+6)^{-\frac{1}{2}}$$
$$= 2\sqrt{x+6} + \frac{x}{\sqrt{x+6}}$$

$$y' = \frac{3x+12}{\sqrt{x+6}}$$

for stat. pt $y' = 0$

$$\frac{3x+12}{\sqrt{x+6}} = 0$$
$$x = -4$$

& $y = 2(-4)\sqrt{-4+6}$

$= -8\sqrt{2}$

stat pt $(-4, -8\sqrt{2})$

test nature

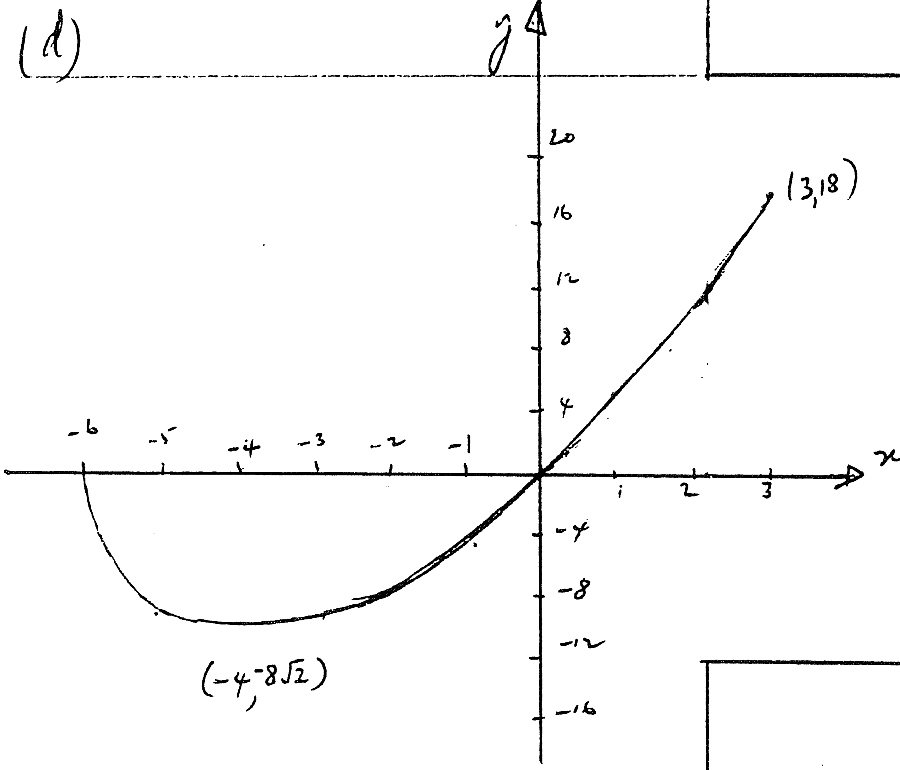
x	-5	-4	-3
y'	-3 <0	0	$3/\sqrt{3}$ >0

Since curve is continuous for $x \geq -6$ & change in gradient from <0 to >0 then local min tp.

(c) when $x = -6$, y' is undefined $\therefore$ tangent is vertical.

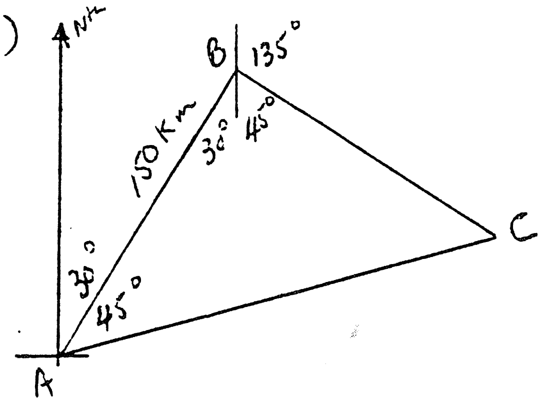
(d) (PTO)

(d)



SECTION F

(i) (a)



(b) $\hat{C} = 60°$

$$\frac{BC}{\sin 45°} = \frac{150}{\sin 60°}$$

$$BC = \frac{150 \sin 45°}{\sin 60°}$$

$$= 150 \times \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}} \text{ Km}$$

$$= \frac{300}{\sqrt{6}} \text{ Km}$$

$$= 50\sqrt{6} \text{ Km}$$

(ii) (a) $A = (x-2)(y-3)$

but $xy = 600$

$$\therefore A = (x-2)\left(\frac{600}{x} - 3\right)$$

$$= 600 - 3x - \frac{1200}{x} + 6$$

$$A = 606 - 3x - \frac{1200}{x}$$

(b) $A = 606 - 3x - 1200x^{-1}$

$$\frac{dA}{dx} = -3 + 1200x^{-2}$$

for stat pt $\frac{dA}{dx} = 0$

$$\frac{1200}{x^2} = 3$$

$$x^2 = 400$$

$$x = 20 \quad (x > 0)$$

$$\frac{d^2A}{dx^2} = -2400x^{-3}$$

when $x = 20$, $\frac{d^2A}{dx^2} = \frac{-2400}{20^3}$

< 0 ∩

∴ local max. tp

Since function is continuous for $x > 0$ & there is only one tp. which is a local max. tp then it is the abs. max. tp.

when $x = 20$, $y = 30$

dimensions are $(x-2)$ by $(y-3)$

ie. 18 cm × 27 cm.