YEAR 11 PRELIMINARY EXAMINATION 2003

MATHEMATICS

Time Allowed – 85 minutes

All questions may be attempted

All questions are of equal value

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each bundle must show your Candidate's Number.

Question 1 (Start a new page)

a) Differentiate f(x) with respect to x, then simplify:

Marks

i) $f(x) = \tan(x^2)$ 2

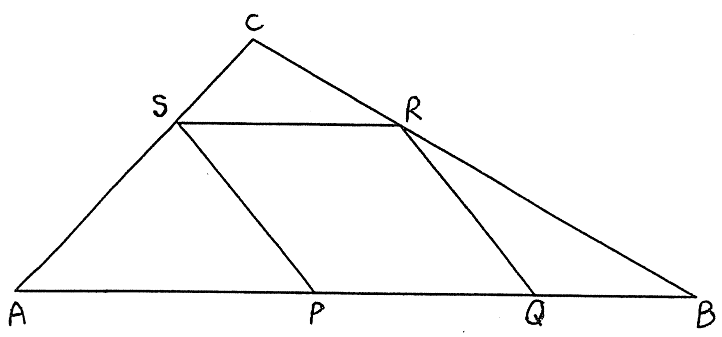
ii) $f(x) = \dfrac{3x-7}{4x+5}$ 2

iii) $f(x) = \ln(e^{3x} + \sin 2x)$ 3

b) Evaluate the following limit:

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$$ 2

c)



Not to scale

In ΔABC, AP = PQ = QB, and PQRS is a rhombus.

Copy the diagram onto your working paper showing the information given.

i) Show that ΔRQB is isosceles 1

ii) Show that $\angle$RQP is double the $\angle$RBQ 2

iii) Hence prove that $\angle$ACB is a right angle 3

Question 2 (Start a new page)

a) Evaluate $\log_{\sqrt{3}} 243$ **2**

b) Solve for x :

i) $\log 4 + \log 8 + \log 16 + \log 32 = x \log 2$ **2**

ii) $3^{2x+1} - 26(3^x) - 9 = 0$ **3**

c) If $\cos\theta = k$ and θ is in the first quadrant, express in terms of k the value of

i) $\sin\theta$ **1**

ii) $\tan(180° + \theta)$ **1**

d) Find the equation of the line perpendicular to $4x - 5y - 6 = 0$ passing through (2, -2) **2**

e) Find the length of the radius R if $y = 4x - 5$ is a tangent to $(x-1)^2 + (y+2)^2 = R^2$ **2**

f) Prove $\dfrac{\tan\theta}{\sec\theta - 1} - \dfrac{\tan\theta}{\sec\theta + 1} = 2\cot\theta$ **2**

Question 3 (Start a new page)

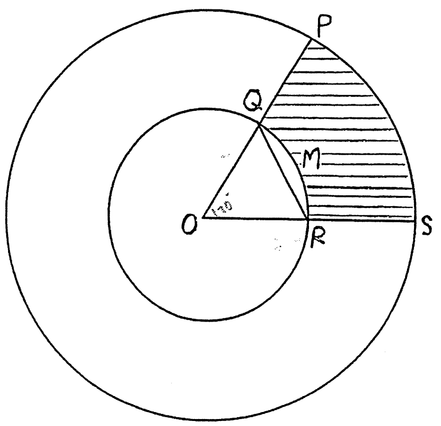
a) If α and β are the roots of the equations $x^2 - 4x - 6 = 0$ find the values of $\alpha^2 + \beta^2$ **2**

b) OPS and OQR are sectors of two concentric circles centre O as shown with $\angle QOR = 70°$, $OQ = 1.5$ cm and $OS = 2.4$ cm. Find to 2 decimal places:

i) the area of the shaded region PQRS **2**

ii) perimeter of the shaded region PQRS **2**

iii) area of the minor segment RMQ **2**



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c) Find the equation of the locus of a point equidistant from the point (4, 3) and the line $y = -1$. 2
Sketch the graph showing all the essential features. 2

d) If $\sqrt{2x+5} + \sqrt{2x+3} = 6$ and $(2x+5) - (2x+3) = 2$

i) Show $\sqrt{2x+5} - \sqrt{2x+3} = \frac{1}{3}$ 1

ii) Using (i) or otherwise solve $\sqrt{2x+5} + \sqrt{2x+3} = 6$ 2

Question 4 (Start a new page)

a) Solve : $|2x-1| = 3$ 2

b) Sketch the graph of $y = \frac{x-2}{x+1}$ 2

c) Consider $f(x) = \sqrt{25-x^2}$

i) Find $f'(x)$ 1

ii) At what point of $f(x)$ is the gradient $\frac{3}{4}$? 2

iii) Find the equation of the tangent at this point. 2

d) i) Find the equation of the tangent to $y = \sin x$ at $x = 0$ 2

ii) Find the coordinates of the turning points of $y = \cos x + \frac{x^2}{2} - 1$ 4
and determine their nature.

END

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1 (a) (i) $f'(x) = 2x \sec^2 x^2$

(ii) $f'(x) = \dfrac{(4x+5)3 - (3x-7).4}{(4x+5)^2}$

$= \dfrac{12x+15-12x+28}{(4x+5)^2}$

$= \dfrac{43}{(4x+5)^2}$ ②

(iii) $f'(x) = \dfrac{3e^{3x} + 2\cos 2x}{e^{3x} + \sin 2x}$ ③

(b) $\lim\limits_{x \to 2} \dfrac{x^2+x-6}{x^2-4} = \lim\limits_{x \to 2} \dfrac{(x+3)(x-2)}{(x-2)(x+2)}$ ②

$= \lim\limits_{x \to 2} \dfrac{x+3}{x+2}$

$= \dfrac{5}{4}$

(c) (i) — Diagram

(ii) $RQ = PQ$ (sides rhombus PQRS

But $PQ = BQ$

$\therefore RQ = QB$ ①

$\therefore \triangle RQB$ is isosceles (two equal sides)

(iii) $\therefore \angle QRB = \angle RBQ$ (equal angles opposite equal sides) ①

$\therefore \angle RQP = \angle QRB + \angle RBQ$ (Exterior angle $\triangle RBQ$ is equal to the sum of the two interior opposite angles) ①

$= 2\angle RBQ$

(iv) $\angle SPA = \angle RQP$ (corresponding angles in parallel lines $RQ \parallel SP$ from rhombus PQRS ③

follow similar proof in Part (ii) $SP = AP$

$\angle SAP = \angle PSA$ (equal angles opposite equal sides $SP = AP$)

$\therefore \angle SAP = \dfrac{180^\circ - 2\angle RBQ}{2}$ (Angle sum $\triangle ASP$)

$= 90^\circ - \angle RBQ$

$\therefore \angle RBQ + \angle SAP = 90^\circ$

$\therefore \angle ACB = 90^\circ$ (Angle sum $\triangle CAB$)

Qu 2

a) $\log_{\sqrt{3}} 243 = \log_{\sqrt{3}} (\sqrt{3})^{10}$

$= 10$ ②

b) (i) $\log 4 + \log 8 + \log 16 + \log 32 = x \log 2$

$2\log 2 + 3\log 2 + 4\log 2 + 5\log 2 = x\log 2$ ②

$x = 14$

(ii) $3^{2x+1} - 26(3^x) - 9 = 0$

$3(3^{2x}) - 26(3^x) - 9 = 0$

Let $u = 3^x$

$3u^2 - 26u - 9 = 0$

$(3u+1)(u-9) = 0$ ①

$\therefore 3^x = 9$ or $3^x = 3^{-1}$ ①

$\therefore x = 2$ or $x = -1$ ①

c) $\cos\theta = k$

(i) $\therefore \sin\theta = \sqrt{1-k^2}$ $\quad 0 < \theta < 90^\circ$ ①

(ii) $\tan(180^\circ + \theta) = \tan\theta$ ①

$= \dfrac{\sqrt{1-k^2}}{k}$

d) $5x + 4y = 5 \times 2 + 4 \times -2$ ②

$5x + 4y - 2 = 0$

e) $R = \left| \dfrac{4x - y - 5}{\sqrt{16+1}} \right|$ $\quad (2, -2)$

$= \left| \dfrac{8 + 2 - 5}{\sqrt{17}} \right|$

$= \dfrac{5}{\sqrt{17}}$ units ②

(iv) $LHS = \dfrac{\tan\theta}{\sec\theta - 1} - \dfrac{\tan\theta}{\sec\theta + 1}$

$$= \frac{\tan\theta[\sec\theta + 1] - \tan\theta(\sec\theta - 1)}{\sec^2\theta - 1}$$

$$= \frac{2\tan\theta}{1 + \tan^2\theta - 1}$$

$$= \frac{2\tan\theta}{\tan^2\theta}$$

$$= \frac{2}{\tan\theta}$$

$$= 2\cot\theta$$

2

$\therefore LHS = RHS.$

3 (a) $x^2 - 4x - 6 = 0$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4^2 - 2\times(-6)$$

$$= 28$$

2

(b) (i) $A = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$ $\qquad \theta = \frac{70\pi}{180^\circ}$ rad.

$$= \frac{1}{2}(2.4^2 - 1.5^2)\,\frac{7\pi}{18}$$

$$= 2.14\ cm^2$$

2

(ii) $RQ = \frac{70}{360}\times 2\pi \times 1.5$ $\qquad PS = \frac{70}{360}\times 2\pi\times 2.4$

$= 1.83\ cm$ $\qquad = 2.93\ cm$

Permeter segment ORQ $= 1.83 + 2.93 + (2.4 - 1.5)\times 2$

$= 6.56\ cm$

2

(iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$$= \frac{1}{2}\cdot 1.5^2\left(\frac{7\pi}{18} - \sin\left(\frac{7\pi}{18}\right)\right)$$

$$= 0.32\ cm^2$$

2

e) $PS = PM$

$PS^2 = PM^2$

$(x-4)^2 + (y-3)^2 = (y - -1)^2$ ①

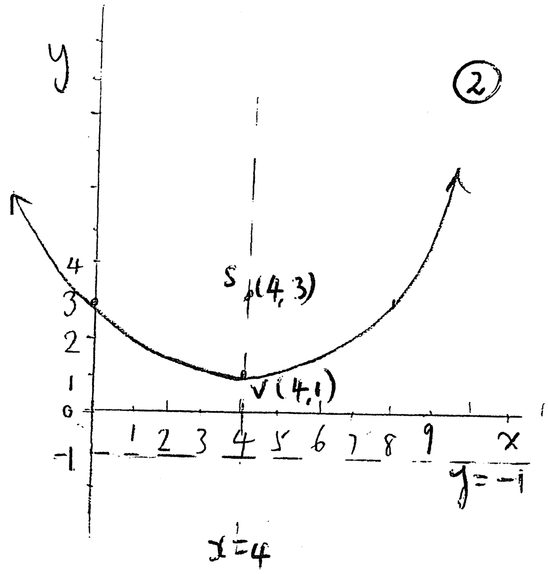
$(x-4)^2 + y^2 - 6y + 9 = (y+1)^2$

$(x-4)^2 + \cancel{y^2} - 6y + 9 = \cancel{y^2} + 2y + 1$

$(x-4)^2 + 9 = 8y + 1$

$(x-4)^2 = 8y - 8$

$(x-4)^2 = 8(y-1)$ ①



Locus is a parabola with vertex $(4,1)$ directrix at $y=-1$ focus at $(4,3)$

d i) $(2x+5) - (2x+3) = 2$

$(\sqrt{2x+5} - \sqrt{2x+3})(\sqrt{2x+5} + \sqrt{2x+3}) = 2$

$(\sqrt{2x+5} - \sqrt{2x+3}) \times 6 = 2$

$\therefore \sqrt{2x+5} - \sqrt{2x+3} = \frac{1}{3}$ ①

$\sqrt{2x+5} + \sqrt{2x+3} = 6$

ii) $\therefore 2\sqrt{2x+5} = \frac{19}{6}$

$\sqrt{2x+5} = \frac{19}{6}$

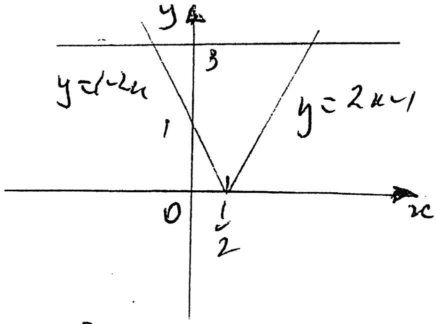
$2x+5 = \frac{361}{36}$

$2x = \frac{181}{36}$

$x = \frac{181}{72} \sim 2\frac{37}{72}$ ②

4

(a)

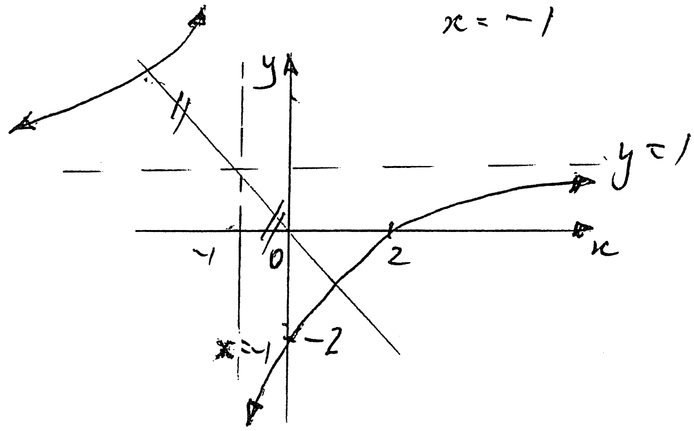


$\therefore \quad 2x-1 = 3$

$x = 2$ OR $1-2x = 3$

$x = -1$ (2)

(b)



(2)

(c) $f(x) = \sqrt{25-x^2}$

(i) $f'(x) = \dfrac{-x}{\sqrt{25-x^2}}$ (1)

(ii) $\therefore \dfrac{-x}{\sqrt{25-x^2}} = \dfrac{3}{4}$. $x < 0$.

$16x^2 = 9(25-x^2)$

$25x^2 = 9 \times 25$

$x = -3$.

$y = 4 \Rightarrow$ Point $(-3, 4)$ (2)

(iii) Eqn Tangent $y - 4 = \frac{3}{4}(x+3)$

$3x - 4y + 25 = 0$ (2)

(d) (i) $y = \sin x$

$y' = \cos x$

At $x = 0$ $y' = 1$ $y = 0$

$\therefore$ Eqn Tangent $y = x$ (2)

(ii) $y = \cos x + \frac{x^2}{2} - 1$

$y' = -\sin x + x$ ①

$y'' = -\cos x + 1$

For turning points

$y' = 0$

$-\sin x + x = 0$

$\sin x = x$

$x = 0$ only. from part (i) as $y = x$ is tangent to $y = \sin x$. ①

$y = 1 - 1$

$= 0$

$\Rightarrow$ turning point $(0, 0)$

Test gradients for nature of turning point as y, y' are continuous for all x.

x	-0.1	0	0.1
y'	-1.7×10^{-4}	0	1.7×10^{-4}
	\	—	/

①

$\therefore$ relative minimum at $(0, 0)$

①