JAMES RUSE AGRICULTURAL HIGH SCHOOL
YEAR 11 MATHEMATICS (2 UNIT)
PRELIMINARY EXAMINATION 2005

QUESTION 1 — Marks

(a) Solve for k: $\left(4^2\right)^{3k} = 1$ — 1

(b) Simplify $\log_4 128$ — 2

(c) Factorise and simplify: $\dfrac{x^3 - 27}{x^2 - 9}$ — 2

(d) Solve for x: $\dfrac{3x-5}{2} - \dfrac{9x-1}{5} < -6$ — 2

(e) Simplify $\dfrac{\log_7 8}{\log_7 2}$ — 2

(f) Rationalize the denominator: $\dfrac{4\sqrt{2}-3}{5+\sqrt{2}}$ — 2

(g) Simplify $\sqrt{(x+1)^2}$ — 1

(h) Solve for x: $|5x-2| = 6x - 12$ — 2

(i) Simplify: $\tan(90° + \theta)$ — 1

QUESTION 2 (START A NEW PAGE) — Marks

(a) Differentiate with respect to x:

(i) $9x - x^2\sqrt{x}$ — 2

(ii) $\left(x^2 - 1\right)^{30}$ — 2

(iii) $\dfrac{4x-9}{x^3}$ — 2

(iv) $e^{2x}\left(e^{4x} - 1\right)$ — 2

(v) $x(x+2)^6$ — 2

(vi) $\sin^2 x$ — 1

(b) Show that $\dfrac{d}{dx}\left(\dfrac{10}{\sqrt{x+3} - \sqrt{x-2}}\right) = \dfrac{1}{\sqrt{x+3}} + \dfrac{1}{\sqrt{x-2}}$ — 2

(c) Show that $f(x)$ is a monotonic decreasing function if $f(x) = \dfrac{1}{(x+2)^4}$ for $x > -2$. — 2

QUESTION 3 (START A NEW PAGE) Marks

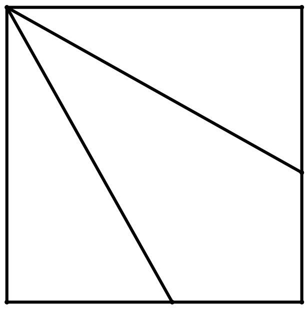
(a) Find the equation of the normal to the curve $y = 4\ln\left(x^2 - 4\right)$ at the point where $x = 3$. 3

(b) Find an expression without logarithms if: $2\log_2(xy) - 3\log_2(x + y) = 4$. 2

(c) If α and β are the roots of $4x^2 - 5x + 7 = 0$, find the value of $\alpha^2 + \beta^2$. 2

(d) Shade the region defined by: $(x + 3)^2 + y^2 \le 9$. 1

(e) ABCD is a square with $CX = CY$. 4
Prove that $\angle AXC = \angle AYC$.



(f) Find the coordinates of the inflexion point on the curve $P(x) = x^3 - 6x^2 + 11x - 6$. 3
Fully justify your answer.

QUESTION 4 (START A NEW PAGE) Marks

(a) Show that quadrilateral $ABCD$ is a parallelogram if its vertices are the points: 3
$A(-14,18)$, $B(-8,27)$, $C(8,12)$ and $D(2,3)$.

(b) Find the value of x, correct to the nearest minute in the domain $-90° \le x \le 90°$, if 2
$2\sin x + 3\cos x = 0$.

(c) The area A of an irregular shape is given by: $A = 150 - 32y + 2y^2$ where $y = 5 + 4x - x^2$ and x is the distance measured from a fixed point O.

(i) Show that when $x = 0.5$, $\dfrac{dA}{dx} = -15$. 2

(ii) Find the value of $\dfrac{dA}{dx}$ when $x = 1.5$ and $x = 2.5$. 2

(iii) Find the values of x for the relative minimum and relative maximum values of the area A. 4

(iv) Graph the area function A versus x in the domain $0 \le x \le 4$. 2

2

2

Y11 PRELIMINARY 2005 (MATHS 2U)

(a) $k = 0$ (1)

(b) $\log_4 128 = \dfrac{\log_2 128}{\log_2 4}$ (1)

$= \dfrac{7}{2}$ (1)

(c) $\dfrac{x^3-27}{x^2-9} = \dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$ (1) (1)

$= \dfrac{x^2+3x+9}{x+3}$ provided $x \neq 3$

(d) $\dfrac{3x-5}{2} - \dfrac{9x-1}{5} < -6$

$5(3x-5) - 2(9x-1) < -60$

$15x - 25 - 18x + 2 < -60$

$-3x - 23 < -60$

$-3x < -37$ (2)

$x > 12\frac{1}{3}$

(e) $\dfrac{\log_7 8}{\log_7 2} = \log_2 8$ (1)

$= 3.$ (1)

(f) $\dfrac{(4\sqrt{2}-3)}{(5+\sqrt{2})} \cdot \dfrac{(5-\sqrt{2})}{(5-\sqrt{2})}$

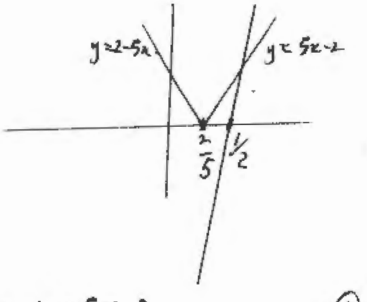
$= \dfrac{20\sqrt{2} - 8 - 15 + 3\sqrt{2}}{25-2}$

$= \dfrac{23\sqrt{2} - 23}{23}$ (2)

$= \sqrt{2} - 1$

(g) $\sqrt{(x+1)^2} = |x+1|$ (1)

(h)



(1)

$\therefore 6x - 12 = 5x - 2$ (1)

$x = 10$ only.

(i) $\tan(90° + \theta) = \tan[90° - (-\theta)]$

$= \cot(-\theta)$

$= -\cot\theta$ (1)

2 (i) $\dfrac{d}{dx}\, 9x - x^2\sqrt{x} = \dfrac{d}{dx}\left(9x - x^{5/2}\right)$

$= 9 - \frac{5}{2}x\sqrt{x}.$ (2)

(ii) $\dfrac{d}{dx}(x^2-1)^{30} = 60x(x^2-1)^{29}$ (2)

(iii) $\dfrac{d}{dx}\dfrac{4x-9}{x^3} = \dfrac{d}{dx}\, 4x^{-2} - 9x^{-3}$

$= -\dfrac{8}{x^3} + \dfrac{27}{x^4}$ (2)

(iv) $\dfrac{d}{dx}\, e^{2x}(e^{4x}-1) = \dfrac{d}{dx}\, e^{6x} - e^{2x}$

$= 6e^{6x} - 2e^{2x}$ (2)

(v) $\dfrac{d}{dx}\, x(x+2)^6 = (x+2)^6 + 6x(x+2)^5$ (2)

OR $(x+2)^5(7x+2)$

(vi) $\dfrac{d}{dx}[\sin x]^2 = 2\sin x\cos x.$ (1)

2(b) $\dfrac{d}{dx}\dfrac{10}{\sqrt{x+3} - \sqrt{x-2}}$

$= \dfrac{d}{dx}\dfrac{10\left[\sqrt{x+3} + \sqrt{x-2}\right]}{(\sqrt{x+3} - \sqrt{x-2})(\sqrt{x+3} + \sqrt{x-2})}$

$= \dfrac{d}{dx}\dfrac{10}{3+2}\left[\sqrt{x+3} + \sqrt{x-2}\right]$ (1)

$= 2\left[\dfrac{1}{2\sqrt{x+3}} + \dfrac{1}{2\cdot\sqrt{x-2}}\right]$

$= \dfrac{1}{\sqrt{x+3}} + \dfrac{1}{\sqrt{x-2}}.$ (1)

(c) $\dfrac{d}{dx}\dfrac{1}{(x+2)^4} = \dfrac{-4}{(x+2)^5}$ (1)

ie. $f'(x) < 0$ as $x+2 > 0$ (1)

for all x $(x+2)^5 > 0$.

$f(x)$ cts $x > -2$.

$\therefore$ monotonic decreasing $x > -2$

3 (a) $y = 4\ln(x^2-4)$

$\dfrac{dy}{dx} = \dfrac{8x}{x^2-4}$ (1)

At $x = 3$ $m = \dfrac{24}{5}.$ 1

m normal $= \dfrac{-5}{24}.$ (1)

$\therefore x = 3$ $y = 4\ln 5.$

Eqn normal.

$y - y_1 = m(x - x_1)$

$y - 4\ln 5 = \dfrac{-5}{24}(x-3)$ (1)

$5x + 24y - 96\ln 5 - 15 = 0.$

(b)

$2\log_2 xy - 3\log_2(x+y) = 4$

$\log_2(xy)^2 - \log_2(x+y)^3 = 4\log_2 2$ (1)

$\dfrac{x^2y^2}{(x+y)^3} = 2^4$

ie $x^2y^2 = 16(x+y)^3$ (1)

(c) $\alpha + \beta = \dfrac{5}{4}$ $\alpha\beta = \dfrac{7}{4}$ (1)

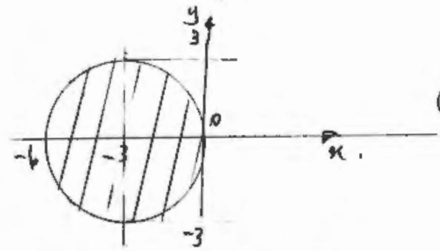
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\dfrac{5}{4}\right)^2 - 2 \times \dfrac{7}{4}$

$= \dfrac{25}{16} - \dfrac{56}{16}$

$= \dfrac{-31}{16}.$ (1)

(d)



(1)

(e) Construction: Join AC.

Proof: In Δ AXC & Δ AYC

AC is common (1)

CX = CY (Data) (1)

∠ACX = ∠ACY (diagonal AC bisects angle of square 45°) (1)

∴ ΔAXC ≡ ΔAYC (SAS)

∴ ∠AXC = ∠AYC (corresponding angles of congruent triangles are equal) (1)

3(i) $P(x) = x^3 - 6x^2 + 11x - 6$

$P'x = 3x^2 - 12x + 11$

$P''x = 6x - 12$

For possible inflexion point

$P''x = 0$

$6x - 12 = 0$ (1)

$x = 2$

Since y, y', y'' cts for all x

that y'' for change in concavity

$y''(1.9) = 6 \times 1.9 - 12$

$= -0.6$ (1)

< 0

$y''(2.1) = 6 \times 2.1 - 12$

$= 0.6$

> 0

∴ change in concavity (1)

∴ Inflexion point: $x = 1$

$y = 0$

$\Rightarrow (1, 0)$

4(a) Midpoint $AC \equiv \left(\frac{-14+8}{2}, \frac{18+12}{2}\right)$

$\equiv (-3, 15)$ (1)

Midpoint $BD \equiv \left(\frac{-8+2}{2}, \frac{27+3}{2}\right)$

$\equiv (-3, 15)$ (1)

∴ Midpoints equal.

∴ ABCD is parallelogram (1)

(diagonals AC & BD bisect each other)

(b) $2\sin x + 3\cos x = 0$ (1)

$\tan x = -\frac{3}{2}$

$x = -56°19'$ (1)

$A = 150 - 32y + 2y^2$ $y = 5 + 4x - x^2$

(c)(i) $\frac{dA}{dx} = \frac{dA}{dy} \cdot \frac{dy}{dx}$

$= (-32 + 4y)(4 - 2x)$ (1)

At $x = 0.5$ $y = 5 + 4 \times 0.5 - 0.25$

$= 6.75$

∴ $\frac{dA}{dx} = (-32 + 4 \times 6.75)(4 - 2 \times 0.5)$

$= -5 \times 3$

$= -15$. (1)

(ii) At $x = 1.5$ $\frac{dA}{dx} = (-32+35)(4-3)$

$y = 8.75$ $= 3$ (1)

At $x = 2.5$ $\frac{dA}{dx} = (-32 + 35)(-1)$

$y = 8.75$ $= -3$ (1)

(iii) For max/min values $\frac{dA}{dx} = 0$

i.e. $-32 + 4y = 0$ And $4 - 2x = 0$

$y = 8$ $x = 2$ (1)

∴ $8 = 5 + 4x - x^2$

$x^2 - 4x + 3 = 0$ (1)

$(x-3)(x-1) = 0$

$x = 1$ or 3 or 2.

Test turning points using gradient $\frac{dy}{dx}$ where A, y are continuous $\{x \mid 0 < x < 4\}$ (1/2)

x	0.5	1	1.5	2	2.5	3	3.5
$\frac{dA}{dx}$	-15	0	3	0	-3	0	15
	\	—	/	—	\	—	/

Since change in gradient from -ve to +ve at $x = 1$ (1/2)

then $x = 1$ is a relative minimum

$A = 22$

Since change in gradient from +ve to -ve at $x = 2$ (1/2)

then $x = 2$ is a relative maximum

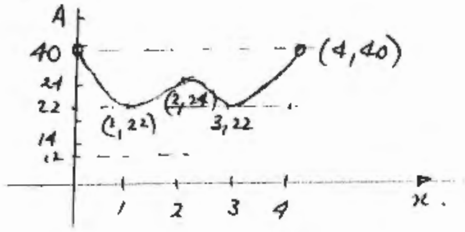
$A = 24$

Since change in gradient from -ve to +ve at $x = 3$ (1/2)

then $x = 3$ is a relative minimum

$A = 22$

(iv)



(1) Endpoints

(1) Graph.