YEAR 11 PRELIM 2007 MATHS (2U)

Question 1 (15 Marks) Begin a SEPARATE sheet of paper **Marks**

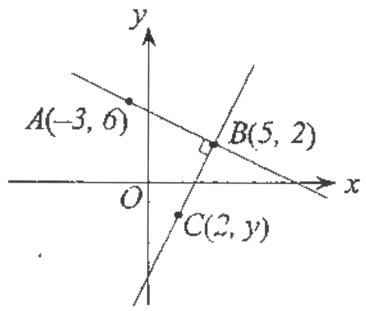
(a) Graph on the number line the solution set of: $|2x \quad 1| \geq 7$. **2**

(b) Simplify:

(i) $\dfrac{3\log_e 8}{\log_e 16}$ **1**

(ii) $e^{2\log_e 2}$. **1**

(c)



NOT TO SCALE

The diagram shows the origin O and the points $A(-3, 6)$, $B(5, 2)$ and $C(2, y)$. The lines AB and BC are perpendicular. A and B lie on the line $x + 2y = 9$ and the length of AB is $4\sqrt{5}$ units.

(i) Find the perpendicular distance from O to AB. **2**

(ii) Show that C has the coordinates $(2, -4)$. **1**

(iii) Does the line AC pass through the origin? Explain. **2**

(d) Sketch the curve $y = 1 + \sin x$ for $0 \leq x \leq 2\pi$. **1**

(e) Shade the region in the Cartesian number plane which represent the simultaneous solution of $y \leq \sqrt{4 - x^2}$ and $y \geq |x| - 2$. **2**

(f) Find the locus of a point $P(x, y)$ which moves in Cartesian number plane such that it is equidistant from points $A(-2, -4)$ and $B(6, 2)$. **3**

Question 2 (15 Marks) Begin a SEPARATE sheet of paper | **Marks**

(a) Simplify fully $\frac{2}{x^2-9}-\frac{1}{x-3}-\frac{4}{x+3}$. **2**

(b) For what values of r will the quadratic equation $x^2+(r-2)x+1=0$ has unreal roots? **3**

(c) Find the derivative of the following:

(i) $\log_e(6x^2-3)$. **1**

(ii) $\sin^3 x$. **1**

(iii) $x^2 \ln x$. **2**

(d) Given that $y=\frac{e^{2x}}{2x+1}$, show that $\frac{dy}{dx}=\frac{4xe^{2x}}{(2x+1)^2}$. **2**

(e) If $f(x) = \frac{1}{\sqrt{2x+7}}$, find $f'(x)$. **2**

(f) Let α and β be the roots of the equation $x^2-rx+s=0$, where r and s are real numbers and not equal to zero. Find

(i) $\alpha+\beta$ **1**

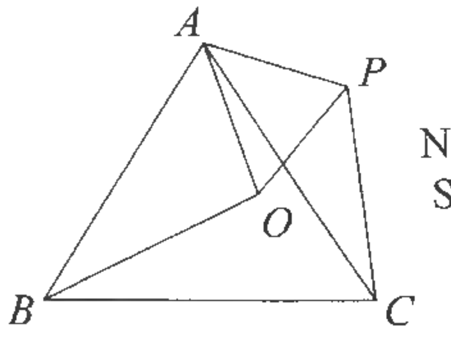
(ii) $\frac{1}{\alpha}+\frac{1}{\beta}$ **1**

<u>Question 3</u> (15 Marks) Begin a SEPARATE sheet of paper **Marks**

(a) Prove that $\dfrac{\cot\theta \operatorname{cosec}\theta}{1+\cot^2\theta} = \cos\theta$. **2**

(b) Solve for x $\quad 9^x - 10(3^x) + 9 = 0$. **2**

(c) In the figure shown triangles ACB and APO are equilateral.



(i) Copy this diagram onto your answer sheet and explain why $\angle BAO = \angle PAC$. **1**

(ii) Prove $\Delta AOB \equiv \Delta APC$. **2**

(d) The function $y = x^3 - 6x^2 + 9x \quad 4$ is defined in the domain $0 \le x \le 4$.

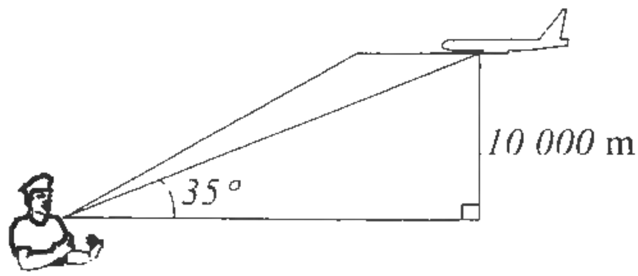
(i) Find the co-ordinates of any stationary points and determine their nature. **3**

(ii) Draw a neat sketch of the function and indicate the appropriate position of the point of inflexion. **3**

(e) Show that $\dfrac{(a+b-2\sqrt{ab})}{\sqrt{a}-\sqrt{b}}$ is a rational number, given that a and b are two distinct positive rational numbers. **2**

Question 4 (15 Marks) Begin a SEPARATE sheet of paper **Marks**

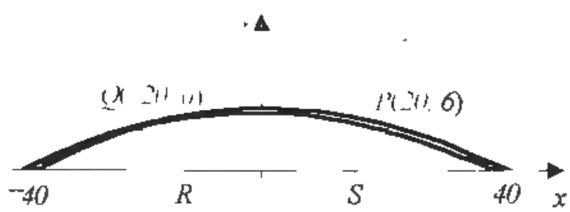
(a) A plane is flying horizontally at a height of 10 000 m above a level stretch of ground at a constant speed of 240 km/h. A man on the ground observes the angle of elevation of the plane (which is flying on a course directly towards him) to be 35°. How far (to the nearest km) in a straight line is the man from the plane 1 minute after he measured the angle of elevation? **3**



(b) A heavy metal arch is to be fixed in the vertical plane as shown in the figure below.

The arch is in the shape of a parabola with equation $y = -\dfrac{x^2}{200} + 8$

Two straight beams are to be fixed at the points P and Q to hold the arch in position. For maximum support the beams must be normal to the parabolic arch.

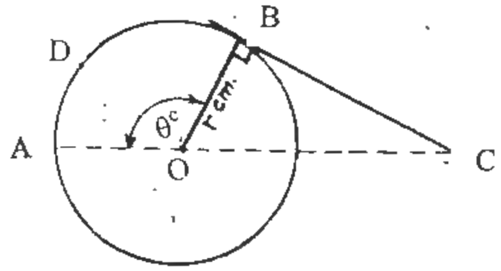


(i) Find the position of the point R where the beam will be fixed at ground level on the x axis. **3**

(ii) Find (to the nearest degree) the angle of inclination of the beam QR. **2**

Question 4 continued

(c) In the figure shown, arc ADB plus the interval BC equals x cm (ie. arc ADB + interval $BC = x$ cm). The radius of the circle is r cm.



(i) If A, O, C are collinear points and angle $AOB = \theta^c$, show that $\theta - \tan\theta = \dfrac{x}{r}$ **3**

(ii) If the area of sector AOB is $\dfrac{3}{8}$ of the area of the circle, find θ and the ratio $\dfrac{x}{r}$. **2**

(iii) Express AC in terms of r and θ. **2**

End of paper

2007 PRELIM. MATHS (2U)

MATHEMATICS: Question 1

Suggested Solutions	Marks	Marker's Comments
a) $\lvert 2x+1\rvert \geq 7$		
$2x+1 \geq 7$ or $-(2x+1) \geq 7$		
$x \geq 3$ or $x \leq -4$	1	
	1	
b (i) $\dfrac{3\log 8}{\log 16} = \dfrac{9\log 2}{4\log 2} = \dfrac{9}{4}$	1	
(ii) $e^{2\log 2} = 4$	1	
c (i) $d = \dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $a=1$, $b=2$, $c=-9$; $x_1=0$, $y_1=0$		
$d = \dfrac{\lvert 1(0)+2(0)-9\rvert}{\sqrt{1^2+2^2}}$	1	
$= \dfrac{\lvert -9\rvert}{\sqrt{5}}$		
$= \dfrac{9}{\sqrt{5}}$ units or $\dfrac{9\sqrt{5}}{5}$ units	1	
ii) $M_{AB} = \dfrac{2-6}{5+3} = \dfrac{-4}{8} = \dfrac{-1}{2}$		
$\therefore M_{CB} = 2$.		
$\dfrac{y_2-y_1}{x_2-x_1} = 2$	1	
$= \dfrac{y-2}{2-5} = 2$		
$= \dfrac{y-2}{-3} = 2$		
$= y-2 = -6$		
$= y = -4$		
$\therefore$ point C is $(2,-4)$		

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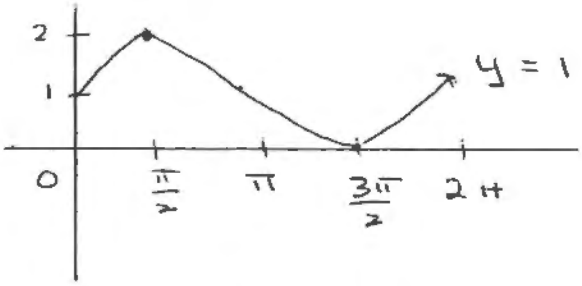
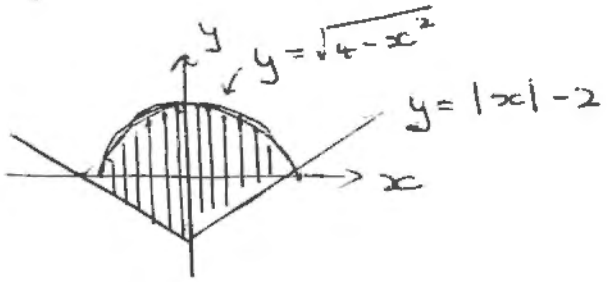
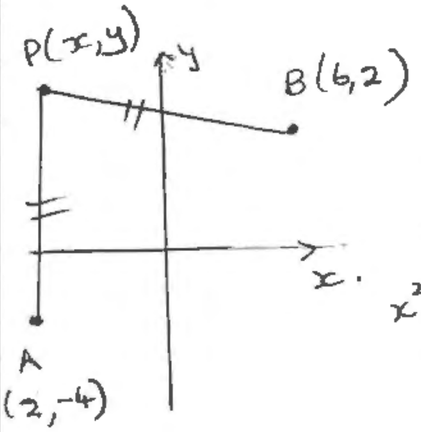
MATHEMATICS: Question 1 continued.

Suggested Solutions	Marks	Marker's Comments
C (iii) $M_{AC} = \frac{-4-6}{2+3} = \frac{-10}{5} = -2$		
Equation $y - y_1 = m(x - x_1)$		
$y - 6 = -2(x+3)$		
$y = -2x$	1	
Yes $(0,0)$ lies on the line	1	
d) 	1	
e) 	1	graph
	1	shading.
f) 		
$\boxed{PA = PB}$	1	
$\sqrt{(x+2)^2 + (y+4)^2} = \sqrt{(x-6)^2 + (y-2)^2}$		
$(x+2)^2 + (y+4)^2 = (x-6)^2 + (y-2)^2$	1	
$x^2 + 4x + 4 + y^2 + 8y + 16 = x^2 - 12x + 36 + y^2 - 4y + 4$		
$4x + 4 + 8y + 16 + 12x - 36 + 4y - 4$		
$16x + 12y - 20 = 0$.		
or $4x + 3y - 5 = 0$	1	
or $y = \frac{-4x}{3} + \frac{5}{3}$		

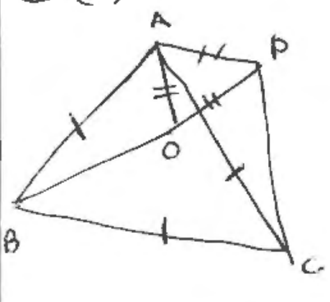
MATHEMATICS: Question 2

Suggested Solutions	Marks	Marker's Comments
a) $\frac{2}{x^2-9} - \frac{1}{x-3} - \frac{4}{x+3}$		
$= \frac{2-(x+3)-4(x-3)}{x^2-9}$	1	
$= \frac{-5x+11}{x^2-9}$ or $\frac{-5x+11}{(x+3)(x-3)}$	1	
b) $\Delta = b^2-4ac < 0$		
$=(r-2)^2 - 4(1)(1) < 0$ ($a=1$, $b=r-2$, $c=1$)	1	
$(r-2)^2 < 4$		
$r-2 < \pm 2$	1	
$r-2<2$ or $r-2>-2$ $r<4$ $\quad$ $r>0$		
	1	
c) (i) $\log_e(6x^2-3)$		
$\frac{f'(x)}{f(x)} = \frac{12x}{6x^2-3}$ or $\frac{12x}{3(2x^2-1)}$ or $\frac{4x}{2x^2-1}$	1	
(ii) $\sin^3 x = (\sin x)^3$		
$\frac{dy}{dx} = 3(\sin x)^2 . \cos x$		
$= 3\cos x(\sin x)^2$		
$= 3\cos x . \sin^2 x$	1	
iii) $x^2 \ln x$		
let $u = x^2$ so $u' = 2x$ $v = \ln x$ so $v' = \frac{1}{x}$		
Product rule $uv' + vu'$		
$= 2x\ln x + x^2 . \frac{1}{x}$	1	
$= 2x\ln x + x.$	1	

MATHEMATICS: Question 2 continued.

Suggested Solutions	Marks	Marker's Comments
d) $y = \dfrac{e^{2x}}{2x+1}$		
quotient rule		
$u = e^{2x}$ so $u' = 2e^{2x}$		
$v = 2x+1$ so $v' = 2$		
$\dfrac{vu' - uv'}{v^2} = \dfrac{(2x+1).2e^{2x} - 2e^{2x}}{(2x+1)^2}$	1	
$= \dfrac{2e^{2x}[(2x+1-1)]}{(2x+1)^2}$		
$= \dfrac{2e^{2x}(2x)}{(2x+1)^2}$		
$= \dfrac{4xe^{2x}}{(2x+1)^2}$	1	
e) $\dfrac{1}{\sqrt{2x+7}} = (2x+7)^{-\frac{1}{2}}$		
$\dfrac{dy}{dx} = -\frac{1}{2}(2x+7)^{-\frac{3}{2}}.2$	1	
$= \dfrac{-1}{\sqrt{(2x+7)^3}}$	1	
f) i) $\alpha + \beta = \dfrac{-b}{a} = \dfrac{-(-r)}{1} = r$.	1	
ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{r}{s}$	1	

MATHEMATICS: Question 3

Suggested Solutions	Marks	Marker's Comments
a) $\dfrac{\cot\theta \csc\theta}{1+\cot^2\theta} = \cos\theta$		
LHS $\dfrac{\dfrac{\cos\theta}{\sin\theta} \times \dfrac{1}{\sin\theta}}{\csc^2\theta}$		$\left(\frac{1}{2}\right)$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\cos\theta}{\sin^2\theta} \div \csc^2\theta$		$\left(\frac{1}{2}\right)$ $\csc = \dfrac{1}{\sin\theta}$
$= \dfrac{\cos\theta}{\sin^2\theta} \times \sin^2\theta$		$\left(\frac{1}{2}\right)$ $1+\cot^2\theta = \csc^2\theta$
$= \cos\theta$ $= RHS$		$\left(\frac{1}{2}\right)$ Prove LHS = RHS
b) $9^x - 10(3^x) + 9 = 0$ $(3^x)^2 - 10(3^x) + 9 = 0$ Let $u = 3^x$	1	
$u^2 - 10u + 9 = 0$ $(u-9)(u-1) = 0$ $u = 9$ or $u = 1$ $3^x = 9$ or $3^x = 1$	1	
$x = 2$ or $x = 0$		
c (i)  $B\hat{A}O = 60 - \angle AOC$. $P\hat{A}C = 60 - \angle AOC$. Hence both angles are equal to $60 - A\hat{O}C$	1	
ii) $AB = AC$ (given data) $AO = AP$ (given data) $\angle BAO = \angle PAC$ (from part (i))	1	
$\therefore \triangle AOB \equiv \triangle APC$ (SAS)	1	

MATHEMATICS: Question 3 continued'

Suggested Solutions	Marks	Marker's Comments

d) i) $y = x^3 - 6x^2 + 9x - 4$

At TP $y' = 0$

$y' = 3x^2 - 12x + 9 = 0$ — 1

$= 3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

when $x = 1$ $y = 1 - 6 + 9 - 4 = 0$ so TP $(1, 0)$ — 1 – TP

when $x = 3$ $y = 27 - 54 + 27 - 4 = -4$ so TP $(3, -4)$

Nature $y'' = 6x - 12$

at $x = 1$ $y'' = -ve$ so $\frown$ max TP — 1 – nature.

x	0	1	2
$\frac{dy}{dx}$	+	||||	−

at $x = 3$ $y'' = +ve$ so $\smile$ min. TP

x	2	3	4
$\frac{dy}{dx}$	−	|||	+

ii) sketch

domain when $x = 0$, $y = 4$
$x = 4$ $y = 0$.

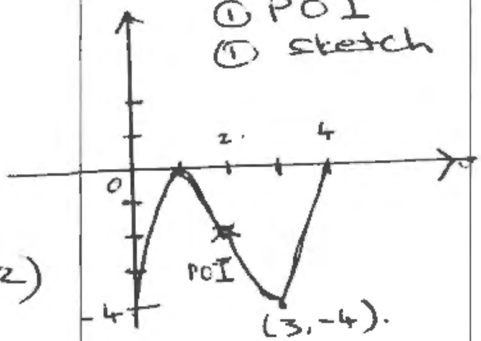
point of inflexion occurs $y'' = 0$

$6x = 12$
$x = 2$

when $x = 2 \Rightarrow y = 8 - 24 + 18 - 4 = -2$

$\therefore$ pt of inflexion occurs at $(2, -2)$

① Correct domain
① POI
① sketch



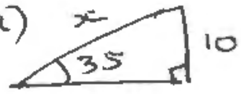
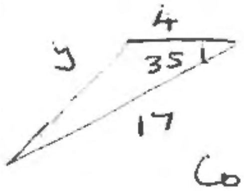
e) $\dfrac{(a + b - 2\sqrt{ab})}{\sqrt{a} - \sqrt{b}}$

$= \dfrac{(\sqrt{a} - \sqrt{b})^2}{\sqrt{a} - \sqrt{b}} \times \dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}}$ — ① recognise $a + b - 2\sqrt{ab} = (\sqrt{a} - \sqrt{b})^2$

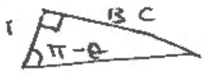
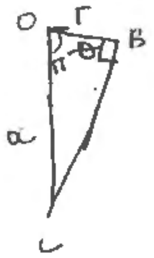
$= \dfrac{(\sqrt{a} - \sqrt{b})(a - b)}{a - b}$

$= \sqrt{a} - \sqrt{b}$ — ①

MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments
a)  $\sin 35 = \frac{10}{x}$		
$x = 17.43446796$		
$x = 17\text{km}.$	1	
 Note $1\text{hr} = 240\text{ km}$ $\therefore 1\text{min} = 4\text{ km}$		
Cosine Rule		
$y^2 = 4^2 + (17.43446796)^2 - 2\times 4\times(17.43446796)\cos 35$	1	
$= 205.7055324$		
$y = 14.3425532$	1	
$y = 14\text{ km}$ (nearest km).		
b) $y = \frac{-x^2}{200} + 8$		
$y' = \frac{-2x}{200} = \frac{-x}{100}$		
at point R $(-20, 6)$		
$y' = \frac{20}{100} = \frac{1}{5}$ so $m = \frac{1}{5}$	1	
$\therefore M_{normal} = M_{QR} = -5$.		
Equation of QR $y - y_1 = m(x - x_1)$		
$y - 6 = -5(x + 20)$		
$y = -5x - 94$	1	
Point R is the x intercept so put $y = 0$		
$0 = -5x - 94$		
$5x = 94$		
$x = -18.8$	1	
$\therefore$ pt R is $(-18.8, 0)$		
ii) $\tan\theta = -5$	1	
$\theta = -79^\circ$		
with positive x axis $\theta = 180 - 79$		
$= 101^\circ$	1	

MATHEMATICS: Question 4 continued.

Suggested Solutions	Marks	Marker's Comments
c (i) $x = \text{arc } ADB + BC.$		
$= r\theta + $ 		
$\tan(\pi - \theta) = \frac{BC}{r}$		
so $BC = r\tan(\pi-\theta)$.		
$\therefore x = r\theta + r\tan(\pi - \theta)$	1	
$x = r(\theta + \tan(\pi-\theta))$		$\tan(\pi - \theta) = -\tan\theta$
$= r(\theta - \tan\theta)$		1
so $\theta - \tan\theta = \frac{x}{r}$	1	
ii) Area		
$\frac{1}{2}r^2\theta = \frac{3}{8}\pi r^2$	1	
$\frac{1}{2}\theta = \frac{3}{8}\pi$		
$\theta = \frac{3\pi}{4}$		
from (i) $\frac{x}{r} = \theta - \tan\theta$		
$\frac{x}{r} = \frac{3\pi}{4} - (-1)$		
$\frac{x}{r} = \frac{3\pi + 1}{4}$ or $\frac{3x+4}{4}$	1	
iii) length $AC = r + OC.$		
find length OC		
 $\cos(\pi-\theta) = \frac{r}{oc}$		
$OC = \frac{r}{\cos(\pi-\theta)} = \frac{r}{-\cos\theta}$ or $-r\sec\theta$.	1	
$\therefore$ length $AC = r + (-r\sec\theta)$		
$= r(1 - \sec\theta)$		
or		
$r\left(1 - \frac{1}{\cos\theta}\right)$	1	