Student Number: ____________________

Class Teacher (*circle*): MC CG DL GP

ESTO TESTIS SOL

YEAR 11 MATHEMATICS

Assessment Task 4

Preliminary Examination

September 2010

Instructions

- **Time allowed: 2 hours plus 5 minutes reading time**
- There are six questions, each worth 12 marks.
- The mark value of each part is indicated in [...] next to that part.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Question 1 **Start a new page** **[12 Marks]**

(a) Find the value of m to three significant figures if: [2]

$$m = \sqrt{\frac{3.7 \times 8.4}{0.32}}$$

(b) Factorise the expression $54 + 2x^3y^3$ [2]

(c) Find the integers a and b such that $\frac{\sqrt{3}}{3+\sqrt{3}} = a + b\sqrt{3}$. [3]

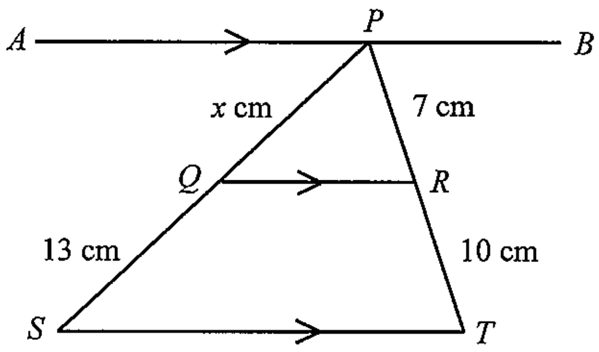
(d) If $\tan \alpha = 4$ and $\cos \alpha < 0$, find the exact value of $\sin \alpha$ for $0 \le \alpha \le 360°$. [3]

(e) Solve the equation $\frac{p}{4} - \frac{p+3}{2} = 3$. [2]

Question 2 **Start a new page** **[12 Marks]**

(a) Solve the inequation $x^2 + 5x - 6 > 0$. **[3]**

(b) In the diagram below $AB \parallel QR \parallel ST$ and the lengths are as marked.



NOT TO SCALE

Find the value of x to one decimal place, giving reasons for your answer. **[2]**

(c) For the parabola $(x+3)^2 = 12(y-2)$

(i) Find the coordinates of the vertex. **[1]**

(ii) Find the focal length. **[1]**

(iii) Find the coordinates of the focus. **[1]**

(iv) State the equation of the directrix. **[1]**

(v) Sketch the parabola labelling the vertex, focus and directrix. **[1]**

(d) For what value(s) of p does $3x^2 - 4x + p = 0$ have equal roots? **[2]**

Question 3 **Start a new page** **[12 Marks]**

(a) Solve $|2x-4|<6$. [2]

(b) Differentiate the following:

(i) $5x^4+2$ [1]

(ii) $y=(2x-5)^7$ [2]

(iii) $f(x)=\dfrac{x+3}{2x-4}$ [2]

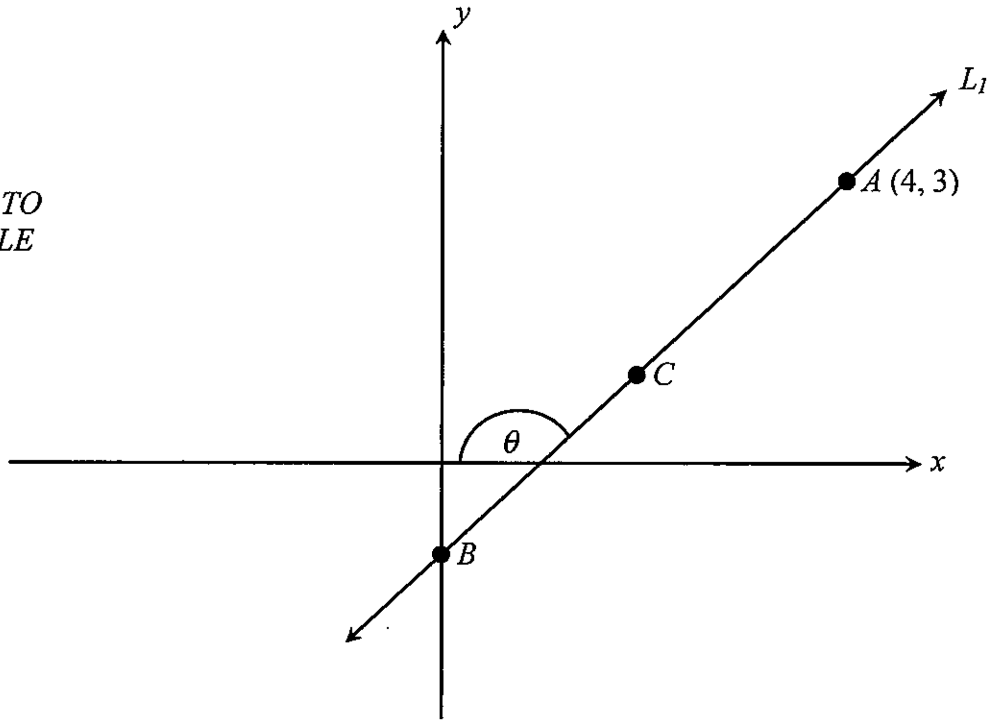
(c) Simplify $\dfrac{\tan A\sec A}{1+\tan^2 A}$. [2]

(d) (i) On the same set of axes, sketch graphs of $x^2+y^2=4$ and $x+y=1$. Note: It is not necessary to find the point(s) of intersection of both graphs. [2]

(ii) On your graph from (i) above, shade the region where the pair of inequalities $x^2+y^2\geq 4$ and $x+y\leq 1$ hold simultaneously. [1]

Question 4 **Start a new page** **[12 Marks]**

(a) Points $A(4, 3)$, B and C lie on line L_1 as illustrated in the diagram below.



(i) The equation of line L_1 is $x - y - 1 = 0$. Find the coordinates of B. **[1]**

(ii) Find the size of θ, the obtuse angle between the x-axis and L_1. **[2]**

(iii) Find the coordinates of point C, the mid-point of AB. **[2]**

(iv) Find the length of AB. **[1]**

(v) Find the equation of the circle of which AB is a diameter. **[2]**

(b) (i) On the same set of axes, sketch the graphs of $y = \cos x$ and $y = \tan x$ for $0° \leq x \leq 360°$, showing all important features. **[3]**

(ii) Hence or otherwise, determine the number of solutions to the equation $\tan x - \cos x = 0$ in the domain $0° \leq x \leq 360°$. **[1]**

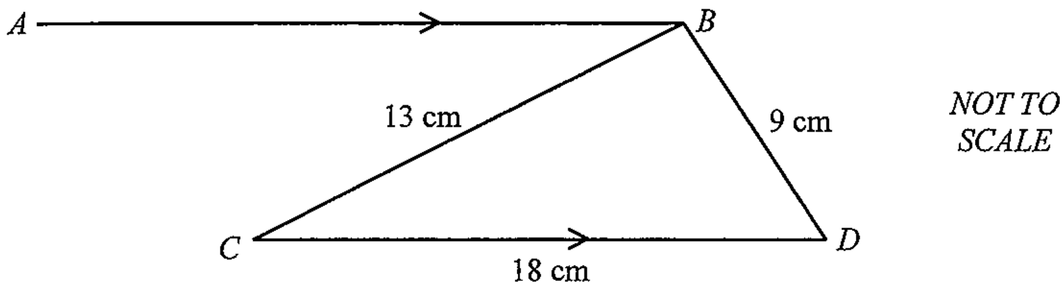
Question 5 **Start a new page** **[12 Marks]**

(a) Given the function $f(x) = x(x+3)^2$,

(i) Find $f'(x)$. **[2]**

(ii) Find the value(s) of x such that $f'(x) = 0$. **[2]**

(b) In the diagram below, AB is parallel to CD and the lengths of BC, BD and CD are as shown.



(i) Find the size of $\angle BDC$ to the nearest degree. **[2]**

(ii) Hence find the size of $\angle ABD$, giving reasons for your answer. **[1]**

(c) Find $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$. **[2]**

(d) Find the shortest distance between the two parallel lines l_1: $x - 3y + 6 = 0$ and l_2: $x - 3y - 2 = 0$. **[3]**

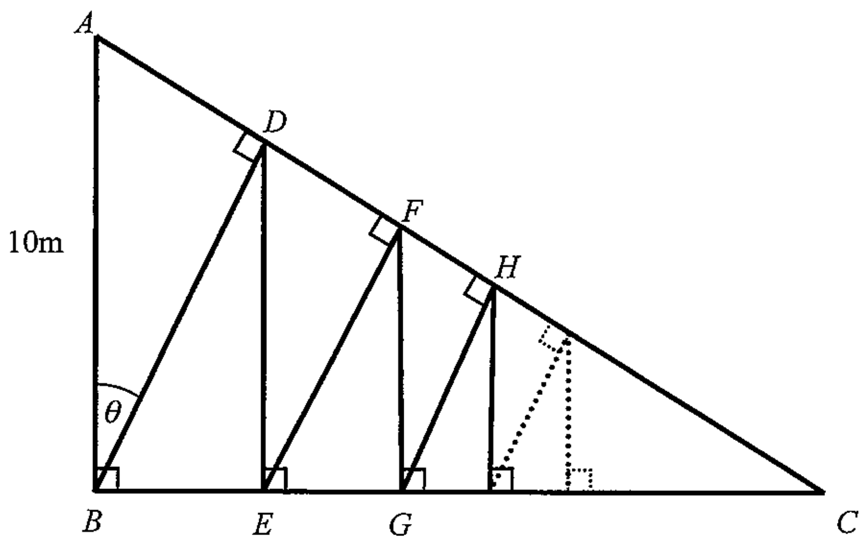
Question 6 **Start a new page** **[12 Marks]**

(a) Find the values of a, b and c if: **[3]**

$$x^2 + x + 1 \equiv a(x-1)^2 + b(x-1) + c$$

(b) If the two roots of the equation $2x^2 - 5(p-1)x + 12 = 0$ are consecutive integers, find the value(s) of p. **[3]**

(c) The diagram below shows a roof truss, comprising a number of right-angled triangles. $\angle ABD = \theta$ and the length of AB is 10m.



(i) Prove that $\triangle ABD \;|||\; \triangle BDE$. **[2]**

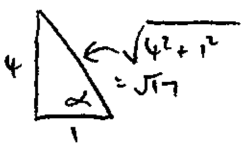
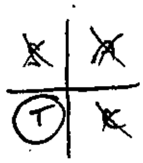
(ii) Show that $BD = 10\cos\theta$. **[1]**

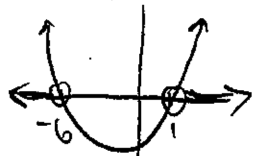
(iii) Find an expression for the length of EF in terms of $\cos\theta$. **[2]**

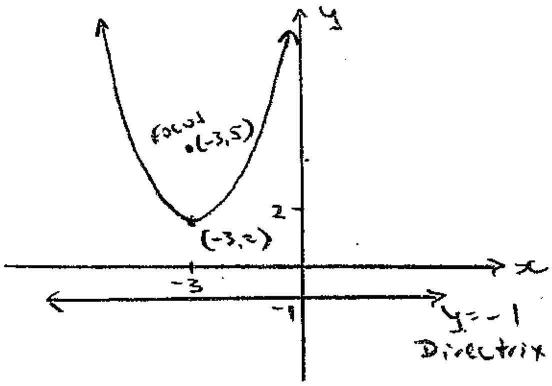
(iv) If $\theta = 30°$, find the exact length of EF. **[1]**

End of Assessment Task

YEAR 11 MATHEMATICS – PRELIMINARY EXAM SOLUTIONS

Qn	Solutions	Marks	Comments+Criteria
1	(a) $m = \sqrt{\dfrac{3.7 \times 8.4}{0.32}}$		
	$= 9.855201672$	1	
	$= 9.86$ to 3 sig figures	1	
	(b) $54 + 2x^3y^3$		
	$= 2(27 + x^3y^3)$	1	
	$= 2\left[(3)^3 + (xy)^3\right]$		
	$= 2(3 + xy)(9 - 3xy + x^2y^2)$	1	
	(c) $\dfrac{\sqrt{3}}{3+\sqrt{3}} = a + b\sqrt{3}$		
	$\dfrac{\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}} = \dfrac{\sqrt{3}(3-\sqrt{3})}{9-3}$	1	for conjugate
	$= \dfrac{3\sqrt{3}-3}{6}$		
	$= \dfrac{\cancel{3}^{1}(\sqrt{3}-1)}{\cancel{6}_{2}}$		or 1 for progress if incorrect
	$= \dfrac{\sqrt{3}-1}{2}$		
	$= -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}$		
	$\therefore a = -\dfrac{1}{2}$	1	for a
	$b = \dfrac{1}{2}$	1	for b
	(d) $\tan\alpha = 4$ $\cos\alpha < 0$		
	 Quadrant 3 	1	unknown side
	$\therefore \sin\alpha < 0$	1	Correct ratio
	$\sin\alpha = -\dfrac{4}{\sqrt{17}}$	1	for –ve (Q3)

Qn	Solutions	Marks	Comments+Criteria
1	(e) $\frac{p}{4} - \frac{p+3}{2} = 3$		
	$p - 2(p+3) = 3\times 4$	1	for denominator
	$p - 2p - 6 = 12$		
	$-p - 6 = 12$		
	$-p = 18$		
	$p = -18$	1	correct answer
2	(a) $x^2 + 5x - 6 > 0$		
	$(x+6)(x-1) > 0$	1	factors
			
	$\therefore x < -6$ or $x > 1$	1	for each soln
	(b) $\frac{x}{13} = \frac{7}{10}$ (Ratio of intercepts on one transversal equals the ratio of intercepts on any other transversal)		1 for reason
	$10x = 13\times 7$		1 off for any error
	$x = \frac{13\times 7}{10}$		
	$= 9.1$	1	correct answer
	(c) $(x+3)^2 = 12(y-2)$		
	(i) Vertex: $(-3, 2)$	1	Correct answer only
	(ii) Focal Length: $4a = 12$		
	$\therefore a = 3$		
	Focal Length $= 3$	1	Correct answer

Qn	Solutions	Marks	Comments+Criteria
2(c)	(iii) Parabola is concave up: $\therefore$ Focus $= (-3, 2+3)$ $= (-3, 5)$	1	for answer
	(iv) Directrix: $y = 2-3$ $y = -1$	1	for answer - must be eqn.
	(v) 	1	for sketch
	(d) $3x^2 - 4x + p = 0$ Equal roots $\Delta = 0$	1	set $\Delta = 0$
	$\Delta = b^2 - 4ac$ $0 = (-4)^2 - 4(3)(p)$ $0 = 16 - 12p$ $16 - 12p = 0$ $12p = 16$ $p = 16/12$ $= 4/3$	1	answer
3	(a) $\lvert 2x-4 \rvert < 6$ Either $2x - 4 < 6$ or $2x - 4 > -6$ $2x < 10$ $2x > -2$ $x < 5$ $x > -1$ $\therefore -1 < x < 5$	1	for each case -1 not continuous domain

Qn	Solutions	Marks	Comments+Criteria
3	(b) (i) $\frac{d}{dx} 5x^4 + 2$		
	$= 20x^3$	1	Correct answer
	(ii) $y = (2x-5)^7$		
	$\frac{dy}{dx} = 7(2x-5)^6 . 2$	1	power down, reduce power
	$= 14(2x-5)^6$	1	Differentiate ( )
	(iii) $f(x) = \frac{x+3}{2x-4}$		
	Quotient Rule $u = x+3 \quad v = 2x-4$ $u' = 1 \quad v' = 2$	1	for set up
	$f'(x) = \frac{vu' - uv'}{v^2}$ $= \frac{(2x-4) - 2(x+3)}{(2x-4)^2}$		
	$= \frac{2x-4-2x-6}{(2x-4)^2}$	1	Correct expansion
	$= \frac{-10}{(2x-4)^2}$		
	(c) $\frac{\tan A \sec A}{1+\tan^2 A} = \frac{\frac{\sin A}{\cos A} \times \frac{1}{\cos A}}{\sec^2 A}$	1	for substitutions - $\sec^2 A$ key
	$= \frac{\frac{\sin A}{\cos^2 A}}{1/\cos^2 A}$		
	$= \frac{\sin A}{\cancel{\cos^2 A}} \times \frac{\cancel{\cos^2 A}}{1}$		
	$= \sin A.$	1	answer

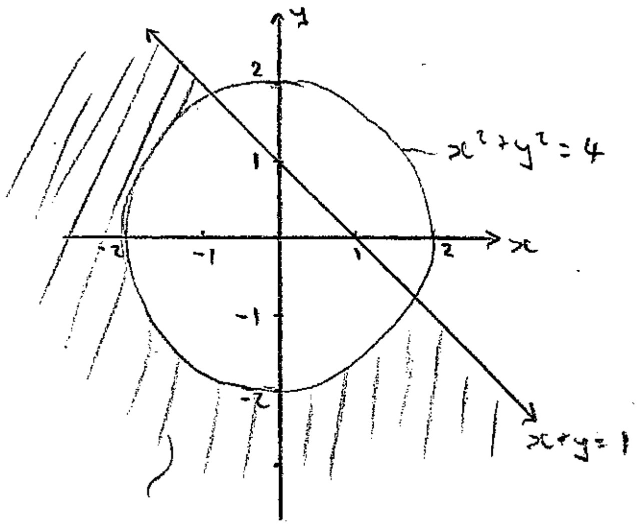
or $\frac{\tan A \sec A}{1+\tan^2 A}$

$= \frac{\tan A \sec A}{\sec^2 A}$

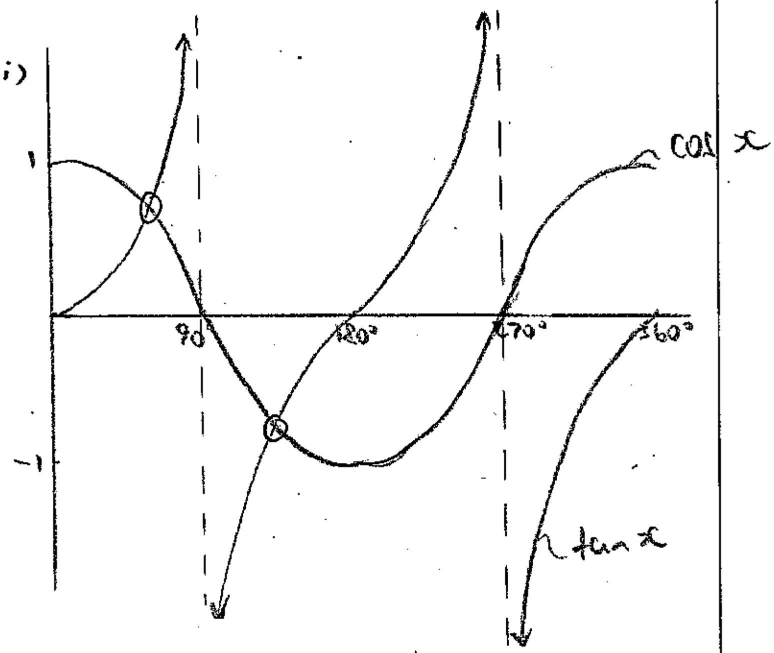
$= \frac{\tan A}{\sec A}$

$= \frac{\frac{\sin A}{\cos A}}{1/\cos A} = \frac{\sin A}{\cancel{\cos A}} \times \frac{\cancel{\cos A}}{1}$

$= \sin A$

Qn	Solutions	Marks	Comments+Criteria
3	(d) 	1	for each graph
	and $x^2+y^2 \geq 4$ $x+y \leq 1$	1	Correct region
4	(a) (i) $x-y-1=0$ $B - x=0$ $\therefore 0-y-1=0$ $y=-1$ $\therefore B(0,-1)$	1	Answer - must be coordinates
	(ii) $x-y-1=0$ i.e. $y=x-1$ $\therefore m=1$ $m=\tan\alpha$ (acute angle) $1=\tan\alpha$ $\therefore \alpha=45^\circ$	1	for gradient
	$\theta = 180-45$ $=135^\circ$	1	for answer (recognise $180-\theta$)
	(iii) $C\left(\frac{0+4}{2}, \frac{-1+3}{2}\right)$ $=(2,1)$	1 1	$\left.\begin{matrix}x\\y\end{matrix}\right\}$ values.
	(iv) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(4-0)^2+(3-(-1))^2}$ $=\sqrt{4^2+4^2}$	1	at substitution point

$=\sqrt{32}$
$=4\sqrt{2}$ units

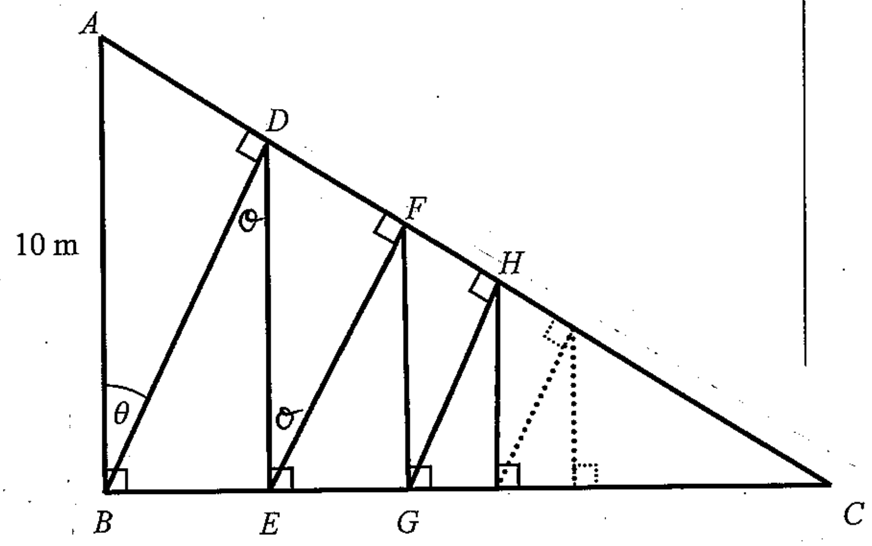
Qn	Solutions	Marks	Comments+Criteria
4	(a) (v) Ctr is $C(2,1)$ $AB = 4\sqrt{2}$ units $=$ diameter $\therefore$ radius $= 2\sqrt{2}$ units Eqn of circle: $(x-2)^2 + (y-1)^2 = (2\sqrt{2})^2$ $(x-2)^2 + (y-1)^2 = 8$	1 1	for centre for radius
	(b) (i)  		1 for shape cos x. 1 for shape tan x 1 for axes value including asymptotes
	(ii) Solns to $\tan x - \cos x = 0$ i.e. $\tan x = \cos x$ $\therefore$ 2 solutions (intersection of graphs)	1	as per graph intersection
5	(a) (i) $f(x) = x(x+3)^2$ $= x(x^2+6x+9)$ $= x^3 + 6x^2 + 9x$ $f'(x) = 3x^2 + 12x + 9$ 2 differential -1 for error or product rule $f'(x) = vu' + uv'$ $u = x$ $v = (x+3)^2$ $u' = 1$ $v' = 2(x+3)$ $f'(x) = (x+3)^2 + 2x(x+3)$ $= x^2 + 6x + 9 + 2x^2 + 6x$ $= 3x^2 + 12x + 9$		1 for setup 1 for substitution

Qn	Solutions	Marks	Comments+Criteria
5(a)	(ii) $f'(x) = 0$		
	i.e. $3x^2 + 12x + 9 = 0$		
	$3(x^2 + 4x + 3) = 0$		
	$x^2 + 4x + 3 = 0$		
	$(x+3)(x+1) = 0$	1	factoring (progress)
	$x = -3$ or -1	1	correct answer
(b)	(i) By Cosine Rule		
	Let $\angle BDC = \theta$		
	$\cos\theta = \dfrac{18^2 + 9^2 - 13^2}{2 \times 18 \times 9}$	1	sub into cosine rule
	$= \dfrac{59}{81}$		
	$\theta = \cos^{-1}\left(\dfrac{59}{81}\right)$		
	$= 43.2°$	1	answer
	$= 43°$ (nearest degree)		
	(ii) $\angle ABD = 180 - 43$		
	$= 137°$ (Cointerior angles are supplementary on parallel lines)		1 for angle no penalty if no reason or incorrect reason
(c)	$\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$		
	$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$	1	factoring
	$= \lim\limits_{x \to 2} (x+2)$		
	$= 2 + 2$		
	$= 4$	1	answer

Qn	Solutions	Marks	Comments+Criteria
5	(d) $l_1 : x - 3y + 6 = 0$ $l_2 : x - 3y - 2 = 0$ Find a point on l_1 Let $x = 0$ $\quad 0 - 3y + 6 = 0$ $3y = 6$ $y = 2$ $\therefore (0, 2)$ lies on l_1	1	for any correct point on one line.
	$d = \left\| \dfrac{ax + by + c}{\sqrt{a^2 + b^2}} \right\|$ $= \left\| \dfrac{1(0) + (-3)(2) + (-2)}{\sqrt{(1)^2 + (-3)^2}} \right\|$	1	Substitution
	$= \left\| \dfrac{0 - 6 - 2}{\sqrt{10}} \right\|$ $= \left\| \dfrac{-8}{\sqrt{10}} \right\|$ $= \dfrac{8}{\sqrt{10}}$ units	1	answer
	$= \dfrac{8\sqrt{10}}{10}$ $= \dfrac{4\sqrt{10}}{5}$ units		
6	(a) $x^2 + x + 1 \equiv a(x-1)^2 + b(x-1) + c$ RHS $= a(x^2 - 2x + 1) + b(x-1) + c$ $= ax^2 - 2ax + a + bx - b + c$ $= ax^2 - (2a - b)x + (a - b + c)$ Equating coefficients $a = 1$ $-(2a - b) = 1$ $-2a + b = 1$ $-2(1) + b = 1$ $b = 3$ $a - b + c = 1$ $1 - 3 + c = 1$ $c = 3$ $\therefore a = 1$ $b = 3$ $c = 3$	1	for a, b, c Award marks for progress / method

Qn	Solutions	Marks	Comments+Criteria
	Or $x^2+x+1 \equiv a(x-1)^2+b(x-1)+c$		
	Sub $x=1$		
	$(1)^2+(1)+1 \equiv a(1-1)^2+b(1-1)+c$		
	$3 = c$		
	Equate coefficients of x^2		
	$x^2 = ax^2$		Award marks for
	$\therefore a = 1$		progress/method
	Sub $x=0$		
	$0+0+1 \equiv 1(0-1)^2+b(0-1)+3$		
	$1 \equiv 1 - b + 3$		
	$b = 3$	1	for a, b, c
	$\therefore a=1, b=3, c=3.$		
	(b) $2x^2-5(p-1)x+12=0$		
	Two roots are consecutive integers		
	Let the roots be $\alpha, \alpha+1$	1	for $\alpha, \alpha+1$
	$\therefore \alpha(\alpha+1) = \frac{c}{a}$		
	$\alpha^2+\alpha = \frac{12}{2}$		
	$\alpha^2+\alpha-6=0$		
	$(\alpha+3)(\alpha-2)=0$	1	factored quadratic
	$\therefore \alpha = 2$ or -3		
	Sum of roots:		
	$\alpha+\alpha+1 = \frac{-b}{a}$		or 1 for
	$2\alpha+1 = \frac{-[-5(p-1)]}{2}$		progress
	$2\alpha+1 = \frac{5p-5}{2}$		
	$4\alpha+2 = 5p-5$		
	$5p = 4\alpha+7$		
	$p = \frac{4\alpha+7}{5}$		
	when $\alpha=2$ or $\alpha=-3$		
	$p = \frac{4(2)+7}{5} = 15/5 = 3$ $p = \frac{4(-3)+7}{5} = \frac{-12+7}{5} = -5/5 = -1$		

$\therefore p = 3$ or -1 1   for answer

Qn	Solutions	Marks	Comments+Criteria
6	(c) 		
	(i) $\angle ADB = \angle BED = 90°$ (Given) $AB \parallel DE$ (Corresponding angles $\angle ADB = \angle BED = 90°$) $\therefore \angle ABD = \angle BDE = \theta$ (Alternate angles are equal on parallel lines) $\therefore \angle ABD \parallel\!\!\parallel \triangle BDE$ (Equiangular)		1 for progress 1 for answer including statement
	(ii) In $\triangle ABD$ $\cos\theta = \frac{BD}{10}$ $BD = 10\cos\theta$ as req'd	1	must show sufficient work
	(iii) In $\triangle BDE$ $\cos\theta = \frac{DE}{BD}$ $= \frac{DE}{10\cos\theta}$ $\therefore DE = 10\cos^2\theta$	1	for DE
	Similarly: $\triangle DFE \parallel\!\!\parallel \triangle BDE$ as above In $\triangle DEF$ $\cos\theta = \frac{EF}{DE}$ $= \frac{EF}{10\cos^2\theta}$ $\therefore EF = 10\cos^3\theta$	1	for EF
	(iv) $\theta = 30°$ $\therefore EF = 10\cos^3 30$		

$= 10 \times \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{30\sqrt{3}}{8} = \frac{15\sqrt{3}}{4}$ m 1 Correct substitution into (iii) above