STUDENT NUMBER: ____________________________

YEAR 11 MATHEMATICS

September 2011

PRELIMINARY EXAMINATION

Time allowed: 2 hours plus 5 minutes reading time

INSTRUCTIONS

- This examination paper contains two sections with a total value of 65 marks.

  Section I: Objective Response Questions (5 questions of 1 mark each)
  Answer on the Objective-Response Answer Sheet provided.

  Section II: Extended Response Questions (4 questions of 15 marks each)
  Start each question on a new page.

- A table of standard integrals is provided.
- Board-approved calculators may be used.
- Geometric equipment and mathematical curve-drawing templates are allowed.
- Marks may not be awarded for untidy or careless work.
- More marks will be awarded for questions involving higher-order thinking.
- You may tear off the Objective-Response Answer Sheet.

SECTION I : Objective Response Questions **5 Marks**
Answer on the Objective-Response Answer Sheet provided.

1. The equation of the line through the point (9, 7) and parallel to the x-axis is

   (A) $x = 9$ (B) $y = 7$

   (C) $y = 9$ (D) $x = 7$

2. The function $f(x) = x^2 - 2x$ is

   (A) even (B) odd

   (C) neither (D) positive definite

3. The gradient of a line perpendicular to $3x - 4y + 7 = 0$ is

   (A) $\frac{3}{4}$ (B) $\frac{-3}{4}$

   (C) $\frac{4}{3}$ (D) $\frac{-4}{3}$

4. $81^{-\frac{3}{4}}$ is **not** equal to

   (A) $\frac{1}{81^{\frac{3}{4}}}$ (B) $\frac{1}{27}$

   (C) $\frac{1}{27^4}$ (D) $0.\dot{0}3\dot{7}$

5. Express the equation of the parabola $x^2 = 2(2y - 1)$ in the form $(x - h)^2 = 4a(y - k)$ and hence write down the focal length.

SECTION II: Extended Response Questions **60 Marks**

Start each question on a new page.

Question 1 (*Start a new page.*) **15 Marks**

(a) Evaluate, correct to three significant figures **[2]**

$$\frac{13^5}{9^6+11^4}+1$$

(b) Factorise $16x^2 - 25$. **[2]**

(c) Solve $2x + 3 \le 8$. **[2]**

(d) Find all the value(s) of x in the interval $0^0 \le x \le 360^0$ for which $\tan x = \dfrac{1}{\sqrt{3}}$. **[2]**

(e) For what value(s) of k does $3x^2 + 2x + k = 0$ have real roots? **[2]**

(f) Simplify $\dfrac{5}{x^2+x} + \dfrac{2}{x^2-1}$. **[3]**

(g) Simplify $\tan\theta\cos\theta$. **[2]**

Question 2 (*Start a new page.*) **15 Marks**

(a) Differentiate

(i) $x^2 - 3x$ **[2]**

(ii) $\dfrac{1}{x}$ **[2]**

(b) (i) Sketch the graph of $y = |2x - 6|$, showing all relevant features. **[2]**

(ii) On the same set of axes, sketch the line $y = -1$. **[1]**

(iii) Hence state number of solutions to the equation $|2x - 6| = -1$. **[1]**

(c) Given $x^2 - (k+5)x + 5k = 0$, find the value(s) of k such that one root is two more than the other. **[3]**

(d) A parabola has focus (2, 1) and directrix $y = 5$.

(i) Find the vertex. **[1]**

(ii) Sketch the parabola and write down its equation. **[3]**

Question 3 (*Start a new page.*) **15 Marks**

(a) If $f(x) = 1 - x^2$,

(i) find $f(x+a)$ **[1]**

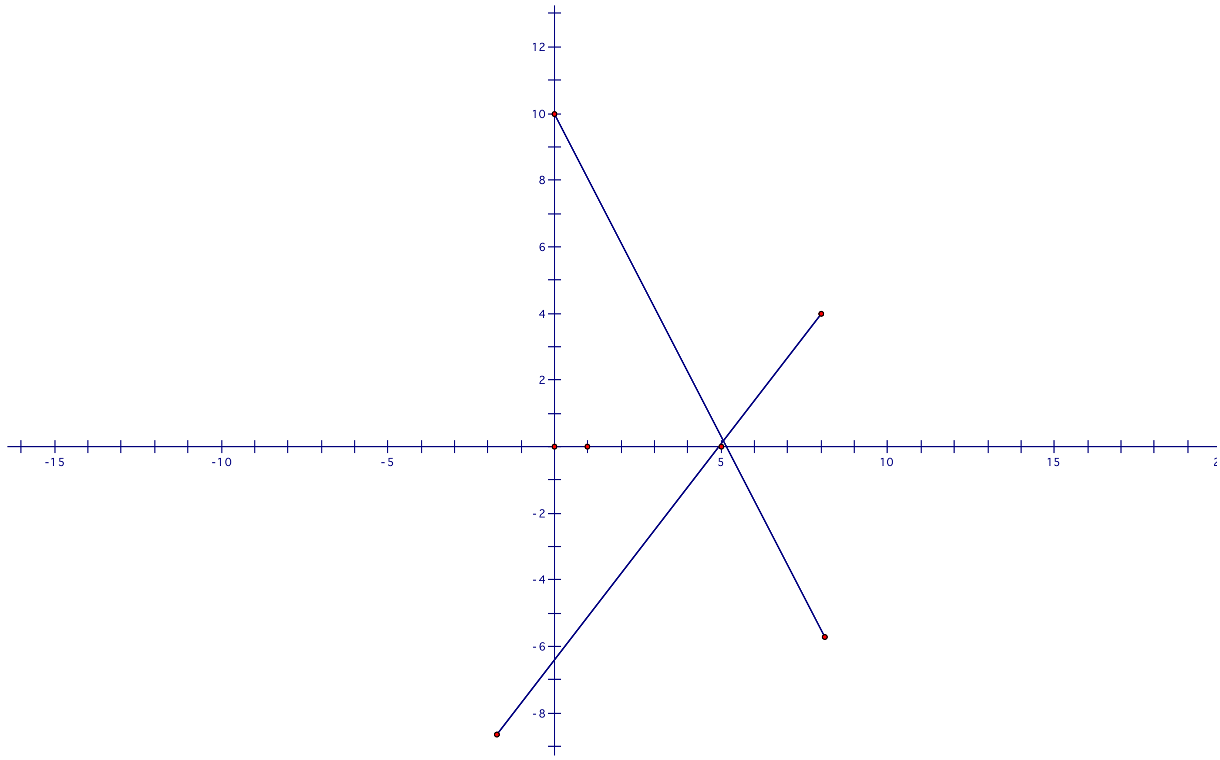
(ii) find, in simplest form, $\dfrac{f(x+a) - f(x)}{a}$. **[2]**

(b) (i) Show that $1 + \dfrac{2}{x} = \dfrac{x+2}{x}$. **[1]**

(ii) Find the domain and range of the curve $y = \dfrac{x+2}{x}$. **[2]**

(c) Given that the sides of a right-angled triangle are x, $x + 1$ and $x + 2$, find the value of x. **[3]**

(d) In the diagram below, the point B(8, 4) lies on line L_1 and the point C(0, 10) lies on the line L_2. The lines L_1 and L_2 meet at the point A(5, 0). The point M lies on the y-axis.



<u>Question 3 - Continued</u>

(i) Show that the gradient of AB is $\frac{4}{3}$. **[1]**

(ii) Find the angle that the line AB makes with the positive direction of the x-axis. **[1]**

(iii) Show that the equation of the line AB is $3y = 4x - 20$. **[1]**

(iv) The line AB cuts the y-axis at M. Find the co-ordinates of the point M. **[1]**

(v) Find the area of Δ CMA to one decimal place. **[2]**

Question 4 (*Start a new page*.) **15 Marks**

(a) Simplify $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$. **[2]**

(b) (i) Find the perpendicular distance between the line $3x + 4y - 3 = 0$ and the circle $x^2 + y^2 = 16$. **[2]**

(ii) Is the line a tangent or secant to the circle, or neither? Give reasons for your answer. **[1]**

(c) Write down a possible equation of a parabola that is negative definite. **[1]**

(d) Given the points A (2, 4) and B (-4, 2), find the equation of the locus of the point P(x, y) and describe the locus geometrically if $\angle APB$ is a right-angle. **[3]**

Question 4 -Continued

(e) In the diagram below, AD = 2cm, OC = 3 cm and $\angle DAB$ = 30°. O is the centre of the circle.

(i) Show that $\angle OCB$ = 56° (to the nearest degree) **[2]**

(ii) Find the area of triangle OCB. **[2]**

END OF EXAMINATION

Objective Response Answers

Student Number:__________________

Tear off this sheet and hand it in separately.

Questions 1-4 : **Circle** the correct answer

Question 5: Write the correct answer in the box provided

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5				

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ \ n \neq -1;\ \ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ \ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

SECTION I

Year 11 2U Kambala Preliminary Examination solutions Sept 2011

Qn	Solutions	Marks	Comments+Criteria
1.	parallel to x axis (B) $\therefore y = 7$		
2	$f(x) = \underline{x^2 - 2x}$ $f(-x) = (-x)^2 - 2x^{-x}$ $= \underline{x^2 + 2x}$ $\therefore$ NEITHER (C)		
3.	m in general form $= -\frac{a}{b}$ $= \frac{3}{4}$ $\therefore$ perpendicular $= -\frac{4}{3}$ (D)		
4	$81^{-3/4} = \frac{1}{81^{3/4}} = \frac{1}{\sqrt[4]{81^3}} = \frac{1}{3^3} = \frac{1}{27}$ $\boxed{\text{NOT-C}}$ (C)		
5.	$x^2 = 2(2y-1)$ $= 4(y - \frac{1}{2})$ $x^2 = 4a(y-k)$ $4a = 4$ $\underline{a = 1}$		

QUESTION 1

Year 11 2U Kambala Preliminary Examination solutions Sept 2011

Qn	Solutions	Marks	Comments+Criteria
(a)	$1.67\|992$ $=1.68$	1 1	always write down your answer from calc before rounding. If wrong answer but correct rounding 1 – 2
(b)	$16x^2 - 25$ $(4x-5)(4x+5)$	1 1	
(c)	$2x+3 \leq 8$ $2x \leq 8-3$ $2x \leq 5$ $x \leq 2\frac{1}{2}$	1 1	
(d)	$\tan x = \frac{1}{\sqrt{3}}$  positive $\therefore$ Q1+3 acute angle $x = 30^\circ$ Q3 $180+30 = 210^\circ$ $x = 30^\circ$ or 210°	1 1	
(e)	$\Delta \geq 0$ $3x^2+2x+K=0$ $b^2-4ac \geq 0$ $4 - 4\times3\times K \geq 0$ $-12K \geq -4$ $K \leq \frac{-4}{-12}$ $K \leq \frac{1}{3}$		Real roots $\Delta \geq 0$ many wrote equal $\div$ by $-$ change inequality sign

QUESTION 1 CONTINUED

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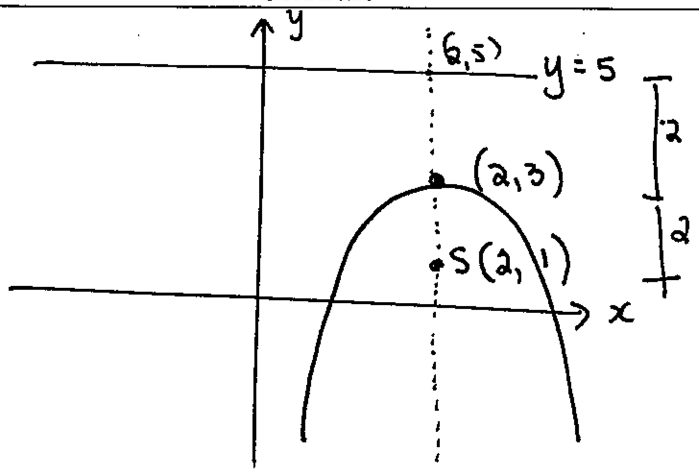
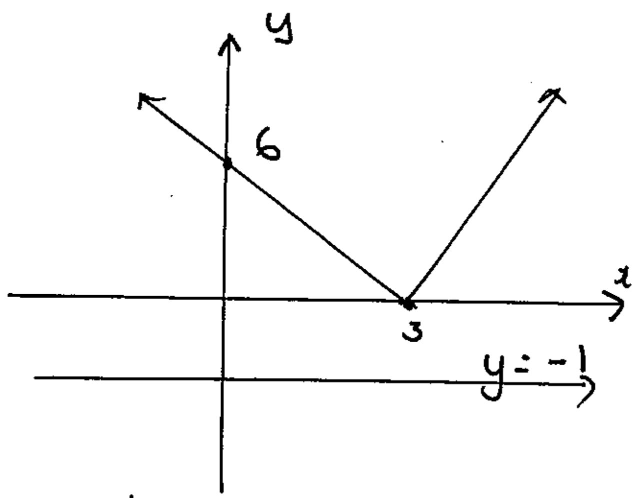
Qn	Solutions	Marks	Comments+Criteria
(f)	$\dfrac{5}{x^2+x} + \dfrac{2}{x^2-1}$	max 1	Much easier to factorise
	$\dfrac{5}{x(x+1)} + \dfrac{2}{(x+1)(x-1)}$	1	4 not factorised and wrong answer. Those who didn't often ended up with <u>NOT</u> the simplest fraction
	$\dfrac{5(x-1)+2x}{x(x+1)(x-1)}$		
	$\dfrac{5x-5+2x}{x(x+1)(x-1)}$	1	Could have different forms of the denominator
	$\dfrac{7x-5}{x(x+1)(x-1)}$	1	
(g)	$\tan\theta \cdot \cos\theta$		An easy 2 marks.
	$\dfrac{\sin\theta}{\cancel{\cos}\theta} \cdot \cancel{\cos}\theta = \sin\theta$	2	

Q2

Year 11 2U Kambala Preliminary Examination solutions Sept 2011

Qn	Solutions	Marks	Comments+Criteria
(a)	(i) $y = x^2 - 3x$ $y' = 2x - 3$	(2)	
	(ii) $y = \frac{1}{x} = x^{-1}$	1	
	$y' = -1x^{-2}$ $= -\frac{1}{x^2}$	1 (2)	
(b)	$x^2 - (K+5)x + 5K = 0$ Let first root $= \alpha$ second root $= \alpha + 2$ • $\alpha + \alpha + 2 = -\frac{b}{a}$ $2\alpha + 2 = K + 5$ (1) $2\alpha = K + 3$ $2\alpha - 3 = K$ • $\alpha(\alpha + 2) = \frac{c}{a} = 5K$ (2)	1 for both eqn.	
	$\alpha^2 + 2\alpha = 5K$ $\alpha^2 + 2\alpha = 5(2\alpha - 3)$ subst. $\alpha^2 + 2\alpha = 10\alpha - 15$ $\alpha^2 - 8x + 15 = 0$ $(\alpha - 5)(\alpha - 3) = 0$ $\alpha = 3$ or 5 $K = 2\alpha - 3$ or $2\alpha - 3$ $= 2 \times 5 - 3$ $\quad 2 \times 3 - 3$ $= 10 - 3$ $\quad 6 - 3$ $= 7$ $\quad 3$		

Qn	Solutions	Marks	Comments+Criteria
(c) (i)	 Vertex $(2,3)$ $a = 2$ $(x-2)^2 = -4a(y-3)$ $(x-2)^2 = -8(y-3)$		no penalty for inconsistent scale or for incorrect y intercept. 1. correct directrix 1. correct eqn 1. focal length
	 $\lvert 2x-6 \rvert = -1$ has no solutions.		

Q3

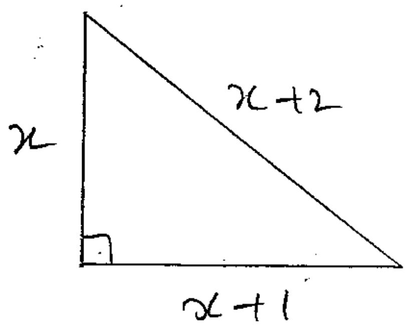
Year 11 2U Kambala Preliminary Examination solutions Sept 2011

Qn	Solutions	Marks	Comments+Criteria
(a) (i)	$C(10,6)$ $A(5,0)$ $m = \frac{0-6}{5-10} = \frac{6}{5}$	1	
(ii)	$y-0 = \frac{6}{5}(x-5)$ $5y = 6x-30$ $0 = 6x-5y-30$	1	
(iii)	cuts y axis $x=0$ $0 = 0-5y-30$ $5y=-30$ $\therefore y=-6$ $M(0,-6)$	1	
(iv)	$A = \frac{1}{2}bh = \frac{1}{2}\times 16\times 5$ $= 40u^2$	1	
(v)	$B(0,10)$ $A(5,0)$ $m = \frac{0-10}{5-0} = -2$ $\therefore$ -ve $\therefore$ obtuse angle acute angle $63°26'$ (n.m) obtuse $180 - 63°26'$ $116°34'$		

Q3

Year 11 2U Kambala Preliminary Examination solutions Sept 2011

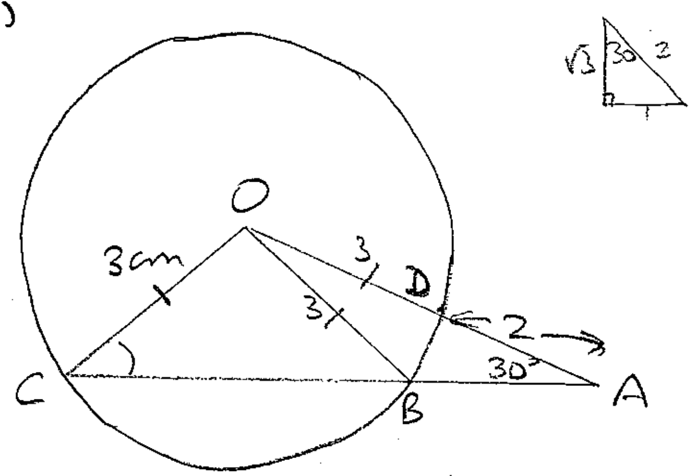
Qn	Solutions	Marks	Comments+Criteria
(b)	LHS: $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta}$		
	$\dfrac{1+\cancel{\sin\theta}+1-\cancel{\sin\theta}}{(1-\sin\theta)(1+\sin\theta)}$	1	
	$\dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta = RHS$	1	
(c)	$f(x) = 1-x^2$		
	$f(x+a) = 1-(x+a)^2$	1	
	$= 1-(x^2+2ax+a^2)$		
	$= 1-x^2-2ax-a^2$		
	$\dfrac{f(x+a)-f(x)}{a} = \dfrac{\cancel{1}\cancel{-x^2}-2ax-a^2\cancel{-1}\cancel{+x^2}}{a}$	1	
	$= \dfrac{\cancel{a}(-2x-a)}{\cancel{a}}$		
	$= -(2x+a)$	1	
(d)	PA PB		
	$\left(\dfrac{y-4}{x-2}\right)\cdot\left(\dfrac{y-2}{x+4}\right) = -1$		
	$\dfrac{y^2-6y+8}{x^2+2x-8} = -1$		
	$y^2-6y+8 = -1(x^2+2x-8)$		
	$y^2-6y+\cancel{8} = -x^2-2x+\cancel{8}$		
	$x^2+2x+y^2-6y=0$		
	$x^2+2x+\boxed{1^2}+y^2-6y+(-3)^2 = 1+9$		
	$(x+1)^2+(y-3)^2 = 10$		
	Circle centre $(-1,3)$ $r=\sqrt{10}$		

Qn	Solutions	Marks	Comments+Criteria
4	(a) 		
	$(x+2)^2 = x^2 + (x+1)^2$	1	
	$\cancel{x^2} + 4x + 4 = x^2 + \cancel{x^2} + 2x + 1$		
	$0 = x^2 - 2x - 3$		
	$(x-3)(x+1) = 0$	1	
	$x = 3,\ x = -1.$		
	but $x > 0$, $\therefore x = 3$ only.	1	
	(b) (i) $LHS = 1 + \frac{2}{x} = \frac{x}{x} + \frac{2}{x}$		
	$= \frac{x+2}{x}$		
	$= RHS$	1	proof
	(ii) $y = \frac{x+2}{x} = 1 + \frac{2}{x}$		
	D: all real x except $x = 0$	1	
	R: all real y except $y = 1$.	1	

Qn	Solutions	Marks	Comments+Criteria
4 ctd	(c) (i) $3x+4y-3=0$ $x^2+y^2=16$ $(0,0)$ x y $a=3,\ b=4,\ c=-3.$ $d=\dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $=\dfrac{\lvert 3\times 0+4\times 0-3\rvert}{\sqrt{3^2+4^2}}$ $=\dfrac{\lvert -3\rvert}{\sqrt{9+16}}$ $=\dfrac{3}{\sqrt{25}}$ $\therefore d=\dfrac{3}{5}$ units	1 1	correct substitution into correct formula
	(ii) $d=\dfrac{3}{5}$ and $r=4$ units $d<r$ $\therefore$ the line is a secant.	1	proof
	(d) $\dfrac{8^x-2^x}{2^x}$ $=\dfrac{(2^3)^x-2^x}{2^x}$ $=\dfrac{(2^x)^3-2^x}{2^x}$ $=\dfrac{\cancel{2^x}((2^x)^2-1)}{\cancel{2^x}}$	1 1	use of index laws factorisation

$= 2^{2x}-1$

and simplification

Qn	Solutions	Marks	Comments+Criteria
4 ctd	(e)  		
	(i) In $\triangle COB$, $\dfrac{\sin 30}{3} = \dfrac{\sin(\angle OCB)}{5}$	1	Correct substitution into sine rule
	$\sin(\angle OCB) = 5 \times \dfrac{\frac{1}{2}}{3}$ $\sin(\angle OCB) = \dfrac{5}{6}$ $\therefore \angle OCB = \sin^{-1}\left(\dfrac{5}{6}\right)$ $= 56°26'33.68''$ $= 56°$ (nearest deg.)	1	
	(ii) $\triangle OCB$ is isosceles (equal radii). $OC = OB = 3$ cm $\therefore$ base $\angle$s $=$. $\angle COB = 180 - 2 \times 56$ $= 68°$	1	
	A of $\triangle OCB = \frac{1}{2} ab \sin C$ $= \frac{1}{2} \times 3 \times 3 \times \sin 68°$ $= 4.1723\ldots$	1	

$= 4.17$ cm^2 (2dp's)