Student Number: ______________________

2013

PRELIMINARY EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen. Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 6–9.

Total Marks – 65

Section I Pages 2–3

5 marks

- Attempt Questions 1–5
- Allow about 10 minutes for this section

Section II Pages 4–7

60 marks

- Attempt Questions 6–9
- Allow about 1 hours 50 minutes for this section

Section I

5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5.

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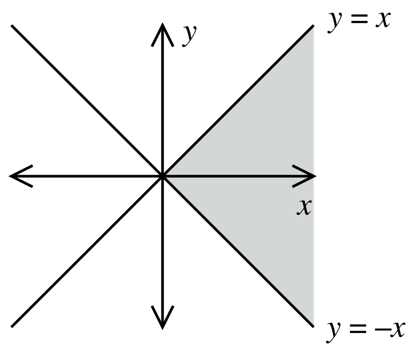
1 What is the value of $\sqrt{9.87^2 - 1.23^2}$, correct to 3 significant figures?

(A) 2.94 (B) 8.64 (C) 9.79 (D) 9.95

2



Which two inequalities best describe the shaded region shown in the graph above?

(A) $y \geq x$ and $y \geq -x$ (B) $y \geq x$ and $y \leq -x$ (C) $y \leq x$ and $y \geq -x$ (D) $y \leq x$ and $y \leq -x$

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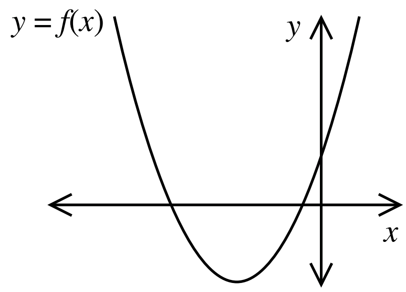
3 The diagram below shows the graph of a quadratic function.



Which of the following statements is true?

(A) $a > 0$ and $\Delta > 0$ (B) $a > 0$ and $\Delta < 0$ (C) $a < 0$ and $\Delta > 0$ (D) $a < 0$ and $\Delta < 0$

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4 What is the angle of inclination of the line $3x - 5y - 7 = 0$ with the positive direction of the x-axis?

(A) 30°58' (B) 59°2' (C) 120°58' (D) 149°2'

5 Which of these expressions is equal to $\tan(180 - \theta)^\circ \times \tan(90 - \theta)^\circ$?

(A) 1 (B) -1 (C) $\tan^2\theta$ (D) $-\tan^2\theta$

Section II

60 marks
Attempt Questions 6–9
Allow about 1 hours 50 minutes for this section

Start each question on a new page. Extra writing pages are available.

All necessary working should be shown in every question.

Question 6 (15 marks) Start a new page

(a) Solve the following:

(i) $\frac{x}{3} - \frac{2x-1}{4} = 1$ **2**

(ii) $|2x-9| = 3x - 1$ **3**

(b) Express $\frac{10-\sqrt{3}}{10+\sqrt{3}}$ as a fraction with a rational denominator in simplest form. **2**

(c) Simplify: $\frac{x^2-81}{x^3+27} \times \frac{x+3}{x-9}$ **2**

(d) If $\sin A = \frac{4}{5}$ and $\cos A < 0$, find the exact value of $\tan A$. **2**

(e) α and β are the roots of the equation $2x^2 - 3x + 5 = 0$. Find the value of:

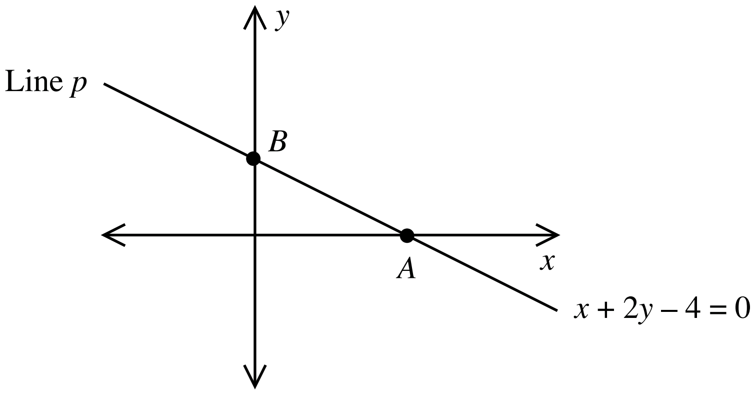
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

Question 7 (15 marks) Start a new page

(a) In the diagram below, the line p has equation $x + 2y - 4 = 0$, and cuts the x-axis and y-axis at A and B respectively.



(i) Copy the diagram onto your writing paper.

(ii) Find the coordinates of A and B. **2**

(iii) Find the gradient of AB. **1**

(iv) Show that the equation of the line q, perpendicular to the line AB and passing through the point A is $2x - y - 8 = 0$. **3**

(v) Find the area of the triangle enclosed by line q and the coordinate axes. **2**

(b) Solve the following:

(i) $3x^2 - 5x - 28 < 0$ **2**

(ii) $x^6 - 28x^3 + 27 = 0$ **3**

(c) State the domain and range of the function $y = \sqrt{2x-6}$. **2**

Question 8 (15 marks) Start a new page

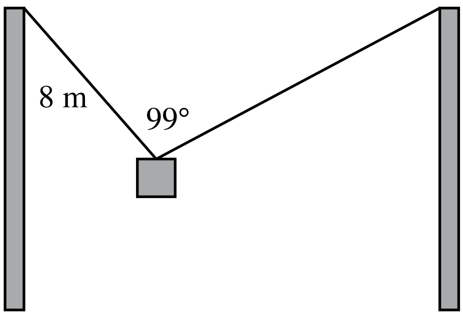
(a) For what value(s) of k will the equation $x^2 + (k + 3)x + 4k = 0$ have:

(i) roots which are equal? **2**

(ii) roots which are reciprocals of each other? **2**

(b) Find the values of A, B and C if $x^2 + x + 1 \equiv A(x - 2)^2 + B(x - 2) + C$. **3**

(c) A rope 20 metres long is tied to the top of two posts of equal height fixed into level ground. A weight is attached to the rope so that it is 8 metres from one end of the rope. The angle between the two parts of the rope is 99°, as shown in the diagram below. **3**



Find the distance between the two posts. Give your answer to correct to 1 decimal place.

(d) A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 5x - 4 & \text{for } x \geq 0 \\ px + q & \text{for } x < 0 \end{cases}$$

(i) Evaluate $f(2)$. **1**

(ii) Find an expression for $f(3) + f(-2)$. Give your answer in simplest form. **2**

(iii) Find the values of p and q so that $f(x)$ is an even function. **2**

Question 9 (15 marks) Start a new page

(a) Show that $\dfrac{x+2}{x+1}=1+\dfrac{1}{x+1}$. **2**

(b) Solve the equation $2\sin 2x - 1 = 0$ in the domain $0° \le x \le 360°$. **3**

(c) (i) Factorise $x^2 + 3x + 2$. **1**

(ii) Prove that $\dfrac{\sin^2 x + 3\sin x + 2}{\cos^2 x} = \dfrac{2+\sin x}{1-\sin x}$. **3**

(d) (i) Fully factorise the expression $2x^2 + 4x - 6$. **1**

(ii) Find the minimum value of the function $f(x) = 2x^2 + 4x - 6$. **1**

(iii) Hence, or otherwise, draw the graph of the function $y = \left|2x^2 + 4x - 6\right|$. Use at least half a page for your graph. Ensure that you label the key features of the graph. **2**

(iv) On the same diagram, draw the graph of the function $y = x + 2$. **1**

(v) Hence state the **number** of solutions for the equation $x + 2 = \left|2x^2 + 4x - 6\right|$. **1**

End of paper

2 UNIT MATHEMATICS
2013 PRELIMINARY EXAMINATION

SECTION I

1 $\sqrt{9.87^2 - 1.23^2}$ $= \sqrt{9.87^2 - 1.23^2}$ 1. **C**

$= \sqrt{95.904}$

$= 9.793058766$

≈ 9.79

$\therefore$ Answer: **C**

2 Choosing (1,0) as a test point: 2. **C**

For $y = x$, we have $0 < 1$, so the region is defined as $y \le x$.

For $y = -x$, we have $0 > -1$, so the region is defined as $y \ge -x$.

$\therefore$ Answer: **C**

3 Parabola is concave up, so $a > 0$. 3. **A**

Parabola has two zeros, so $\Delta > 0$.

$\therefore$ Answer: **A**

4 4. **A**

$$\text{Gradient} = \frac{-a}{b} = \frac{-3}{-5} = \tfrac{3}{5}$$

$$\tan\theta = m = \tfrac{3}{5}$$

$$\theta = 30.96375653 = 30°58'$$

$\therefore$ Answer: **A**

5 $\tan(180 - \theta) \times \tan(90 - \theta)$ $= -\tan\theta \times \cot\theta$ 5. **B**

$$= -\frac{\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= -1$$

$\therefore$ Answer: **B**

SECTION II

QUESTION 6

(a) (i)

$$\frac{x}{3}-\frac{2x-1}{4}=1$$
$$\frac{4x}{12}-\frac{3(2x-1)}{12}=1$$
$$\frac{4x-3(2x-1)}{12}=1$$
$$\frac{4x-6x+3}{12}=1$$
$$\frac{-2x+3}{12}=1$$
$$-2x+3=12$$
$$-2x=9$$
$$x=-\tfrac{9}{2}$$

(ii) When $2x-9\geq 0$ (or $x\geq 4\tfrac{1}{2}$):

$$2x-9=3x-1$$
$$2x=3x+8$$
$$-x=8$$
$$x=-8$$

But since $x\geq 4\tfrac{1}{2}$, there is no solution in this case.

When $2x-9<0$ (or $x<4\tfrac{1}{2}$):

$$-(2x-9)=3x-1$$
$$-2x+9=3x-1$$
$$-2x=3x-10$$
$$-5x=-10$$
$$x=2$$

This is a valid solution.

$\therefore$ The only solution is $x=2$.

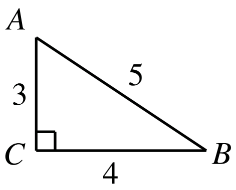
(b)
$$\frac{10-\sqrt{3}}{10+\sqrt{3}}=\frac{10-\sqrt{3}}{10+\sqrt{3}}\times\frac{10-\sqrt{3}}{10-\sqrt{3}}$$
$$=\frac{(10-\sqrt{3})^2}{(10+\sqrt{3})(10-\sqrt{3})}$$
$$=\frac{100-20\sqrt{3}+3}{(10)^2-(\sqrt{3})^2}$$
$$=\frac{103-20\sqrt{3}}{100-3}$$
$$=\frac{103-20\sqrt{3}}{97}$$

(c) $\frac{x^2-81}{x^3+27}\times\frac{x-9}{x+3} = \frac{(x+9)(x-9)}{(x+3)(x^2-3x+9)}\times\frac{x+3}{x-9}$

$= \frac{(x+9)\cancel{(x-9)}}{\cancel{(x+3)}(x^2-3x+9)}\times\frac{\cancel{x+3}}{\cancel{x-9}}$

$= \frac{x+9}{x^2-3x+9}$

(d)



Since $\sin A > 0$ and $\cos A < 0$, A lies in the 2nd quadrant.

$\therefore \tan A < 0.$

$\therefore \tan A = -\frac{4}{3}$

(e) $2x^2 - 3x + 5 = 0$

$\therefore a = 2, b = -3, c = 5$

(i)

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

(ii)

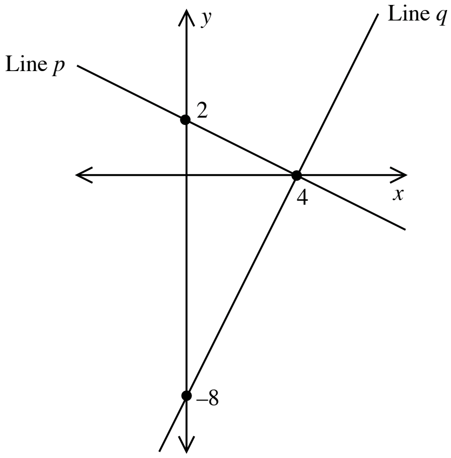
$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

(iii)

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\tfrac{3}{2})^2 - 2(\tfrac{5}{2}) = \tfrac{9}{4} - 5 = -\tfrac{11}{4}$$

QUESTION 7

(a) (i)



(ii) When $y = 0$:

$$x + 2(0) - 4 = 0$$
$$x - 4 = 0$$
$$x = 4$$

$\therefore A \equiv (4,0)$

When $x = 0$:

$$(0) + 2y - 4 = 0$$
$$2y - 4 = 0$$
$$2y = 4$$
$$y = 2$$

$\therefore B \equiv (0,2)$

(iii) Gradient of $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2-0}{0-4}$$
$$= -\tfrac{1}{2}$$

or

Gradient of $AB = \dfrac{-a}{b}$

$$= \frac{-1}{2}$$
$$= -\tfrac{1}{2}$$

(iv) Since line q is perpendicular to line p the gradient of line q is 2.

Equation of q:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 4)$$
$$y = 2x - 8$$
$$2x - y - 8 = 0$$

or

Equation of line p is $x + 2y - 4 = 0$.

$\therefore$ Equation of line q is $2x - y + k = 0$.

Since line q passes through (4,0):

$$
\begin{aligned}
2(4)-(0)+k&=0\\
8-0+k&=0\\
8+k&=0\\
k&=-8
\end{aligned}
$$

$\therefore$ Equation of line q is $2x - y - 8 = 0$.

(v) When $x = 0$:

$$
\begin{aligned}
&2(0)-y-8=0\\
&-y-8=0\\
&y=-8
\end{aligned}
$$

$\therefore$ Line q cuts the y-axis at (0,–8).

Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 8$

$= 16$ square units

(b) (i) Consider $3x^2 - 5x - 28 = 0$

$(3x + 7)(x - 4) = 0$

$\therefore x = -2\frac{1}{3}$ or 4

$\therefore$ The solution to the inequality $3x^2 - 5x - 28 < 0$ is $-2\frac{1}{3} < x < 4$.

(ii) $x^6 - 28x^3 + 27 = 0$

$(x^3)^2 - 28x^3 + 27 = 0$

Let $t = x^3$

$t^2 - 28t + 27 = 0$

$(t - 1)(t - 27) = 0$

$\therefore t = 1$ or 27

$\therefore x^3 = 1$ or 27

$\therefore x = 1$ or 3

(c) For the domain:

$$
\begin{aligned}
2x-6&\geq 0\\
2x&\geq 6\\
x&\geq 3
\end{aligned}
$$

For the range:

$y \geq 0$

QUESTION 8

(a) (i) For equal roots, $\Delta = 0$. Now:

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (k+3)^2 - 4(1)(4k) \\ &= k^2 + 6k + 9 - 16k \\ &= k^2 - 10k + 9\end{aligned}$$

Therefore we need:

$$\begin{aligned}k^2 - 10k + 9 &= 0 \\ (k-1)(k-9) &= 0 \\ k &= 1 \text{ or } 9\end{aligned}$$

(ii) For roots as reciprocals, their product of the roots is 1. Now:

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{4k}{1} \\ &= 4k\end{aligned}$$

Therefore we need:

$$\begin{aligned}4k &= 1 \\ k &= \tfrac{1}{4}\end{aligned}$$

(b) $$\begin{aligned}x^2 + x + 1 &\equiv A(x-2)^2 + B(x-2) + C \\ &\equiv A(x^2 - 4x + 4) + B(x-2) + C \\ &\equiv Ax^2 - 4Ax + 4A + Bx - 2B + C \\ &\equiv Ax^2 - 4Ax + Bx + 4A - 2B + C \\ &\equiv Ax^2 + (-4A + \mathrm{B})x + (4A - 2B + C)\end{aligned}$$

Therefore:

$$A = 1$$

and:

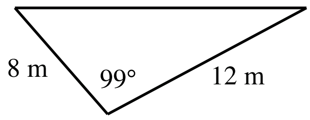
$$\begin{aligned}-4A + B &= 1 \\ -4(1) + B &= 1 \\ -4 + B &= 1 \\ B &= 5\end{aligned}$$

and:

$$\begin{aligned}4A - 2B + C &= 1 \\ 4(1) - 2(5) + C &= 1 \\ 4 - 10 + C &= 1 \\ -6 + C &= 1 \\ C &= 7\end{aligned}$$

$\therefore\ A = 1, B = 5, C = 7$

(c) Since the rope is 20 m long, the two portions of rope must be 8 m and 12 m.



Let d be the distance between the posts.
Using the cosine rule:

$$\begin{aligned} d^2 &= 8^2 + 12^2 - 2\times 8\times 12\times \cos 99° \\ &= 238.0354173 \\ d &= 15.42839646 \\ &= 15.4 \end{aligned}$$

∴ The posts are 15.4 m apart.

(d) (i) $f(2) = 5(2) - 4$
$= 6$

(ii) $f(3) + f(-2) = [5(3) - 4] + [p(-2) + q]$
$= 15 - 4 - 2p + q$
$= 11 - 2p + q$

(iii) An even function is symmetrical about the y axis.
The line $y = px + q$ has a gradient which is the same magnitude but opposite sign compared to $y = 5x - 4$, and touches the y axis at the same point.
∴ $p = -5, q = -4$

QUESTION 9

(a)

$$\begin{aligned} \text{RHS} &= 1 + \frac{1}{x+1} \\ &= \frac{x+1}{x+1} + \frac{1}{x+1} \\ &= \frac{x+1+1}{x+1} \\ &= \frac{x+2}{x+1} \\ &= \text{LHS} \end{aligned}$$

or

$$\begin{aligned} \text{LHS} &= \frac{x+2}{x+1} \\ &= \frac{x+1+1}{x+1} \\ &= \frac{x+1}{x+1} + \frac{1}{x+1} \\ &= 1 + \frac{1}{x+1} \\ &= \text{RHS} \end{aligned}$$

(b)

$$2\sin 2x - 1 = 0$$
$$2\sin 2x = 1$$
$$\sin 2x = \tfrac{1}{2}$$
$$2x = 30°, 150°, 390° \text{ or } 510°$$
$$x = 15°, 75°, 195° \text{ or } 255°$$

(c) (i) $x^2 + 3x + 2 = (x + 1)(x + 2)$

(ii) LHS = $\dfrac{\sin^2 x + 3\sin x + 2}{\cos^2 x}$

$= \dfrac{(\sin x + 1)(\sin x + 2)}{1 - \sin^2 x}$

$= \dfrac{\cancel{(\sin x + 1)}(\sin x + 2)}{\cancel{(1 + \sin x)}(1 - \sin x)}$

$= \dfrac{2 + \sin x}{1 - \sin x}$

= RHS

(d) (i) $2x^2 + 4x - 6 = 2(x^2 + 2x - 3)$
$= 2(x + 3)(x - 1)$

(ii) Zeros of function are –3 and 1.
$\therefore$ Minimum value exists when $x = -1$.
Minimum value $= 2(-1)2 + 4(-1) - 6$
$= 2 - 4 - 6$
$= -8$

or

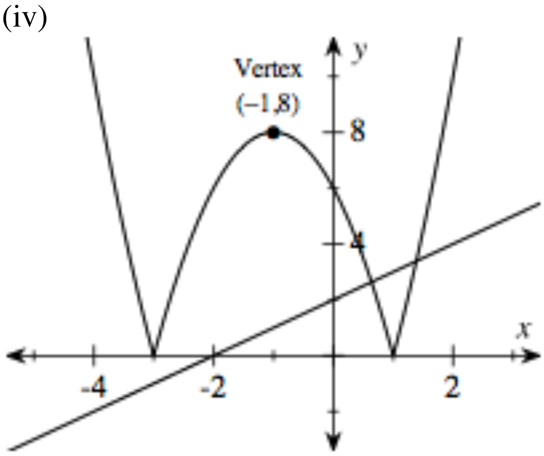
For minimum value, $x = \dfrac{-b}{2a}$

$= \dfrac{-4}{2(2)}$

$= -1$

Minimum value $= 2(-1)2 + 4(-1) - 6$
$= 2 - 4 - 6$
$= -8$

(iii), (iv)



(iv) The equation $x + 2 = \left|2x^2 + 4x - 6\right|$ has 2 solutions.