KAMBALA

Student Number: _____

Task 4
September 2016

Preliminary HSC Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- Answer Questions 1 – 10 on the multiple choice answer sheet
- Answer Questions 11 – 14 on the paper provided
- A reference sheet is provided with this paper
- In Questions 11 – 14 show relevant mathematical reasoning and calculations

Total marks – 70

Section I
10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11 - 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the answer sheet for Questions 1 – 10.

---

1 Evaluate $\sqrt{\dfrac{13.24 \times 3.7}{0.45}}$, giving your answer correct to four significant figures.

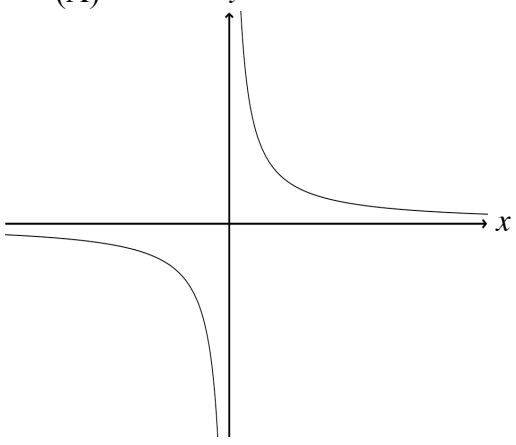
(A) 10.43 (B) 10.433 (C) 10.434 (D) 10.4337

2 The exact value of $\sin 60^\circ \tan 30^\circ$ is:

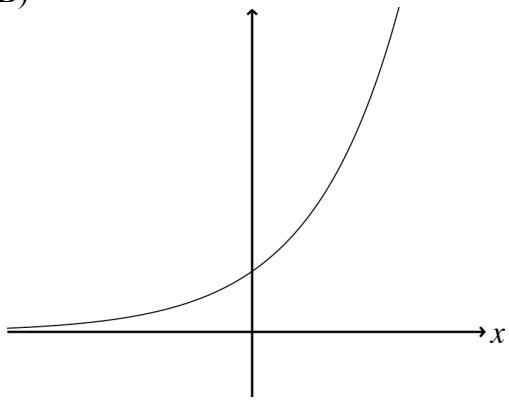
(A) $\dfrac{3}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{1}{2\sqrt{3}}$ (D) $\dfrac{1}{2}$

3 Which of the following graphs represent a function that is neither odd nor even?

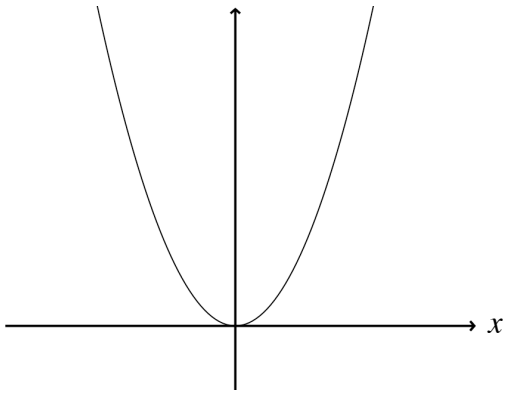
(A)



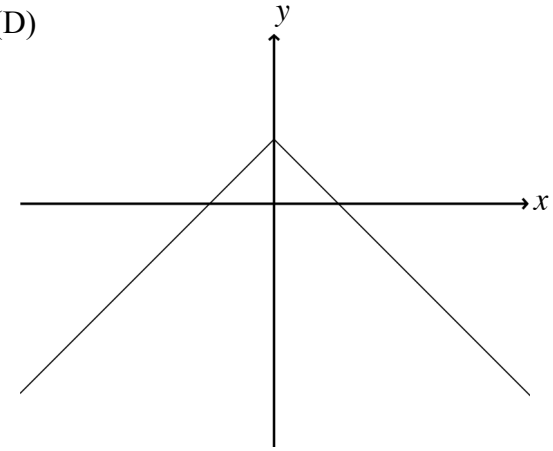
(B)



(C)



(D)

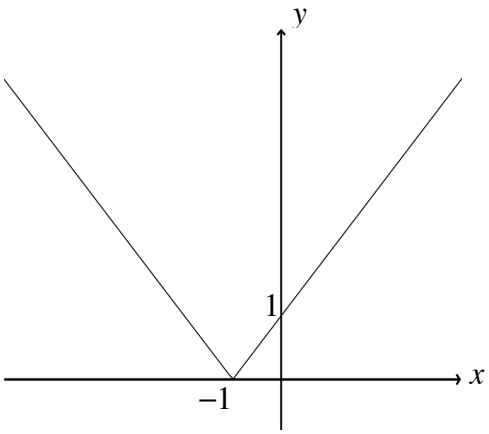


4 Let $f(x)=\sqrt{x-5}$. Which of the following is the domain of $f(x)$?

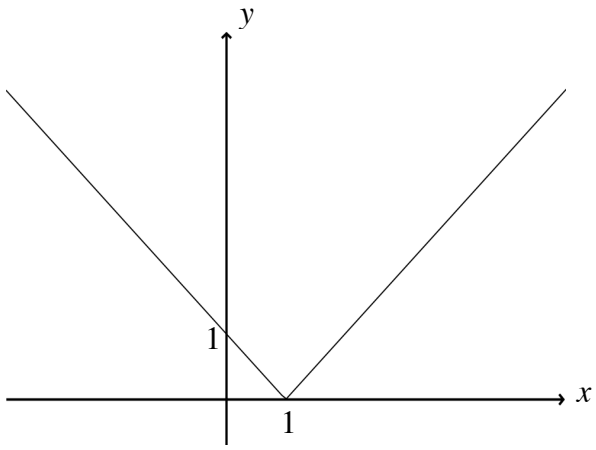
(A) All real x (B) $x \geq 5$ (C) $x \leq 5$ (D) $-5 \leq x \leq 5$

5 Which graph shows $y=|x+1|$?

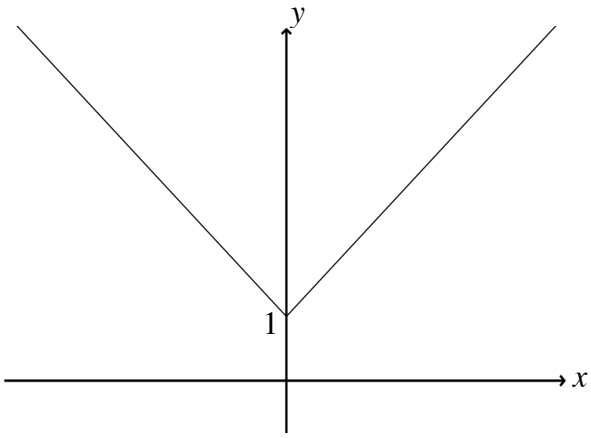
(A)



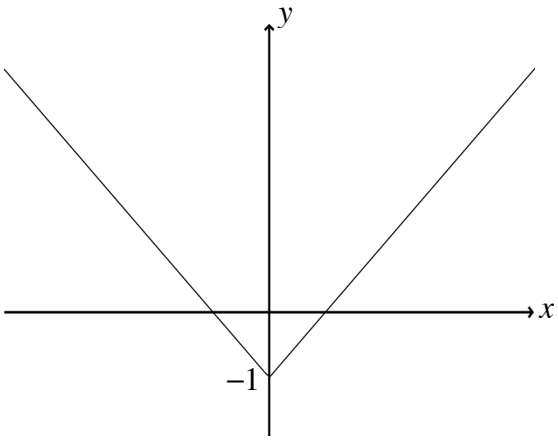
(B)



(C)



(D)



6 The solutions to the inequality $x^2+3x-4 \leq 0$ are represented by:

(A) $x \leq -4,\ x \geq 1$ (B) $x \leq -1,\ x \geq 4$ (C) $-4 \leq x \leq 1$ (D) $-1 \leq x \leq 4$

7 The locus of a point that moves so that its distance from the *y*-axis is always three times its distance from the *x*-axis is:

(A) $y=\pm 3x$ (B) $y=\pm \dfrac{x}{3}$ (C) $y=\pm 3x^2$ (D) $y=\pm \dfrac{x^2}{3}$

8 If $\operatorname{cosec}\theta = -\frac{5}{3}$ and $\cos\theta > 0$, then the value of $\cot\theta$ is:

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) $-\frac{4}{3}$ (D) $\frac{4}{3}$

9 In order to find the gradient of $f(x) = x^2 - 3x + 2$ using first principles, which one of the following expressions would, need to be simplified?

(A) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 3(x+h) + 2 - x^2 - 3x + 2}{h}$

(B) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 3(x+h) + 2 + x^2 - 3x + 2}{h}$

(C) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 3(x+h) + 2 - x^2 + 3x - 2}{h}$

(D) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 3(x+h) + 2 - x^2 - 3x - 2}{h}$

10 The expression $\frac{x^2 - 5x + 6}{4 - x^2}$ is equivalent to:

(A) $\frac{6-5x}{4}$ (B) $\frac{x-3}{2+x}$ (C) $\frac{x-3}{x-2}$ (D) $\frac{3-x}{2+x}$

End of Section I

Section II

60 Marks
Attempt Questions 11 - 14
Allow about 1 hour 45 minutes for this section

Answer each question on the writing paper provided. Start each question on a new page.
In Questions 11 - 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) The two shorter sides of a right-angled triangle have sides of lengths $2+\sqrt{3}$ and $2-\sqrt{3}$. Find the exact length of the hypotenuse. **2**

(b) Find the value of x given that $\cos 30° = \sin(x+20)°$. **1**

(c) Simplify $\dfrac{2^{x-1}\times 2^3}{2^{4-3x}}$. **2**

(d) Given that $f(x)=x^2+1$:

(i) Evaluate $f(-5)$. **1**

(ii) Find the value(s) of x such that $f(x)=5$. **2**

(e) Find the values of k for which $3x^2-4x+k=0$ has no real roots. **2**

(f) If α and β are the roots of the quadratic equation $2x^2-7x+12=0$, find the value of:

(i) $\alpha+\beta$. **1**

(ii) $\alpha\beta$. **1**

(iii) $\alpha^2+\beta^2$. **2**

(g) For what value(s) of x is the graph of $f(x)=1+\dfrac{1}{x-2}$ discontinuous? **1**

Question 12 (15 marks) **Start a new page.**

(a) If $2x^2 + 3x - 5 \equiv A(x+1)^2 + B(x+1) + C$, find A, B and C. **3**

(b) A parabola has equation $y^2 = 8x$.

(i) Find the

(I) focal length. **1**

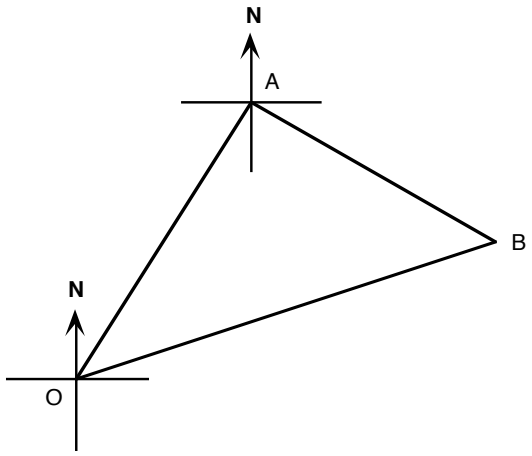
(II) coordinates of the vertex. **1**

(III) coordinates of the focus. **1**

(IV) equation of the directrix. **1**

(ii) Sketch the parabola indicating the above features. **1**

(c) A ship starts from O and sails 80 kilometres on a bearing of 035° to A. It then changes course and sails 55 kilometres on a bearing of 110° to B.



(i) Copy the diagram, marking on it the information supplied. **1**

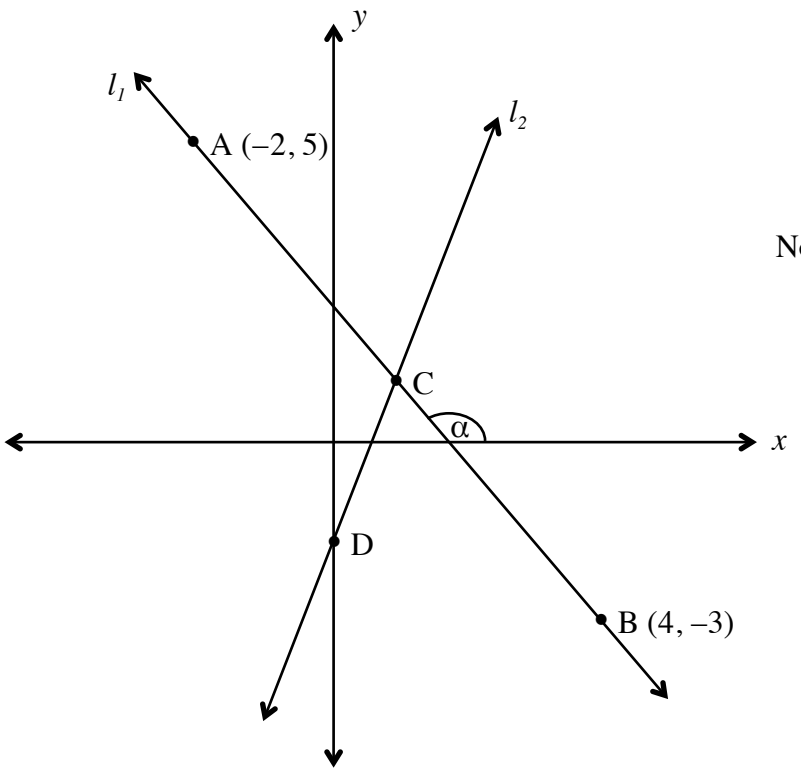
(ii) Show that $\angle OAB = 105^\circ$. **1**

(iii) Calculate the distance of B from O, correct to 1 decimal place. **2**

(d) Solve the equation $x^6 - 7x^3 - 8 = 0$ for x. **3**

Question 13 (15 marks) **Start a new page.**

(a) Points A $(-2, 5)$ and B $(4, -3)$ lie on the line l_1 as shown in the diagram below.
C is the point of intersection of lines l_1 and l_2 and is the midpoint of the interval AB.



(i) Show that the gradient of line l_1 is $-\frac{4}{3}$. **1**

(ii) Hence, show that the equation of line l_1 is $4x + 3y - 7 = 0$. **2**

(iii) Find the size of α, the angle of inclination of line l_1, to the nearest degree. **1**

(iv) Find the coordinates of C, the midpoint of AB. **2**

(v) The equation of line l_2 is $2x - y - 1 = 0$. Find the coordinates of D, the point where line l_2 crosses the y-axis. **1**

(vi) Find the length of AD, leaving your answer in simplest surd form. **2**

(vii) The line through AD has equation $3x + y + 1 = 0$. Find the shortest distance between the point C and this line. **2**

(viii) Hence, or otherwise, find the area of $\triangle ACD$. **1**

Question 13 continues on the next page

Question 13 (continued)

(b) (i) A point $P(x,y)$ moves so that its distance from the point $A(1,\ 5)$ is equal to its distance from the point $B(4,-1)$. **2**

Show that the locus of P is a straight line with equation $2x-4y+3=0$.

(ii) Describe geometrically the locus of P. **1**

Question 14 (15 marks) **Start a new page.**

(a) A function is defined by the rule:

$$f(x)=\begin{cases} x^2+4 & x\geq 0 \\ \dfrac{1}{x} & x<0 \end{cases}$$

(i) Find $f\left(k^2\right)$ **1**

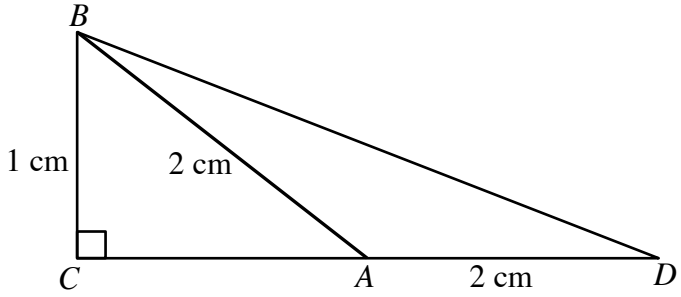
(ii) Sketch the graph of $y=f(x)$ showing all its features. **2**

(b) Consider the equation $x^2+(m-3)x+m=0$. Find the values of m for which the equation has two real and different roots. **3**

(c) Solve the equation $2\sin(\theta-60)^\circ=-\sqrt{3}$ for $-180^\circ\leq\theta\leq 180^\circ$. **3**

(d) The locus of a point $P(x, y)$ is a parabola with equation $x^2-8x+4y-12=0$. Find the coordinates of the vertex of the parabola. **2**

(e) Triangle ABC is right-angled at C with $AB = 2$ cm and $BC = 1$ cm. D is a point on CA produced such that $AD = 2$ cm as shown in the diagram below.



NOT TO SCALE

(i) Show that $\angle ADB=15^\circ$. **2**

(ii) Hence show that the exact value of $\tan 15^\circ=2-\sqrt{3}$. **2**

End of Section II

Student Number: _____

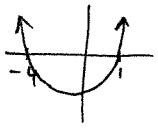
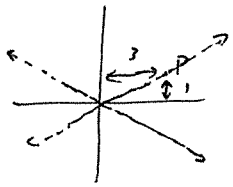
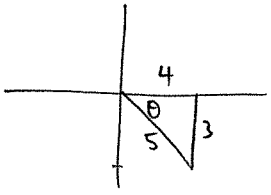
Mathematics

Task 4 Preliminary HSC Examination
September 2016

Section I

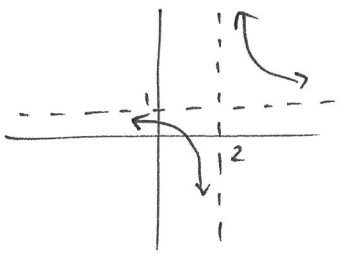
Multiple-Choice Answer Sheet
Circle your response

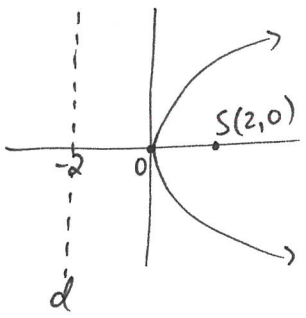
1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D

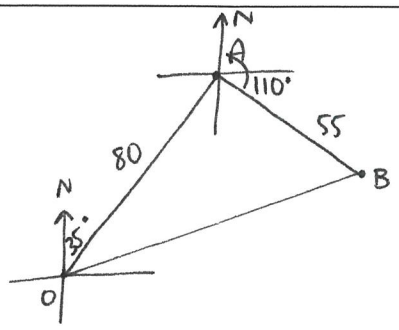
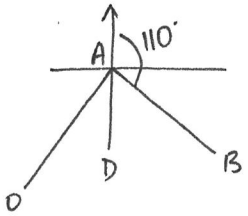
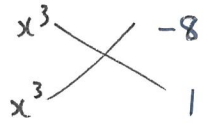
Qn	Solutions	Marks	Comments & Criteria
	Section 1 :		
1.	(A) 10.43	1	
2.	$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$ (D)	1	
3.	(B)	1	
4.	$x - 5 \geq 0$ $x \geq 5$ (B)	1	
5.	(A)	1	
6.	$(x+4)(x-1) \leq 0$  $-4 \leq x \leq 1$ (C)	1	
7.	 $y = \pm\frac{1}{3}x$ (B)	1	
8.	$\sin = \frac{-3}{5}$  $\tan\theta = \frac{-3}{4}$ $\cot\theta = \frac{-4}{3}$ (C)	1	

Qn	Solutions	Marks	Comments & Criteria
9.	$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 - 3x + 2)}{h}$ $\lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 2 - x^2 + 3x - 2}{h}$ (C)		
10.	$\frac{(x-3)(x-2)}{(2-x)(2+x)}$ $\frac{(x-3)(x-2)}{-1(x-2)(2+x)}$ $\frac{(x-3)}{-1(2+x)}$ $-1 \times \frac{(x-3)}{2+x}$ $\frac{3-x}{2+x}$ (D)		

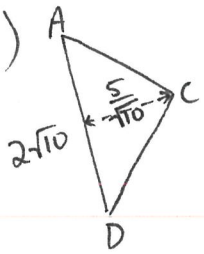
Qn	Solutions	Marks	Comments & Criteria
Q11	Section II:		
(a)	$c^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$ $c^2 = 4 + 4\sqrt{3} + 3 + 4 - 4\sqrt{3} + 3$ $c^2 = 14$ $c = \sqrt{14}$	1 for working 1 for exact area	
(b)	$\cos 30^\circ = \sin(90^\circ - 30^\circ)$ $\cos 30^\circ = \sin 60^\circ$ $\sin 60 = \sin(x+20)$ $x = 40$	1 for answer	
(c)	$2^{(x-1)} + 3 - (4-3x)$ $= 2^{4x-2}$	1 for working 1 for answer	
(d) i)	$f(-5) = (-5)^2 + 1$ $= 25 + 1$ $= 26$	1 for answer	
ii)	$5 = x^2 + 1$ $x^2 = 4$ $x = \pm 2$	1 for working 1 for answer	

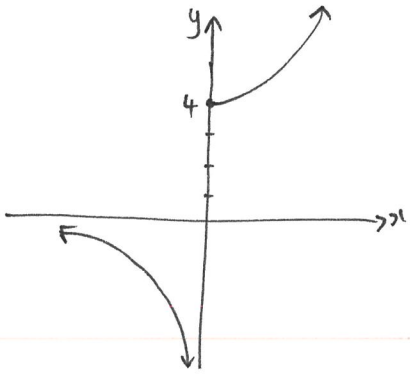
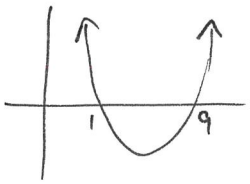
Qn	Solutions	Marks	Comments & Criteria
(e)	$b^2 - 4ac < 0$ $16 - 12k < 0$ $-12k < -16$ $k > \frac{4}{3}$		1 for working 1 for answer
(f) i)	$\alpha + \beta = \frac{-b}{a}$ $= \frac{7}{2}$		1 for answer
ii)	$\alpha\beta = \frac{c}{a}$ $= \frac{12}{2}$ $= 6$		1 for answer
iii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{7}{2}\right)^2 - 2(6)$ $= \frac{49}{4} - 12$ $= \frac{1}{4}$		1 for working 1 for answer
(g)	$x = 2$ 		1 for answer (must be an equation)

Qn	Solutions	Marks	Comments & Criteria
Q12 (a)	$A(x+1)^2 + B(x+1) + C$ $A(x^2+2x+1) + Bx + B + C$ $Ax^2 + 2Ax + A + Bx + B + C$ $Ax^2 + (2A+B)x + (A+B+C)$ $A = 2$ $2A + B = 3$ $4 + B = 3$ $B = -1$ $A + B + C = -5$ $2 - 1 + C = -5$ $C = -6$		1 for each A, B & C. ↳ if incorrect values, progress marks may be awarded.
(b) i)	$y^2 = 8x$		
	(i) focal length: $4a = 8$ $a = 2$	1	
	(ii) vertex: $(0,0)$	1	
	(iii) focus: $(2,0)$	1	
	(iv) directrix: $x = -2$	1	
ii)		1	

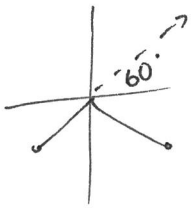
Qn	Solutions	Marks	Comments & Criteria
(c) i)		1	for all information
ii)	 $\angle DAB = 180^\circ - 110^\circ$ $= 70^\circ$ $\angle OAD = 35^\circ$ (alternate angles) $\therefore \angle OAB = 70 + 35$ $= 105^\circ$	1	
iii)	$x^2 = 80^2 + 55^2 - 2 \times 80 \times 55 \times \cos 105$ $x^2 = 11702.6076$ $x = 108.2$ km	1 for working 1 for answer	
(d)	$(x^3)^2 - 7x^3 - 8 = 0$  $(x^3 - 8)(x^3 + 1) = 0$ $x^3 = 8$ $x^3 = -1$ $x = 2$ $x = -1$	1 for factorisation 1 for each answer	

Qn	Solutions	Marks	Comments & Criteria
Q13 a)	i) $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-3-5}{4+2}$ $= \frac{-8}{6}$ $= \frac{-4}{3}$		1 for working to show gradient
	ii) $y+3 = \frac{-4}{3}(x-4)$ $3y+9 = -4x+16$		1 for working
	$4x+3y-7=0$		1 for answer
	iii) $m = \tan\theta$ $\frac{-4}{3} = \tan\theta$		
	$\theta = 53°$ $\theta = 180-53 = 127°$		1 for answer
	iv) $M = \left(\frac{-2+4}{2}, \frac{5-3}{2}\right)$		1 for working
	$= (1,1)$		1 for answer
	v) $2x-y-1=0$ let $x=0$ $-y-1=0$ $y=-1$		
	$D: (0,-1)$		1 for coordinates

Qn	Solutions	Marks	Comments & Criteria
	vi) $d = \sqrt{(-2-0)^2 + (5+1)^2}$ $d = \sqrt{(-2)^2 + 6^2}$ $d = \sqrt{4+36}$ $d = \sqrt{40}$ $d = 2\sqrt{10}$		1 for working 1 for answer in simplest surd form
	vii) $\dfrac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ $= \dfrac{\lvert 3 + 1 + 1 \rvert}{\sqrt{9+1}}$ $= \dfrac{5}{\sqrt{10}}$		1 for working 1 for answer
	viii)  $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 2\sqrt{10} \times \frac{5}{\sqrt{10}}$ $= 5$ units^2		1 for answer

Qn	Solutions	Marks	Comments & Criteria
(b)	i) $PA^2 = PB^2$ $(x-1)^2 + (y-5)^2 = (x-4)^2 + (y+1)^2$ $x^2 - 2x + 1 + y^2 - 10y + 25 = x^2 - 8x + 16 + y^2 + 2y + 1$ $6x - 12y + 9 = 0$ $2x - 4y + 3 = 0$	1 for working 1 for answer	
	ii) Perpendicular bisector of AB	1 for appropriate geometric description	
Q14 a)	i) $f(k^2) = (k^2)^2 + 4$ $= k^4 + 4$	1 for answer	
	ii) 	1 for each part of the curve -1 for any errors	
(b)	$b^2 - 4ac > 0$ $(m-3)^2 - 4m > 0$ $m^2 - 6m + 9 - 4m > 0$ $m^2 - 10m + 9 > 0$ $(m-9)(m-1) > 0$ $m < 1, \quad m > 9$ 	1 for substitution of Δ 1 for factorisation 1 for answer	

Qn	Solutions	Marks	Comments & Criteria
(c)	$2\sin(\theta - 60^\circ) = -\sqrt{3}$ $\sin(\theta - 60^\circ) = \frac{-\sqrt{3}}{2}$ $-180^\circ \leq \theta \leq 180^\circ$ $-240^\circ \leq \theta - 60 \leq 120^\circ$ $-240^\circ \leq u \leq 120^\circ$ $\sin u = -\frac{\sqrt{3}}{2}$ $u = -60^\circ, -120^\circ$ since $x = u + 60$ $\theta = 0^\circ, -60^\circ$	1 for domain 1 for working with 'u' 1 for answers	
(d)	$x^2 - 8x + 4y - 12 = 0$ $x^2 - 8x = -4y + 12$ $(x-4)^2 = -4y + 12 + 16$ $(x-4)^2 = -4y + 28$ $(x-4)^2 = -4(y-7)$ $V: (4, 7)$	1 for working 1 for vertex	



Qn	Solutions	Marks	Comments & Criteria
(e) i)	$a^2+b^2=c^2$ $a^2+1=4$ $a^2=3$ $a=\sqrt{3}$ $\therefore CD=2+\sqrt{3}$ $\tan\theta=\frac{1}{2+\sqrt{3}}$ $\theta=15^\circ$ $\sin(\angle CAB)=\frac{1}{2}$ $\therefore \angle CAB=30^\circ$ $\angle BAD=150^\circ\ (180-30^\circ)$ $\triangle ABD$ is an isosceles $\therefore \angle ADB=\frac{1}{2}\times 30$ $=15^\circ$	Option 1: 1 to get to CD 1 for showing $\theta=15^\circ$ Option 2: 1 for $\angle CAB=30^\circ$ 1 for recognition of isosceles & $\theta=15^\circ$	
ii)	$\tan 15=\frac{1}{2+\sqrt{3}}$ $\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{4-3}$ $=2-\sqrt{3}$ $\therefore \tan 15=2-\sqrt{3}$	1 for tan ratio statement 1 for rationalising.	