Teacher's Name ________________________

Student Number

Knox Grammar School
2012
Year 11 Yearly Examination

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- Answer in simplest exact form unless otherwise stated
- Show all necessary working in questions 11 – 16

Total Marks – 100

Section I 10 Marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II 90 marks

- Attempt Questions 11 – 16

Examiner: E. Choy

Number of Students in Course: 130

This paper MUST NOT be removed from the examination room

Section I Multiple Choice

10 Marks
Attempt Question 1 – 10.
Allow approximately 15 minutes for this section.

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1. Evaluate $\dfrac{3.23}{0.45+1.2^2}$ correct to 3 significant figures.

(A) 1.71 (B) 8.62

(C) 8.617 (D) 1.708

2. If $\dfrac{2}{2-\sqrt{3}} = a+\sqrt{b}$ then the values of a and b are:

(A) − 4 and 12 (B) 4 and 12

(C) 4 and 3 (D) 2 and 3

3. The solutions to the equation $2x^2 - 7x - 2 = 0$ are:

(A) $\dfrac{-7\pm\sqrt{33}}{4}$ (B) $\dfrac{-7\pm\sqrt{65}}{4}$

(C) $\dfrac{7\pm\sqrt{33}}{4}$ (D) $\dfrac{7\pm\sqrt{65}}{4}$

4. The domain and range for the function $y=\sqrt{7-x}$ is:

(A) $x \le 7$; $y \ge 0$ (B) $0 \le x \le 7$; $y \ge 0$

(C) $x \ge 0$; $y \ge 0$ (D) All real x, All real y

5. A function is defined by $f(x) = x^2 - 4x + 5$.
The point on $y = f(x)$ which has a gradient of 2 is:

(A) (3, –1) (B) (2, 1)

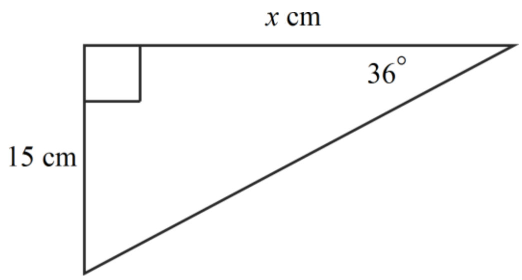
(C) (3, 2) (D) (2, 3)

6. The derivative of $2x^{\frac{1}{2}}$ is :

(A) $-\frac{2}{\sqrt{x}}$ (B) $-\frac{1}{\sqrt{x}}$

(C) $\frac{1}{\sqrt{x^3}}$ (D) $\frac{1}{\sqrt{x}}$

7. The calculation which would be used to find the value of x is:



(A) $15 \tan 36^\circ$ (B) $15 \cos 36^\circ$

(C) $\frac{15}{\tan 36^\circ}$ (D) $\frac{15}{\cos 36^\circ}$

8. The equation $3x^2+2x-1=0$ has roots α and β. The value of $2\alpha+2\beta$ is :

(A) $-\frac{4}{3}$ (B) $-\frac{2}{3}$

(C) $-\frac{1}{3}$ (D) 10

9. The equation $x^2+y^2-6x+6=0$ represents a circle. The coordinates of the centre of the circle are:

(A) (3, –3)

(B) (–3, –3)

(C) (3, 0)

(D) (–6, 0)

10. For what values of k is the expression $-\frac{x^2}{4}-x-k$ negative definite ?

(A) $k < 1$ (B) $k > 1$

(C) $k < 4$ (D) $k > 4$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\sqrt[3]{\dfrac{651}{4\pi}}$ correct to two decimal places. **2**

(b) Simplify $\dfrac{a^2-25}{a-5}$ **1**

(c) (i) Draw a neat sketch of the parabola $y=(2x-1)(x+3)$ noting x and y intercepts. **2**

(ii) Hence, determine the values of x for which $(2x-1)(x+3) < 0$. **1**

(d) If $f(x)=x^2-3x+2$ and $g(x)=x^2-x-4$, for what value of x is $f(x)=g(x)$? **1**

(e) On separate number planes, sketch neatly each of the following curves:

(i) $y=\dfrac{1}{x}$ **1**

(ii) $y=|x|+1$ **1**

(iii) $y=\sqrt{25-x^2}$ **1**

(f) Find the exact value of θ such that $2\cos\theta=1$ where $0^\circ \le \theta \le 90^\circ$. **1**

(g) Solve the pair of simultaneous equations: **2**

$$2x+y=7$$
$$x-2y=1.$$

(h) What are the domain and range of $y=x^2\sqrt{1+x}$? **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

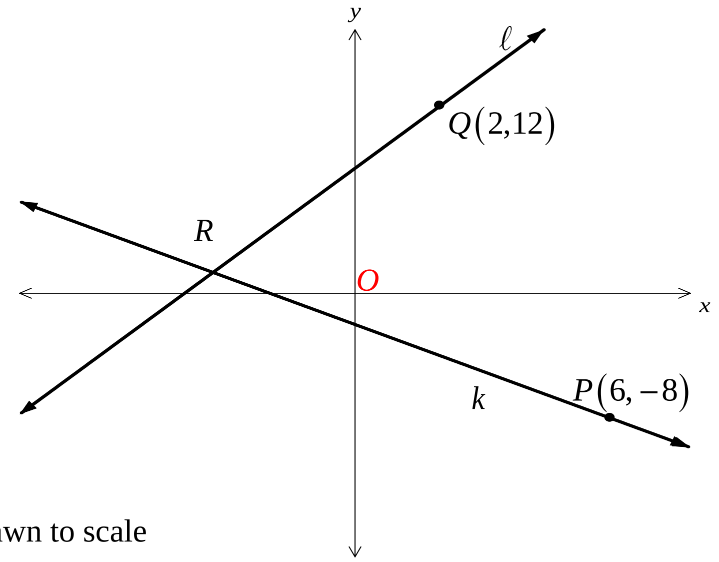


Diagram not drawn to scale

(a) A line ℓ whose equation is $2x - y + 8 = 0$ has the point $Q(2,12)$ on it. Another line k has gradient -2 and passes through the point $P(6,-8)$. The lines k and ℓ intersect at the point R.

(i) Show that the equation of the line k is $2x + y - 4 = 0$. **1**

(ii) Show that the co-ordinates of R are $(-1, 6)$. **2**

(iii) Show that the distance of QR is $3\sqrt{5}$. **1**

(iv) Find the perpendicular distance from P to the line ℓ.
Leave your answer in surd form. **2**

(v) Find the area of ΔPQR. **1**

Question 12 continues on page 6

Question 12 continued

(b) Shade the region in the plane for which the inequations $y \le 4 - x^2$ and $y \ge 0$ hold simultaneously. **2**

(c) Find the locus of the point $P(x, y)$ whose distance from the point $A(4, 0)$ is always twice its distance from the point $B(1, 0)$. **2**

(d) The general form of a line ℓ passing through the point of intersection of $x - 2y + 5 = 0$ and $x + y + 2 = 0$ is $(x - 2y + 5) + k(x + y + 2) = 0$.

(i) Show that the gradient of line ℓ is $\dfrac{1+k}{2-k}$. **2**

(ii) Hence find the equation of the line passing through the point of intersection of $x - 2y + 5 = 0$ and $x + y + 2 = 0$ **and** has gradient 2. **2**
Leave your answer in general form.

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Two ships set sail from a point *O*. The first ship sails for 45 nautical miles on a course bearing N32°W to a point *Y* and the second ship sails for 63 nautical miles on a course bearing N58°E to a point *Z*.

(i) Draw a half-page diagram representing the above information. **1**

(ii) Find, to the nearest nautical mile, the distance *YZ*. **2**

(b) The adjacent sides of a parallelogram have lengths 4 cm and 6 cm and they include an angle of 60°. Calculate the length of the shorter diagonal to 2 decimal places. **2**

(c) Given that $\tan\theta = -\dfrac{5}{12}$ and θ is obtuse, find $\sin\theta$ and $\sec\theta$. **2**

(d) If $\sin\theta = x$, express $\dfrac{1-\cos^2\theta}{\sec^2\theta}$ in terms of x. **2**

(e) (i) Show that $\cos\theta\tan\theta = \sin\theta$ **1**

(ii) Hence solve $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$ for $0^\circ \le \theta \le 360^\circ$. **3**

(f) The distance, *d* km, between two ports P_1 and P_2 can be found from the calculation:

$$d^2 = 900\left[\left(\frac{\sin 44^\circ}{\sin 26^\circ}\right)^2 + \left(\frac{\sin 19^\circ}{\sin 21^\circ}\right)^2\right].$$

(i) Determine the distance, d, from port P_1 to port P_2, correct to two decimal places. **1**

(ii) Determine the time it will take to sail from P_1 to P_2 at an average speed of 10km/h. *Give your answer to the nearest minute*. **1**

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) If $g(x) = x^2 + 3x + 5$, show that $g(a+1) = g(a) + 2(a+2)$ **2**

(b) Find $\lim_{x \to \infty} \dfrac{x^2 - x - 12}{x^2 - 4x}$ **1**

(c) Differentiate:

(i) $x^4 + x^{-1}$ **1**

(ii) $(x^2 - 1)^5$ **2**

(iii) $\dfrac{x-3}{5x+7}$ **2**

(d) Find the equation of the tangent to the curve $y = 2x^2 - 4x + 1$ at the point where $x = 1$. **2**

(e) A function is defined by $f(x) = \begin{cases} x, & \text{for } -1 \le x \le 1 \\ x(2-x), & \text{for } 1 < x \le 2 \end{cases}$

(i) Find $f(0) + f(1.5)$ **1**

(ii) Sketch the graph $y = f(x)$ for $-1 \le x \le 2$. **2**

(iii) The horizontal line $y = k$ meets the graph of $y = f(x)$ *exactly once*. **2**
Find the range of values of k for this to be true.

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) If α and β are the roots of the equation $2x^2+7x+4=0$, find the value of :

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

(b) For the parabola $(y-1)^2=4(x+4)$, find:

(α) (i) The co-ordinates of the vertex. **1**

(ii) The co-ordinates of the focus. **1**

(iii) The equation of the directrix. **1**

(iv) The length of the latus rectum. **1**

(v) Its *x* and *y* intercepts. **2**

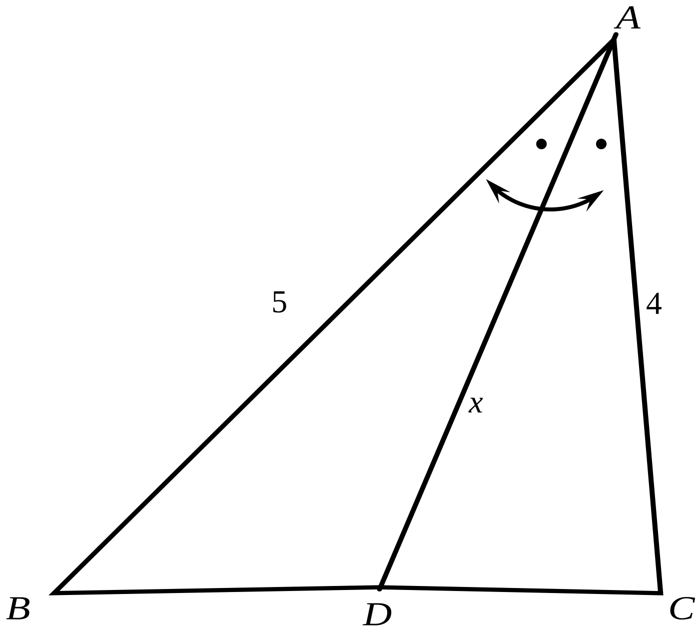
(β) Hence, draw a neat sketch of the parabola. **1**

(c) Find the values of *m* for which the quadratic equation $x^2-3mx+9=0$ has real and distinct roots. **2**

(d) If one of the roots of $x^2+ax+b=0$ is three times the other, prove that $16b=3a^2$. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet

(a)



The above figure shows a triangle ABC such that $AB = 5cm$, $AC = 4cm$ and $\angle BAC = 60^\circ$. D is a point on BC such that AD bisects $\angle BAC$.

(i) Express the area of ΔABD in terms of x. **1**

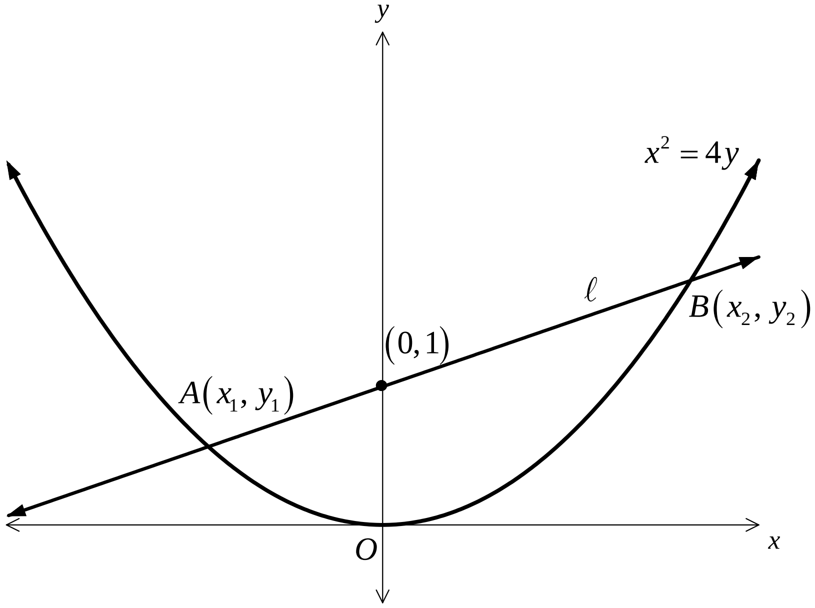
(ii) Hence, using triangles ADC and ABC, show that $x = \dfrac{20\sqrt{3}}{9}$. **2**

(b) For what values of m is $y = mx - 6$ a tangent to the parabola $y = x^2 - 2x + 3$? **3**

Question 16 continues on page 11

Question 16 continued

(c)



A straight line ℓ with gradient m passes through the point $(0,1)$. The line ℓ also cuts the parabola $x^2 = 4y$ at two points $A(x_1, y_1)$ and $B(x_2, y_2)$ as shown above.

(i) Show that the equation of the line ℓ is $y = mx + 1$. **1**

(ii) Line ℓ cuts the parabola at two distinct points. Show that x_1 and x_2 are the roots of the equation $x^2 - 4mx - 4 = 0$. **2**

(iii) Show that $(x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_1x_2$.
Hence, or otherwise find $(x_2 - x_1)^2$ in terms of m. **2**

(iv) From (iii), show that $AB = 4(1 + m^2)$. **3**

End of Examination

Suggested Solution (s) | **Comments**

MULTIPLE CHOICE: SECTION I

1. 1·71 (calc.) 1·708 994 709… ∴ (A)
2. $\frac{2}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{2(2+\sqrt3)}{2^2-(\sqrt3)^2} = \frac{4+2\sqrt3}{1} = 4+\sqrt{12}$
   ∴ a=4 & b=12. So (B)
3. $x = \frac{-(-7)\pm\sqrt{(-7)^2-4\times2\times(-2)}}{2\times2} = \frac{7\pm\sqrt{49+16}}{4} = \frac{7\pm\sqrt{65}}{4}$ ∴ (D)
4. Domain satisfies $7-x \geq 0 \Rightarrow x \leq 7$. So must be (A). For completeness, range satisfies $y \geq 0$.
5. Gradient function is $f'(x) = 2x-4$. Want $2x-4=2 \Rightarrow x=3$. However, (3,−1) does not satisfy the f^n, i.e. $f(3)\neq -1$. In fact $f(3)=2$. So (C).
6. $\frac{d}{dx}(2x^{1/2}) = x^{-1/2} = \frac{1}{\sqrt{x}}$ ∴ (D)
7. $\tan 36° = \frac{15}{x} \Rightarrow x = \frac{15}{\tan 36°}$ ∴ (C)
8. $2\alpha+2\beta = 2(\alpha+\beta) = 2\times -\frac{2}{3}$ or $-\frac{4}{3}$ ∴ (A)
9. $x^2-6x+y^2=-6$
   $\Rightarrow (x-3)^2+y^2=3$
   ∴ centre c(3,0) & so (C)
10. Require $\Delta<0$
   $\Rightarrow (-1)^2-4\times(-\frac14)\times(-k)<0$
   $\Rightarrow 1-k<0$
   $\Rightarrow k>1$ ∴ (B)

END OF SECTION I

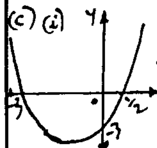
Comments: rationalising denominator; $2\sqrt3=\sqrt4\times\sqrt3=\sqrt{12}$. Quadratic Formula.

1. A 6. D
2. B 7. C
3. D 8. A
4. A 9. C
5. C 10. B

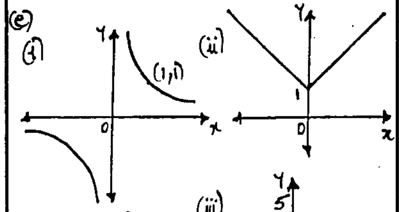
Completing the square. Central form $(x-a)^2+(y-b)^2=r^2$. Δ = discriminant, $b^2-4ac<0$

SECTION II

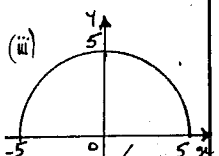
Question 11: (a) 3·73✓ (3·727838 09…)
(b) $\frac{(a-5)(a+5)}{a-5} = a+5$ ✓
(c) (i)



✓ for shape ✓ for intercepts
(ii) $-3 < x < \frac12$ ✓
(d) $x^2-3x+2 = x^2-x-4$
$\Rightarrow -2x=-6 \Rightarrow x=3$ ✓
(e) (i) (ii)



(iii)



1 mark for each shape.

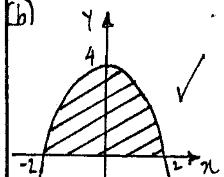
(f) $\cos\theta = \frac12 \Rightarrow \theta = 60°$ ✓ $(0°\leq\theta\leq90°)$
(g) Sub. $x=2y+1$ into $2x+y=7$
$\Rightarrow 2(2y+1)+y=7 \Rightarrow 5y+2=7$
i.e. $y=1$✓ & so $x=3$✓
(h) Domain: $x\geq-1$✓; Range $y\geq0$✓

Question 12:
(a) (i) $y-(-8) = -2(x-6)$
$\Rightarrow y+8=-2x+12$ ✓
$\Rightarrow 2x+y-4=0$
(ii) Sub. (−1,6) into $2x+y-4=0$
LHS $= 2\times(-1)+6-4=0$ = RHS
∴ (−1,6) lies on $2x+y-4=0$
Sub. (−1,6) into $2x-y+8=0$
LHS $= 2\times(-1)-6+8=0$ = RHS
∴ (−1,6) lies on $2x-y+8=0$
As (−1,6) lies on both lines, it must be R, their point of intersection

Comments: 1 for calculation, 1 for rounding. Various methods available. Point-gradient formula. ✓✓ for process; other methods available.

…/Question 12 cont. over

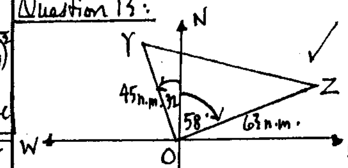
Q12 cont.
(a) (iii) $QR = \sqrt{(2-{}^-1)^2+(12-6)^2}$ ✓
$= \sqrt{3^2+6^2} = \sqrt{45}$ or $3\sqrt5$
(iv) $d_\perp = \frac{|2\times6-1\times(-8)+8|}{\sqrt{2^2+(-1)^2}}$ ✓
$= \frac{28}{\sqrt5}$✓ units $\left(=\frac{28\sqrt5}{5}\text{ optional}\right)$
(v) Area $= \frac12\times$ base $\times \perp$ height
$= \frac12\times QR\times d_\perp$
$= \frac12\times3\sqrt5\times\frac{28}{\sqrt5}$
$= 42u^2$ ✓
(b)



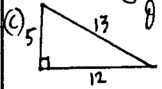
(c) (i) $PA = 2PB \Rightarrow PA^2 = 4PB^2$
$\Rightarrow (x-4)^2+(y-2)^2 = 4\{(x-1)^2+y^2\}$✓
$\Rightarrow x^2-8x+16+y^2-4y+4 = 4(x^2-2x+1+y^2)$
$\Rightarrow 3x^2+3y^2+4y=12$✓ (eventually)
(ii) Circle✓ (centre $(0,-\frac23)$; radius $=\frac23\sqrt{10}$) optional
(d) (i) $(x-2y+5)+k(x+y+2)=0$
$\Rightarrow x-2y+5+kx+ky+2k=0$
$\Rightarrow (1+k)x+(-2+k)y+(5+2k)=0$
$\Rightarrow (2-k)y = (1+k)x+(5+2k)$
$\Rightarrow y = \frac{(1+k)}{(2-k)}x + \frac{(5+2k)}{(2-k)}$ & so
gradient of line is $\frac{1+k}{2-k}$, as required.
(ii) $\frac{1+k}{2-k} = 2 \Rightarrow 1+k = 2(2-k)$
$\Rightarrow 1+k = 4-2k$
$\Rightarrow 3k=3$ or $k=1$✓
∴ eq'n of line is $x-2y+5+x+y+2=0$
$\Rightarrow 2x-y+7=0$✓ (General form)

Comments: distance form. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. $d_\perp = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$. Allow for C.T.E. from (iv). Don't deduct mark if intercepts omitted. OPTIONAL FORM $x^2+y^2+\frac43y=4$ or $x^2+(y+\frac23)^2=\frac{40}{9}$. 1 mark for meaningful progress. Make y the subject y=mx+b.

Question 13:
(a) (i)



(ii) $\angle YOZ = 90°$ ∴ $YZ^2 = 45^2+63^2$✓
$= 5994$✓
∴ $YZ \simeq 77$ n.m. (77.420 927…)
(b) By the cosine rule:
shorter diagonal $= 4^2+6^2-2\times4\times6\times\cos60°$
$= 16+36-24$ or 28
∴ shorter diagonal $=\sqrt{28} = 5\cdot29$cm (2dp)
(c)



θ in 2nd quadrant.
∴ $\sin\theta = \frac{5}{13}$ & $\sec\theta = \frac{1}{\cos\theta} = -\frac{13}{12}$✓
(d) $\frac{1-\cos^2\theta}{\sec^2\theta} = \sin^2\theta\times\cos^2\theta$
$= \sin^2\theta\times(1-\sin^2\theta)$
$= x^2(1-x^2)$ $(=x^2-x^4$ optional$)$
(e) (i) LHS $= \cos\theta\cdot\frac{\sin\theta}{\cos\theta}$✓
$= \sin\theta$
(ii) $8\sin\theta\cdot\cos\theta\cdot\tan\theta = \operatorname{cosec}\theta$
$\Rightarrow 8\sin^2\theta = \frac{1}{\sin\theta}$ using (i)
$\Rightarrow 8\sin^3\theta = 1$✓ or $\sin^3\theta = \frac18$
So $\sin\theta = \frac12 \Rightarrow \theta = 30°, 150°$✓
(f) (i) 54·80 km✓ (54.797 402…)
(ii) time $= \frac{\text{distance}}{\text{speed}}$
$= \frac{54\cdot80}{10}$
$= 5\cdot48$
$\simeq$ 5hr 29 min ✓

Comments: N.B. ∠YOZ=90°. Pyth. thm. Cosine rule available. No penalty for incorrect rounding. 5·291 502… $\frac{1}{\sec^2\theta}=\cos^2\theta$; $\cos^2\theta+\sin^2\theta=1$. 1 mark for meaningful progress. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$. 1 mark for meaningful progress. No penalty for incorrect rounding. $S=\frac{D}{T}$. Deduct mark if 5°29′ given.

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 14:

(a) LHS $= g(a+1) = (a+1)^2 + 3(a+1) + 5$
$= a^2 + 2a + 1 + 3a + 3 + 5$
$= a^2 + 5a + 9$ ✓
RHS $= g(a) + 2(a+2) = a^2 + 3a + 5 + 2a + 4$
$= a^2 + 5a + 9$ ✓
$=$ LHS.

(b) $\lim_{x\to\infty} \frac{x^2 - x - 12}{x^2 - 4x} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$ ✓

Comment: Dominant terms approach. Other methods available

(c)(i) $\frac{d}{dx}(x^4 + x^{-1}) = 4x^3 - x^{-2}$ ✓

(ii) $\frac{d}{dx}\left[(x^2-1)^5\right] = 5(x^2-1)^4 \cdot \frac{d}{dx}(x^2-1)$
$= 10x(x^2-1)^4$ ✓

Comment: chain rule or equivalent algebra.

(iii) $\frac{d}{dx}\left(\frac{x-3}{5x+7}\right) = \frac{(5x+7)\frac{d}{dx}(x-3) - (x-3)\frac{d}{dx}(5x+7)}{(5x+7)^2}$ ✓
$= \frac{(5x+7)(1) - (x-3)(5)}{(5x+7)^2}$
$= \frac{22}{(5x+7)^2}$ ✓

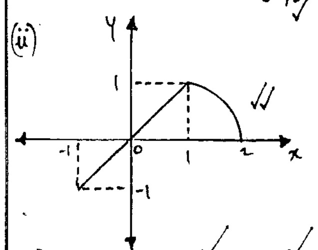
Comment: Quotient Rule or equivalent algebra

(d) Gradient function, $\frac{dy}{dx} = 4x - 4$
$= 0$ when $x = 1$
As gradient is zero, line is horiz.
So eq'n of line is $y = -1$ ✓

Comment: Equivalently $y - (-1) = 0(x-1)$

(e)(i) $f(0) + f(1.5) = 0 + 1.5 \times 0.5$
$= 0.75$, or $\frac{3}{4}$

(ii)



(iii) $-1 \le K < 0$, $K = 1$ ✓ ✓

Question 15:

(a)(i) $\alpha + \beta = -7/2$ ✓ (ii) $\alpha\beta = \frac{4}{2} = 2$ ✓
(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓
$= (-7/2)^2 - 2\times 2$
$= 33/4$ or $8\frac{1}{4}$ ✓

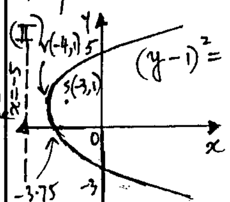
(b) (I) (i) $V(-4,1)$ (ii) $S(-3,1)$ (iii) $x = -5$ ✓

Comment: $(y-k)^2 = 4a(x-h)$

(iv). x intercept corresponds to $y = 0 \Rightarrow$
$x = -15/4$ or -3.75
. y intercepts correspond to $x = 0$
$\Rightarrow y^2 - 2y - 15 = 0 \Rightarrow (y-5)(y+3) = 0$
$\Rightarrow y = -3$ or 5

Comment: No deduction if points given in lieu of intercepts. 2 marks for all three intercepts.

(II)



$(y-1)^2 = 4(x+4)$
✓ shape
✓ features

(c) $(-3m)^2 - 4\times 1\times 9 > 0 \Rightarrow 9m^2 - 36 > 0$ ✓
$\Rightarrow m^2 - 4 > 0 \Rightarrow (m+2)(m-2) > 0$
So $m < -2$ or $m > 2$ ✓

Comment: $\Delta > 0$ i.e., $b^2 - 4ac > 0$

(d) Let α, β be the roots with $\beta = 3\alpha$.
Then $\alpha + \beta = -a$, i.e., $4\alpha = -a$ or $\alpha = -\frac{a}{4}$
Also $\alpha\beta = b$, i.e., $3\alpha^2 = b$. Sub. $\alpha = -\frac{a}{4}$
$\Rightarrow 3\times\left(\frac{-a}{4}\right)^2 = b$ or $3a^2 = 16b$, as required

Comment: $\{\alpha = -\frac{a}{4}$, $3\alpha^2 = b\}$ 1 mark. 1 mark for process.

Question 16: (a) (i) AREA of ΔABD given by
$A = \frac{1}{2}\times 5\times x\times \sin 30° = \frac{5x}{4}$ cm² ✓

Comment: $A = \frac{1}{2}ab\sin C$. Don't deduct mark if units omitted.

(ii) Similarly, area of ΔADC is:
$A = \frac{1}{2}\times 4\times x\times \sin 30° = x$ cm² &
$Ar(\Delta ABD) + Ar(\Delta ADC) = Ar(\Delta ABC)$, i.e.,
$\frac{5x}{4} + x = \frac{1}{2}\times 5\times 4\times \sin 60° \Rightarrow \frac{9x}{4} = 5\sqrt{3}$
or $x = \frac{20\sqrt{3}}{9}$, as required.

Comment: $\sin 60° = \frac{\sqrt{3}}{2}$. 1 mark for process.

(b) Solving simultaneously $\begin{cases} y = mx - 6 \\ y = x^2 - 2x + 3 \end{cases}$
$\Rightarrow x^2 - 2x + 3 = mx - 6$
$\Rightarrow x^2 - (m+2)x + 9 = 0$ Require $\Delta = 0$ (equal roots)
$\Rightarrow [-(m+2)]^2 - 4\times 1\times 9 = 0$
$\Rightarrow (m+2)^2 = 36$ ✓ $\Rightarrow m + 2 = \pm 6$, i.e., $m = -8$ or 4 ✓

Comment: Condition for tangency

... / Question 16 continued over

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 16 cont.

(c)(i) From $y = mx + b$; it is clear the y-intercept is 1 from the diagram. Thus $y = mx + 1$ as required

Comment: Gradient-intercept form. Substituting $(0,1)$ into formula is insufficient to show that it has form $y = mx + 1$.

(ii) Solving $\begin{cases} x^2 = 4y \\ y = mx + 1 \end{cases}$ simultaneously implies $x^2 = 4(mx+1)$ ✓
$\Rightarrow x^2 - 4mx - 4$. However, from graph we know that the x coordinates of the points of intersection of the graphs are x_1 & x_2. Hence x_1, x_2 are the roots of $x^2 - 4mx - 4$, as required

Comment: 1 for explanation

(iii) LHS $= (x_2 - x_1)^2$
$= x_2^2 - 2x_1x_2 + x_1^2$
RHS $= (x_2 + x_1)^2 - 4x_1x_2$
$= x_2^2 + 2x_1x_2 + x_1^2 - 4x_1x_2$
$= x_2^2 - 2x_1x_2 + x_1^2$
$=$ LHS

Comment: 1 mark.

From (ii) $x_1 + x_2 = 4m$ &
$x_1x_2 = -4$
$\therefore (x_2 - x_1)^2 = (4m)^2 - 4(-4)$
$= 16m^2 + 16$. ✓

Comment: Sum of roots. Product of roots.

(iv) From the distance formula
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
& $x_1^2 = 4y_1$, with $x_2^2 = 4y_2$
$\Rightarrow y_2 - y_1 = \frac{1}{4}x_2^2 - \frac{1}{4}x_1^2$
$= \frac{1}{4}(x_2^2 - x_1^2)$ ✓

Comment: (x_1, y_1) & (x_2, y_2) lie on parabola $x^2 = 4y$

$\Rightarrow AB = \sqrt{(x_2 - x_1)^2 + \left\{\frac{1}{4}(x_2^2 - x_1^2)\right\}^2}$
$= \sqrt{(x_2 - x_1)^2 + \frac{1}{16}(x_2 + x_1)^2(x_2 - x_1)^2}$ (Difference of squares.)
$= \sqrt{16m^2 + 16 + \frac{1}{16}(4m)^2(16m^2 + 16)}$
$= \sqrt{(16m^2 + 16) + m^2(16m^2 + 16)}$
$= \sqrt{(16m^2 + 16)(1 + m^2)}$
$= \sqrt{16(m^2 + 1)^2}$
$= 4(m^2 + 1)$, as required

Comment: 2 marks for process. 1/2 for meaningful progress. ; $m^2 + 1 > 0$

(v) If $y_1 = y_2 \Rightarrow m = 0$ ✓
$\therefore AB = 4$ units

τὸ τέλος