



2011
Preliminary Course
FINAL EXAMINATION
Monday, September 12th

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time – 3 hours.
- Write using a black or blue pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question in a new booklet.

Total marks (120)

- Attempt Questions 1- 10.
- All questions are of equal value.

Outcomes to be Assessed:

A student:

- P2** provides reasoning to support conclusions which are appropriate to the context.
- P3** performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.
- P4** chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
- P5** understands the concept of a function and the relationship between a function and its graph.

Question 1 (12 Marks)

Use a Separate Booklet

Marks

- a) Evaluate correct to 1 decimal place $\frac{\sqrt[5]{32}}{6.1+3.2}$ **2**
- b) Factorise completely: $xy - 2yz + 4x - 8z$ **2**
- c) Simplify: $\frac{x}{4} - \frac{3x+1}{6}$ **3**
- d) Write 0.00014 in scientific notation **1**
- e) Find the value of b if $11 + 2\sqrt{18} = 11 + \sqrt{b}$ **1**
- f) Factorise $4m^2 - 9$ **1**
- g) Solve for x : $\frac{1}{x} - 3 = \frac{1}{2x}$ **2**

Question 2 (12 Marks)

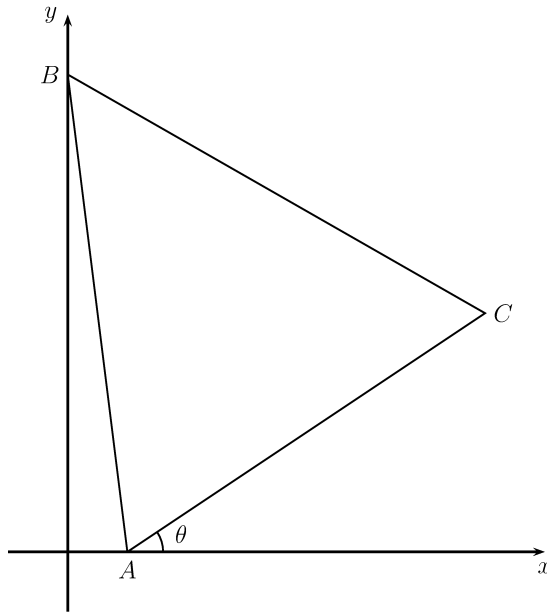
Start a new booklet

Marks

- a) Express $0.4\dot{6}$ as a common fraction. **2**
- b) Mr Dandy's water rate for his property has increased by 8%. The new water rate for his property is now \$256. Find the old water rate (correct to the nearest dollar) **2**
- c) Solve the inequality $|2 - 5x| < 7$. **2**
- d) Express $\frac{1 + \sqrt{3}}{5 - 2\sqrt{3}}$ as a fraction with a rational denominator. **2**
- e) Solve the equation $2 \cos \beta = -\sqrt{3}$ for $0^\circ \leq \beta \leq 360^\circ$ **2**
- f) Find the exact value of $\tan 120^\circ$. **2**

Question 3 (12 marks)

Start a new booklet

Marks

Consider the three points A , B and C on the number plane as shown above.

The equation of the line AC is given by $2x - 3y - 2 = 0$ and the equation of the line BC is given by $4x + 7y - 56 = 0$. The point B has coordinates $(0, 8)$.

- a) Find the coordinates of the point A . **1**

- b) Show that the point C has the coordinates $(7, 4)$. **2**

- c) Find the exact distance of AC in simplest form. **2**

- d) The line AC makes an angle θ with the positive x -axis. Calculate θ to the nearest minute. **2**

- e) Find the equation of the line which is perpendicular to AC and passes through B . **2**

- f) Show that the perpendicular distance from B to AC is $2\sqrt{13}$. **2**

- g) Calculate the area of $\triangle ABC$. **1**

Question 4 (12 Marks)

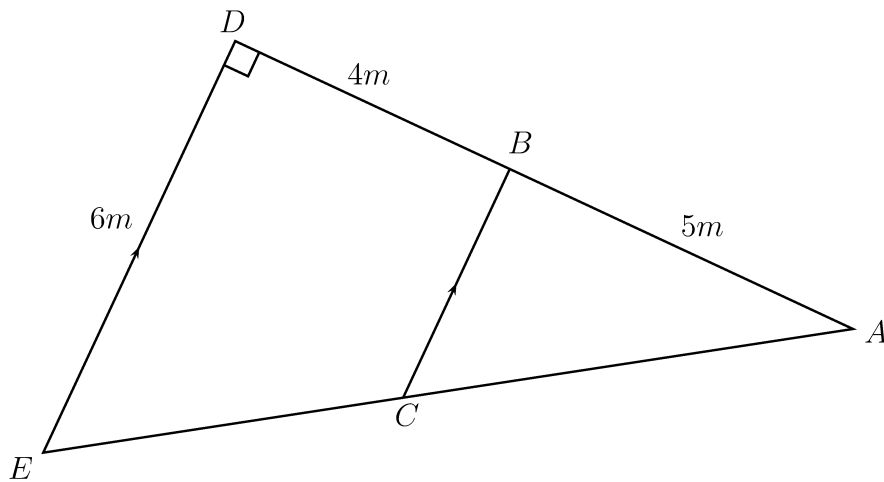
Start a new booklet

Marks

a) Factorise and simplify $\frac{x^3 - 8}{2x^2 - 3x - 2}$

3

- b) In the diagram below triangle ADE is right angled at D . BC is parallel to DE .
 $AB = 5m$, $DB = 4m$ and $DE = 6m$.



- i) Prove that $\triangle ABC$ is similar to $\triangle ADE$. **2**
- ii) Find the length of BC **2**
- iii) Find the area of $DBCE$. **1**
- iv) Show that $AB \times EC = AC \times BD$, giving reasons. **2**
- c) State the domain and range of the curve with equation $y = \frac{3}{x} + 1$ **2**

Question 5 (12 Marks)

Start a new booklet

Marks

- a) The sum of the interior angles of a regular polygon is 2520°
- i) Show that the polygon has sixteen sides. **1**
 - ii) Find the size of each interior angle to the nearest minute. **1**
- b) A given parabola has the equation $y = x^2 - 4x - 12$
- i) By completing the square, write the equation of the parabola in the form $y = (x - h)^2 + k$ **1**
 - ii) State the coordinates of the vertex. **1**
 - iii) Hence or otherwise find the x -intercepts of the parabola $y = x^2 - 4x - 12$. **2**
 - iv) Sketch the parabola showing all relevant features. **2**
- c) The equation $2x^2 - 7x + 12 = 0$ has roots α and β . Find the value of:
- i) $\alpha + \beta$ **1**
 - ii) $\alpha\beta$ **1**
 - iii) $\alpha^2 + \beta^2$ **2**

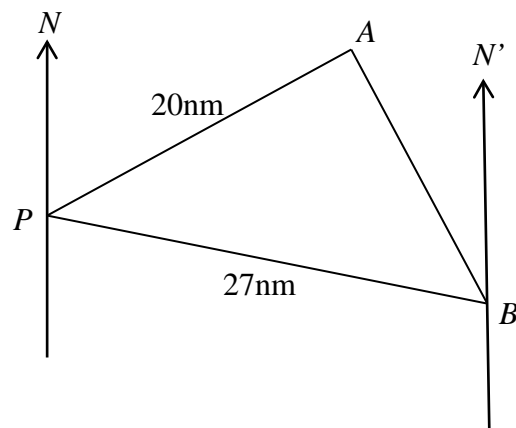
Question 6 (12 Marks)

Start a new booklet

Marks

- a) Find the value of a if $\tan 15^\circ = \cot(a + 30)^\circ$ **1**
- b) Solve for p : $2^{p+2} = 16$ **2**

- c) A ship A has sailed 20 nautical miles from a port P on a bearing of 055° . Ship B has sailed 27 nautical miles from port P on a bearing of 115° .



- i) Copy the diagram above and mark all given information. **1**
- ii) Show that the angle $\angle APB = 60^\circ$. **1**
- iii) Calculate the distance between the two ships to the nearest nautical mile. **2**
- iv) Show that $\angle ABP = 46^\circ$ correct to the nearest degree. **2**
- v) Find the bearing of A from B . **1**
- d) If $f(x) = x^2$ and $g(x) = 2x + 1$, find $f(g(2))$. **2**

Question 7 (12 Marks)

Start a new booklet

Marks

- a) Solve $4^x - 9(2^x) + 8 = 0$ **3**
- b) Consider two functions $f(x)$ and $g(x)$.
- i) If $f(x) = g(x) + g(-x)$ show that $f(x)$ is an even function. **1**
- ii) Is $f(x) = g(x) - g(-x)$ also an even function? Give reasons for your answer. **1**
- c) i) Sketch the graph of $y = |x - 4| - 2$ showing all important features. **2**
- ii) On the same number plane shade the region where
 $y \geq |x - 4| - 2$ and $y < 2$ **2**
- d) Determine the value of a so that $y = ax^2 + 2x + a$ is positive definite. **3**

Question 8 (12 Marks)

Start a new booklet

Marks

a) If $\tan \theta = \frac{7}{24}$ and $180 \leq \theta \leq 360^\circ$, find the exact value of $\sin \theta$

2

b) A function is defined by the rule:

$$f(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x+2, & x \geq 0 \end{cases}$$

Find:

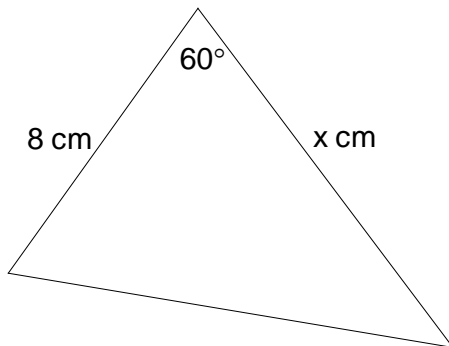
(i) $f(-3)$

1

(ii) $f(b^2)$

1

c) Calculate the value of x in the diagram below, if the area of the triangle is $22\sqrt{3} \text{ cm}^2$.

2

d) The equation of a circle is $(x-1)^2 + (y+2)^2 = r^2$

(i) Write down the coordinates of the centre of this circle.

1

(ii) If $x+y=4$ is a tangent to this circle, find the exact value of r .

2

e) Show that $\tan \theta - \frac{\sin^3 \theta}{\cos \theta} = \sin \theta \cos \theta$

3

Question 9 (12 Marks)

Start a new booklet

Marks

- a) $ABCD$ is a quadrilateral with external angles $\alpha, \beta, \delta, \varphi$.
Explain why $\sin(\alpha + \beta + \delta + \varphi) = 0$. **2**
- b) i) Sketch the graph $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ **1**
- ii) On the same number plane sketch $y = \frac{1}{2}$ **1**
- iii) How many solutions are there for $\cos x = \frac{1}{2}$ in the domain $0^\circ \leq x \leq 360^\circ$ **1**
- iv) For what values of k will $\cos x = k$ have no solutions in the domain $0^\circ \leq x \leq 360^\circ$? **1**
- c) Solve $4\sin^2 \theta - 3 = 0$ for θ , where $0^\circ \leq \theta \leq 360^\circ$. **3**
- d) Given that $x^2 + 4x + 5 \equiv (x + a)^2 + b^2$ find all possible values for a and b . **3**

Question 10 (12 Marks)

Start a new booklet

Marks

a) Find the value(s) of k for which the equation $x^2 + (k + 3)x + 4k = 0$ has:

- | | | |
|------|--|---|
| i) | one root equal to 1 | 1 |
| ii) | roots which are equal but opposite in sign | 2 |
| iii) | equal roots | 2 |
| iv) | no real roots | 1 |

b) In the diagram below, $ABCD$ is a square. X , Y and Z are points on the sides AB , BC and CD respectively such that $XB = YC = ZD$.

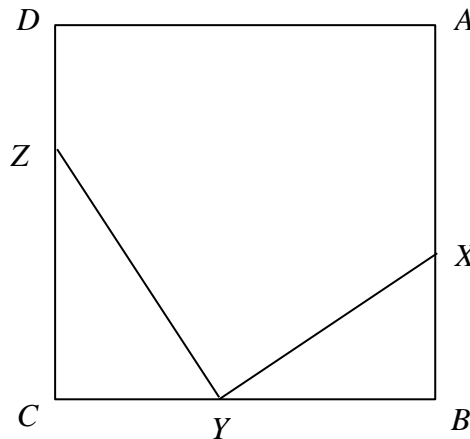


DIAGRAM
NOT TO SCALE

Copy the diagram into your writing booklet.

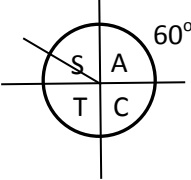
- | | | |
|------|---|---|
| i) | Prove that $\triangle BXY \equiv \triangle CYZ$ | 3 |
| ii) | Hence or otherwise prove that $XY = YZ$ | 1 |
| iii) | Deduce that $\angle XYZ = 90^\circ$ | 2 |

End of Paper

Solutions to Year 11 Mathematics 2011 Preliminary Examination

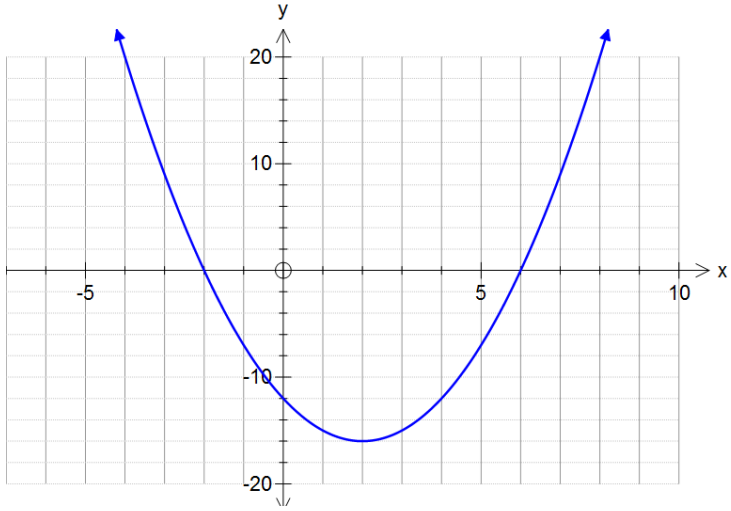
Question 1	Marking Criteria
a) $\frac{\sqrt[5]{32}}{6.1+3.2} = 0.215\dots$ $= 0.2$ (1 decimal place)	2 - correct answer 1 - correct calculator read out, but not rounded correctly
b) $xy - 2yz + 4x - 8z$ $= y(x - 2z) + 4(x - 2z)$ $= (y + 4)(x - 2z)$	2 - correct solution 1 - attempt towards solution using some factorising
c) $\frac{x}{4} - \frac{3x+1}{6}$ $= \frac{3x}{12} - \frac{6x+2}{12}$ $= \frac{-3x-2}{12}$ $= -\frac{3x+2}{12}$	3 - correct solution = $\frac{-3x-2}{12}$ 2 - solution with one error 1 - expressing both fractions with a common denominator
d) 1.4×10^{-4}	1 - correct answer
e) $4 \times 18 = 72$ $b = 72$	1 - correct answer
f) $4m^2 - 9$ $= (2m+3)(2m-3)$	1 - correct factorising
g) $\frac{1}{x} - 3 = \frac{1}{2x}$ $2 - 6x = 1$ $-6x = -1$ $x = \frac{1}{6}$	2 - correct solution 1 - correctly removing denominators or one mistake in otherwise correct solution 1 - if multiplying equation through with $2x^2$ and getting the solutions $x = 0$ $x = \frac{1}{6}$ but not realising that the equation is not defined for $x = 0$

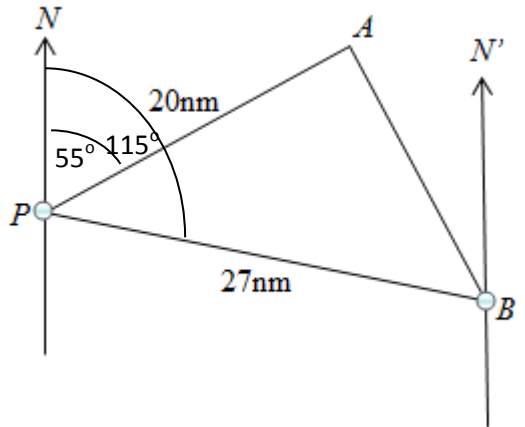
Question 2	
<p>a)</p> <p>let $x = 0.466666\dots$</p> $10x = 4.666\dots$ $\underline{- x = 0.4666\dots}$ $9x = 4.2$ $x = \frac{4.2}{9}$ $x = \frac{42}{90}$ $x = \frac{7}{15}$	<p>2 – correct solution</p> <p>1 – attempt at correct solution</p>
<p>b)</p> <p>Let $x = \text{old water rate}$</p> $1.08x = \$256$ $x = \frac{\$256}{1.08}$ $= \$237.07\dots$ $= \$237(\text{nearest dollar})$	<p>2 – correct solution</p> <p>1 – attempt at correct solution or answer not rounded to nearest dollar</p>
<p>c)</p> $ 2 - 5x < 7$ $2 - 5x < 7 \text{ or } -2 + 5x < 7$ $-5x < 5 \qquad 5x < 9$ $x > -1 \qquad x < \frac{9}{5}$ $\therefore -1 < x < \frac{9}{5}$	<p>2 – correct solution</p> <p>1 - solution with one error but solution must not be over simplified</p>
<p>d)</p> $\frac{1 + \sqrt{3}}{5 - 2\sqrt{3}} \times \frac{5 + 2\sqrt{3}}{5 + 2\sqrt{3}}$ $= \frac{(1 + \sqrt{3})(5 + 2\sqrt{3})}{25 - 12}$ $= \frac{5 + 2\sqrt{3} + 5\sqrt{3} + 6}{13}$ $= \frac{11 + 7\sqrt{3}}{13}$	<p>2 – correct solution</p> <p>1 – multiplying with conjugate surd</p>

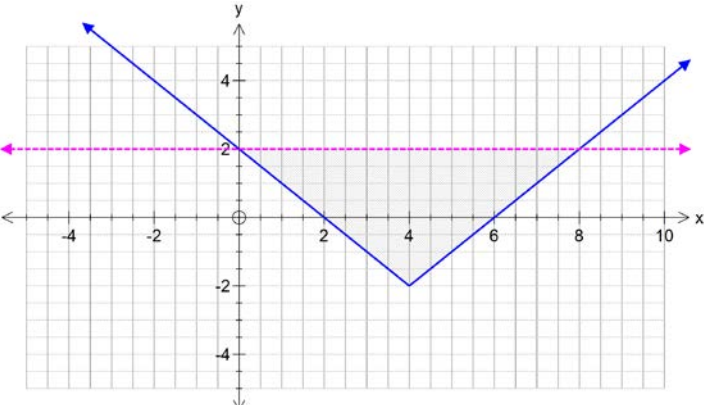
<p>e)</p> $2 \cos \beta = -\sqrt{3}$ $\cos \beta = -\frac{\sqrt{3}}{2}$ $\beta = 180^\circ - 30^\circ, 180^\circ + 30^\circ$ $= 150^\circ, 210^\circ$	<p>2 – correct solution</p> <p>1 – one angle correct or finding acute angle of 30°</p>
<p>f)</p> $\tan 120^\circ = -\tan 60^\circ$ $= -\sqrt{3}$	 <p>2 – correct solution</p> <p>1 – solution with one error</p>
Question3	
<p>a) Let $y = 0$, then</p> $2x - 2 = 0$ $x = 1$ $A = (1, 0)$	<p>1 – correct answer</p>
<p>b) either show C lies on both lines by substitution or solve simultaneously:</p> <p>(1) $2x - 3y - 2 = 0$</p> <p>(2) $4x + 7y - 56 = 0$</p> $x = 7, y = 4$	<p>2 – correct solution</p> <p>1 – solution with one error Or 1 correct substitution</p>
<p>c)</p> $A = (1, 0) \quad C = (7, 4)$ $d = \sqrt{(7-1)^2 + (4-0)^2}$ $d = \sqrt{52}$ $d = 2\sqrt{13}$	<p>2 – correct solution</p> <p>1 – correct substitution into formula Or correct solution but surd not simplified</p>
<p>d)</p> $m_{AC} = \frac{2}{3}$ $\tan \theta = \frac{2}{3}$ $\theta = 33.69006\dots$ $\theta = 33^\circ 41'$	<p>2 – correct angle to the nearest minute</p> <p>1 – correct gradient of AC found</p>

<p>e) $m_{AC} = \frac{2}{3}$, perpendicular gradient = $-\frac{3}{2}$ $B = (0,8)$, using point gradient form $y - 8 = -\frac{3}{2}(x - 0)$ $2y - 16 = -3x$ $3x + 2y - 16 = 0$</p>	<p>2 – Correct equation of straight line in any form 1 - correct perpendicular gradient</p>
<p>$d = \frac{ 2 \times 0 - 3 \times 8 - 2 }{\sqrt{2^2 + 3^2}}$ $= \frac{ -26 }{\sqrt{13}}$ f) $= \frac{26}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$ $= \frac{26\sqrt{13}}{13}$ $= 2\sqrt{13}$</p>	<p>2 – correct solution 1 – correct substitution into formula</p>
<p>$A = \frac{1}{2} \times 2\sqrt{13} \times 2\sqrt{13}$ $= 26 \text{ units}^2$</p>	<p>1 – correct answer</p>
<p>Question 4</p>	
<p>a) $\frac{x^3 - 8}{2x^2 - 3x - 2}$ $= \frac{(x - 2)(x^2 + 2x + 4)}{(2x + 1)(x - 2)}$ $= \frac{(x^2 + 2x + 4)}{(2x + 1)}$</p>	<p>3 – correctly factorised and simplified 2 – correctly factorised but not simplified 1 – correctly factorised numerator or denominator</p>
<p>b) (i) In triangles $\triangle ABC$ and $\triangle ADE$ $\angle A$ is common $\angle ABC = \angle ADE$ (corresponding angles on parallel lines, $DE \parallel BC$) $\therefore \triangle ABC$ is similar to $\triangle ADE$ (equiangular)</p>	<p>2 – correct proof with all reasons given 1 – correct proof but not all reasons given</p>
<p>ii) $\frac{BC}{6} = \frac{5}{9}$ (matching sides in similar triangles in the same ratio) $BC = \frac{30}{9}$ $= 3.\dot{3}m$</p>	<p>2 – correct solution with correct reason 1 – correct ratio given</p>

<p>iii) $DBCE$ is a trapezium:</p> $A = \frac{4m}{2}(6m + 3.3m)$ $= 18.6m^2$	1 – correct answer
<p>iv)</p> $\frac{AB}{BD} = \frac{AC}{CE}$ (intercepts on transversals cut by parallel lines in same ratio) $\therefore AB \times EC = AC \times BD$	<p>2 – correct sides in ratio statement with correct reason</p> <p>1 – incorrect reason OR incorrect sides in ratio statement</p>
<p>c) Domain: all real $x, x \neq 0$ Range: all real $y, y \neq 1$</p>	<p>2 – correct domain and range</p> <p>1 – correct domain or range</p>
Question 5	
<p>a)i)</p> $2520 = (n - 2) \times 180$ $n = 16$	1 – correct solution
<p>ii) $2520 \div 16 = 157^{\circ}30'$</p>	1 – correct answer
<p>b)i)</p> $y = x^2 - 4x - 12$ $y = (x^2 - 4x + 4) - 4 - 12$ $y = (x - 2)^2 - 16$	1- correct expression
<p>ii) $V(2, -16)$</p>	1 – correct coordinates
<p>iii) Let $y = 0$</p> $(x - 2)^2 - 16 = 0$ $(x - 2)^2 = 16$ $(x - 2) = \sqrt{16}$ $x - 2 = \pm 4$ $x = 6 \text{ or } x = -2$	<p>2 – calculating both roots correctly</p> <p>1 – attempt towards finding roots, or factorising original equation $y = (x - 6)(x - 2)$</p>

<p>iv)</p> 	<p>2 – correct graph showing all intercepts and coordinates of vertex</p> <p>1 - correct graph with some of the above information</p>
<p>c)</p> <p>i)</p> $\alpha + \beta = \frac{- -7}{2}$ $= \frac{7}{2}$	<p>1 – correct answer</p>
<p>ii)</p> $\alpha\beta = \frac{12}{2}$ $= 6$	<p>1 – correct answer</p>
<p>iii)</p> $\alpha^2 + \beta^2$ $= (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{7}{2}\right)^2 - 2 \times 6$ $= 0.25$	<p>2 – correct solution</p> <p>1 – correct algebraic expression using only sum and product, but wrong numbers substituted or mistake made in evaluation</p>
<p>Question 6</p>	
<p>a)</p> <p>Using $\tan 15^\circ = \cot(90 - 15^\circ)$</p> $a = 45^\circ$	<p>1 – correct answer</p>

<p>b)</p> $2^{p+2} = 16$ $2^{p+2} = 2^4$ $p + 2 = 4$ $p = 2$	<p>2 – correct solution</p> <p>1 – Writing 16 to the base 2</p>
<p>c) i)</p> 	<p>1 – correct diagram showing sides and bearings</p>
<p>ii)</p> $\angle APB = \angle BPN - \angle APN$ $= 115^\circ - 55^\circ$ $= 60^\circ$	<p>1 – correct answer</p>
<p>iii) Using cosine rule:</p> $x^2 = 20^2 + 27^2 - 2 \times 20 \times 27 \cos 60^\circ$ $x^2 = 589$ $x = 24.26\dots$ $= 24\text{nm (nearest nautical mile)}$	<p>2 – correct solution</p> <p>1 – correct substitution into formula</p>
<p>iv) Using sine rule:</p> $\frac{\sin \theta}{20} = \frac{\sin 60^\circ}{24}$ $\sin \theta = \frac{\sin 60^\circ}{24} \times 20$ $\sin \theta = 0.72168\dots$ $\theta = \sin^{-1}(0.72168\dots)$ $= 46.194\dots^\circ$ $\theta = 46^\circ \text{ (nearest degree)}$	<p>2 – correct solution</p> <p>1 – correct substitution into formula</p>

<p>v)</p> $180^\circ - 115^\circ = 65^\circ$ <p>(cointerior angles on parallel lines)</p> $65^\circ - 46^\circ = 19^\circ$ <p>Bearing of A from B :</p> $360^\circ - 19^\circ = 341^\circ$	<p>1 – correct bearing</p>
<p>d)</p> $g(2) = 5$ $f(5) = 25$ $f(g(2)) = 25$	<p>2 – correct answer</p> <p>1 – finding $g(2)=5$ or applying the function f correctly.</p>
<p>Question 7</p>	
<p>a)</p> $4^x - 9(2^x) + 8 = 0$ $4^x = (2^x)^2$ <p>Let $2^x = u$</p> $u^2 - 9u + 8 = 0$ $(u - 8)(u - 1) = 0$ $u = 8 \quad \text{or} \quad u = 1$ $2^x = 8 \quad 2^x = 1$ $x = 3 \quad x = 0$	<p>3 – correct solution</p> <p>2 – correct substitution and finding of u</p> <p>1 – correct substitution</p>
<p>b)</p> <p>i)</p> $f(-x) = g(-x) + g(-(-x))$ $= g(-x) + g(x)$ $= g(x) + g(-x)$ $= f(x)$ <p>Hence $f(x)$ is even.</p>	<p>1 – correct answer</p>
<p>$f(-x) = g(-x) - g(-(-x))$</p> <p>ii) $= g(-x) - g(x)$</p> <p>$\neq f(x)$</p> <p>Hence this function is not even (However it is an odd function since $f(-x) = -f(x)$)</p>	<p>1 – correct answer</p>
<p>c)i)ii)</p> 	<p>i)</p> <p>2 – correct graph showing all intercepts</p> <p>1 – graph not showing all intercepts or not to scale</p> <p>ii)</p> <p>2 – line $y=2$ drawn correctly (dotted) and correct shading</p> <p>1 – line drawn correctly or somewhat correct shading from incorrect line, or line not dotted.</p>

d)
positive definite means:
 $a > 0$ and $\Delta < 0$

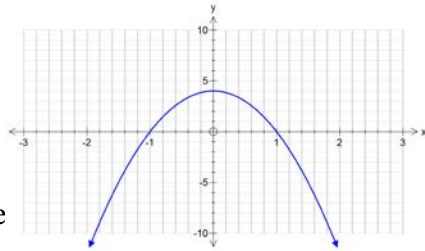
$$\Delta = 4 - 4a^2$$

$$4(1 - a^2) < 0$$

$$(1 - a)(1 + a) < 0$$

$$a < -1, a > 1$$

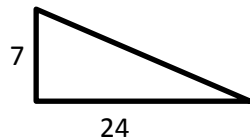
But $a > 0$ (positive de
 $\therefore a > 1$



3 – correct solution
2 - $\Delta < 0$ solved correctly for a
1 – discriminant found and conditions stated

Question 8

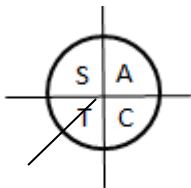
a) $\tan \theta = \frac{7}{24}$



Using right angled triangle

$$\text{Hypotenuse} = \sqrt{7^2 + 24^2}$$

$$= 25$$



$$\therefore \sin \theta = -\frac{7}{25}$$

2 – correct solution
1 – finding the hypotenuse and finding the sine ratio albeit as a positive

1 – correct answer

b)i)

$$f(-3) = \frac{1}{-3}$$

$$= -\frac{1}{3}$$

1 – correct answer

ii)

$$f(b^2) = b^2 + 2$$

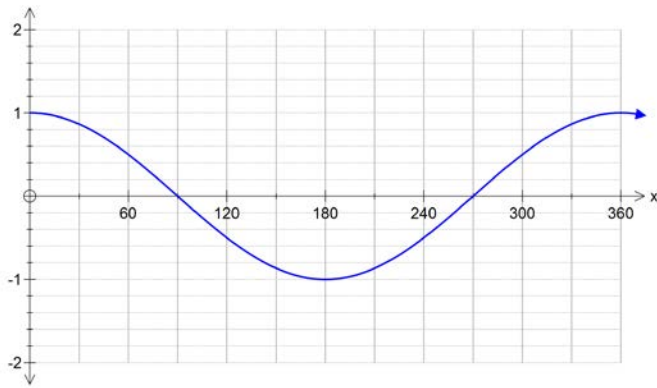
Since
 $b^2 \geq 0$

1 – correct answer

<p>c)</p> $A = \frac{1}{2} ab \sin C$ $22\sqrt{3} = \frac{1}{2} 8x \sin 60^\circ$ $22\sqrt{3} = 4x \frac{\sqrt{3}}{2}$ $22\sqrt{3} = 2x\sqrt{3}$ $x = 11$	<p>2 – correct answer</p> <p>1 – correct substitution into area formula</p>
<p>d)</p> <p>i) C = (1, -2)</p>	<p>1 – correct coordinates</p>
<p>ii)</p> <p>If $x + y = 4$ is a tangent to this circle, it must touch the circle in one point only. So the radius must be equal to the perpendicular distance from the centre to the line $x + y = 4$</p> $r = \frac{ 1 \times 1 + 1 \times (-2) - 4 }{\sqrt{1^2 + 1^2}}$ $= \frac{5}{\sqrt{2}}$ $= \frac{5\sqrt{2}}{2}$	<p>2 – correct solution</p> <p>1 – attempt at solution, substituting into perpendicular distance formula</p>
<p>e)</p> $LHS = \tan \theta - \frac{\sin^3 \theta}{\cos \theta}$ $= \frac{\sin \theta}{\cos \theta} - \frac{\sin^3 \theta}{\cos \theta}$ $= \frac{\sin \theta (1 - \sin^2)}{\cos \theta}$ $= \frac{\sin \theta \cos^2 \theta}{\cos \theta}$ $= \sin \theta \cos \theta$ $= RHS$	<p>3 – correct proof</p> <p>2 – substituting $\frac{\sin \theta}{\cos \theta} = \tan \theta$ and factorising numerator</p> <p>1 – substituting $\frac{\sin \theta}{\cos \theta} = \tan \theta$</p>
<p>Question 9</p>	
<p>a)</p> <p>External angles of a polygon add up to 360°</p> $\sin 360^\circ = 0$	<p>2 – correct explanation</p> <p>1 – realising that external angles add to 360°</p>

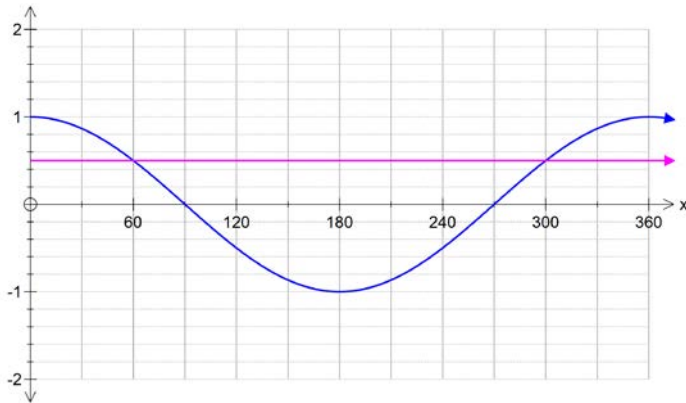
b)

i)



1 – correct graph

ii)



1 – correct line

iii) 2 solutions as there are 2 points of intersection

1 – correct answer

iv) for $k > 1$ or $k < -1$

1 – correct answer

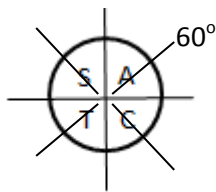
c)

$$4 \sin^2 \theta - 3 = 0$$

$$\sin^2 \theta = \frac{3}{4}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$



3 – correct solution

2 – finding only the solution in the 1st and 2nd quadrant

1 – achieving $\sin^2 \theta = \frac{3}{4}$

d)

$$x^2 + 4x + 5 \equiv (x + a)^2 + b^2$$

$$= x^2 + 2ax + a^2 + b^2$$

$$= x^2 + 2ax + (a^2 + b^2)$$

3 – correct solution

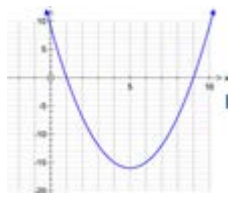
<p>Comparing coefficients $2a = 4 \Rightarrow a = 2$ $a^2 + b^2 = 5$ substituting $a = 2$ $2^2 + b^2 = 5$ $b^2 = 1$ $b = \pm 1$ $\therefore a = 2, b = \pm 1$</p> <p>Alternatively substitute $x = 0$ and $x = 1$ into identity to give a set of equations that can be solved simultaneously.</p>	<p>2 – solution with one error</p> <p>1 – realising that coefficient need to be compared and making an attempt to do so.</p>
Question 10	
<p>a)i) substitute $x = 1$ $1 + (k + 3) + 4k = 0$ $5k + 4 = 0$ $k = -\frac{4}{5}$</p>	<p>1 – correct value for k</p>
<p>ii) The sum of roots will equal 0 $\alpha + \beta = 0$ $-(k + 3) = 0$ $-k - 3 = 0$ $k = -3$</p>	<p>2 – correct solution</p> <p>1 – stating sum of roots equal to 0 OR stating roots as α and $-\alpha$</p>
<p>iii) equal roots means that the discriminant is equal to zero $\Delta = (k + 3)^2 - 4 \times 1 \times 4k$ $(k + 3)^2 - 4 \times 1 \times 4k = 0$ $k^2 + 6k + 9 - 16k = 0$ $k^2 - 10k + 9 = 0$ $(k - 9)(k - 1) = 0$ $\therefore k = 1, k = 9$</p>	<p>2 – correct solution</p> <p>1 – correct substitution into discriminant</p>

iv) no real roots means that the discriminant is negative

$$\Delta < 0$$

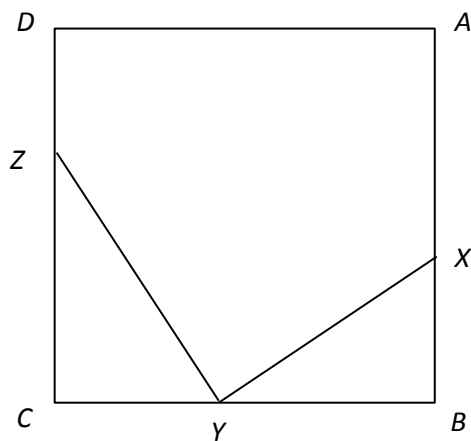
$$(k-9)(k-1) = 0$$

$$1 < k < 9$$



1 – correct solution

b)



i)

In $\triangle BXY$ and $\triangle CYZ$

$\angle C = \angle B = 90^\circ$ ($ABCD$ is a square)

$XB = YC$ (given)

$ZC = YB$ (since CD and CB are equal sides of a square and $ZD = YC$)

$\therefore \triangle BXY \cong \triangle CYZ$ (SAS)

3 – correct proof

2 – correct proof but not all reasons given

1 – one valid statement with a reason

ii)

$XY = YZ$ (corresponding sides in congruent triangles are equal)

1 – correct reason

<p>iii)</p> <p>Since $\triangle ZCY$ and $\triangle XBY$ are congruent all corresponding angles equal and</p> $\angle ZYC = \angle YXB$ <p>$\angle YXB$ is complementary to $\angle XYB$ (right angled triangle and hence</p> <p>$\angle ZYC$ is complementary to $\angle XYB$</p> $\angle XYZ = 180^\circ - (\angle ZYC + \angle XYB) \text{ (straight angle)}$ $\therefore \angle XYZ = 90^\circ$	<p>2 – correct conclusion drawn with all reasons given</p> <p>1 – correct conclusion but not fully explained.</p>
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Communication Marks:

- Q1 no communication marks
- Q2 1 mark – part b – Set up of solution
Let original = x
 $1.08x = \$256$ etc.
- Q3 1 mark – part b – setting out using LHS /RHS
- Q4 1 mark – part b (iii) – for communicating the area of the shape they are finding or wrote out the formula of the shape they are finding.
- Q5 1 mark – part a (ii) – showing correct formula: angle sum = $(n - 2) \times 180$
1 mark – part b (iv) – stating that “the x-intercepts are -2 and 6 ”
- Q6 1 mark – part c (ii) – stating that $\angle APB = \angle BPN - \angle APN$
1 mark – part c (iii) – for showing the answer displayed on the calculator before rounding.
1 mark – part c (iv) – for showing “ $\theta = \sin^{-1}$ ” in their solution.
- Q7 1 mark – part b – showing substitution of $-x$ clearly
1 mark – part d – stating condition for positive definite
- Q8 3 marks – part e – clarity of setting out proof with LHS and RHS
- Q9 2 marks – part a – clear statement of exterior angle sum, followed by $\sin 360^\circ = 0$ and hence $\sin(\alpha + \beta + \delta + \gamma) = 0$
- Q10 1 mark – part a – Stating condition for equal/no real roots
1 mark – part b – Giving reasons at each stage of proofs