Student Number:

Name:

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2012
Preliminary Course
FINAL EXAMINATION
Tuesday, September 11

Mathematics

General Instructions

- Reading Time – 5 minutes.
- Working Time – 3 hours.
- Write using a black or blue pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.

Total marks (100)

Section I

10 marks

- Attempt Questions 1 – 10
- Answer on the multiple choice answer sheet provided
- Allow approximately 15 minutes for this section

Section II

90 marks

- Attempt Questions 11 – 16
- Answer in the booklets provided
- Begin each question in a new booklet
- Allow approximately 2 hours 45 minutes for this section

P2 provides reasoning to support conclusions which are appropriate to the context.

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.

P5 understands the concept of a function and the relationship between a function and its graph.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for your responses to Questions 1 – 10.

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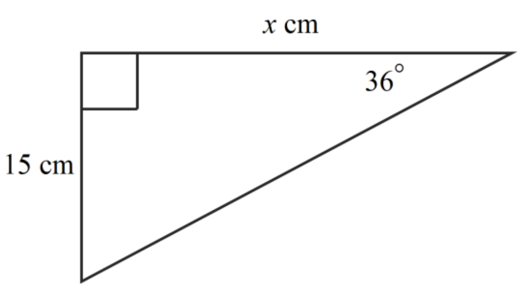
1. What is $\dfrac{3.23 \times 4.96^2}{3.45 + 1.2^2}$ correct to 3 significant figures?

(A) 16·2 (B) 16·3
(C) 24·4 (D) 24·5

2. Which of the following is the factorisation of $x^2 + x - 12$?

(A) $(x+3)(x-4)$ (B) $(x-3)(x-4)$
(C) $(x+6)(x-2)$ (D) $(x-3)(x+4)$

3. Which calculation would be used to find the value of x?



(A) $15\tan 36°$ (B) $15\cos 36°$
(C) $\dfrac{15}{\tan 36°}$ (D) $\dfrac{15}{\cos 36°}$

4. What is the equation of the line that passes through the point (1, 3) and is parallel to the x-axis?

(A) $x=1$ (B) $y=1$ (C) $x=3$ (D) $y=3$

5. What are the values of a and b if $\dfrac{2\sqrt{2}}{2\sqrt{2} - 3} = a - \sqrt{b}$?

(A) −4 and 12 (B) – 4 and 72 (C) −8 and 12 (D) −8 and 72

6. Which of the following are the solutions to the equation $2x^2 - 7x - 2 = 0$?

(A) $\frac{-7 \pm \sqrt{33}}{4}$ (B) $\frac{-7 \pm \sqrt{65}}{4}$

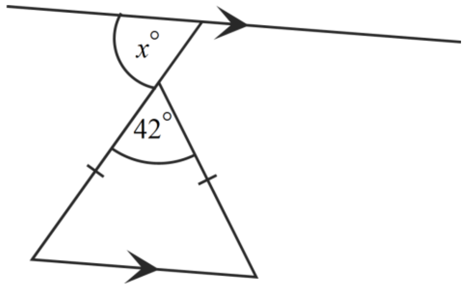
(C) $\frac{7 \pm \sqrt{33}}{4}$ (D) $\frac{7 \pm \sqrt{65}}{4}$

7. What is the domain for the function $y = \sqrt{7 - x}$?

(A) $x \le 7$ (B) $0 \le x \le 7$

(C) $x \ge 0$ (D) All real x

8.



What is the value of x in the above diagram?

(A) 42 (B) 69 (C) 111 (D) 138

9. For what values of k is the expression $\frac{-x^2}{4} - x - k$ negative definite?

(A) $k < 1$ (B) $k > 1$

(C) $k < 4$ (D) $k > 4$

10. What is $\frac{\sin(180° - \theta)}{\sin(90° - \theta)}$ in its simplest form?

(A) 1 (B) 2

(C) $\sin\theta$ (D) $\tan\theta$

End of Section I

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section
Begin each question in a new booklet
All necessary working should be shown
All questions are of equal value

Question 11 (15 Marks) **Marks**

(a) Express $2.\dot{7}$ as a rational number in simplest form. **2**
q

(b) Solve for *x*:

(i) $\dfrac{5}{x+3} = \dfrac{3}{x}$ **2**

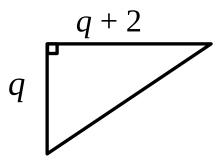
(ii) $|2x+1| = 3x+2$ **3**

(c) Show that the point (1, 4) lies on the line $2x+3y-14=0$. **1**

(d) Show that the three points $A(3,-1)$, $B(5,\ 5)$ and $C(2,-4)$ are collinear. **2**

(e) If $\sin\theta = \dfrac{7}{25}$ and $\cos\theta < 0$ find the exact value of $\cot\theta$. **2**

(f) The diagram shows a right angled triangle in which one of the sides, adjacent to the right angle, is 2 centimetres longer than the other. **3**



Find the length of the shortest side of this triangle if its area is 7.5 cm^2.

End of Question 11

Question 12 **(15 Marks)** ***Start a new booklet*** **Marks**

(a) An interval AB is formed by the points $A(-3,5)$ and $B(2,-3)$ on the number plane.

Draw a clear diagram.

(i) Find the gradient of the interval AB. **1**

(ii) Find the length of the interval AB. **1**

(iii) Show that the equation of the line AB is $8x+5y-1=0$. **1**

(iv) Find the equation of the line through $C(5,8)$ perpendicular to AB. **2**

(v) Show that the perpendicular distance of C from the line AB is $\frac{79}{\sqrt{89}}$. **1**

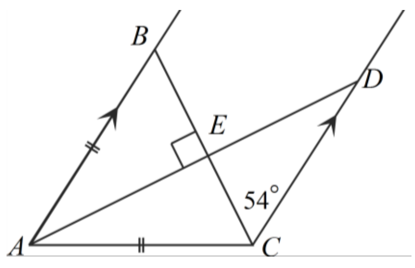
(vi) Calculate the area of the triangle ABC. **1**

(vii) On your diagram, shade the region for which the inequalities $8x+5y-1\geq 0$ and $y\leq 0$ hold simultaneously. **2**

(viii) The points A, B and C are three vertices of the parallelogram $ACBD$. Find the co-ordinates of D. **1**

(b) Simplify $\frac{1}{\text{cosec}^2\theta}+\frac{1}{\sec^2\theta}+\frac{1}{\cot^2\theta}$. **2**

(c) In the diagram, $AB \parallel CD$ and $AB = AC$. The lines AD and BC are perpendicular and $\angle BCD = 54°$.



Calculate, giving clear reasons, the size of $\angle CAD$. **3**

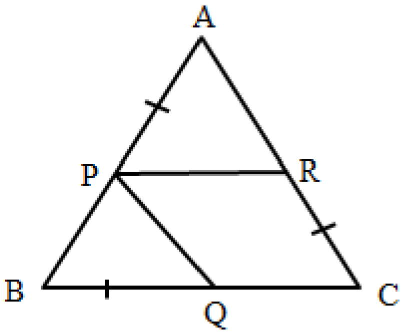
End of Question 12

Question 13 (15 Marks) *Start a new booklet* **Marks**

(a) Write down the exact value of $\tan 120^{\circ}$. **1**

(b) Solve $4\cos^2\alpha = 3$ for the domain $0^{\circ} \le \alpha \le 360^{\circ}$. **3**

(c) Triangle *ABC* is an equilateral triangle. The point *P* lies on the side *AB*, the point *Q* lies on the side *BC* and the point *R* lies on the side *AC* so that *AP* = *BQ* = *CR*.



NOT TO SCALE

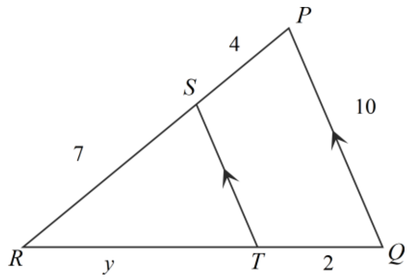
(i) Explain, with reasons, why $AR = BP$. **2**

(ii) Prove $\Delta APR \equiv \Delta BQP$. **2**

(d) (i) Solve simultaneously $y = 2x - 2$ and $y = x^2 - 1$. **2**

(ii) Explain the graphical significance of your answer to part (i). **1**

(e) In the triangle *PQR*, *S* and *T* are points on the sides *PR* and *QR* respectively such that $ST \parallel PQ$.



(i) Prove that the triangles *PQR* and *STR* are similar. **2**

(ii) Find the value of *y*, giving reasons for your answer. **2**

End of Question 13

Question 14 (15 Marks) *Start a new booklet* **Marks**

(a) The equation of a circle is given by $x^2 - 2x + y^2 + 12y - 12 = 0$.

(i) Write the equation in the form $(x-h)^2 + (y-k)^2 = r^2$. **2**

(ii) Hence or otherwise write down the co-ordinates of the centre of the circle. **1**

(iii) Without solving, explain how you would determine the number of times a line cuts this circle. **2**

(b) Two ships leave a port, *P*, at the same time. Ship *A* travels at 12 km/h on a bearing of 213°. Ship *B* travels south-east at a speed of 8 km/h.

(i) Draw a diagram to illustrate the relative positions of the two ships after 3 hours. **1**

(ii) Show that $\angle APB = 78°$. **1**

(iii) Show that the distance between the ships after 3 hours is 39km, to the nearest km. **2**

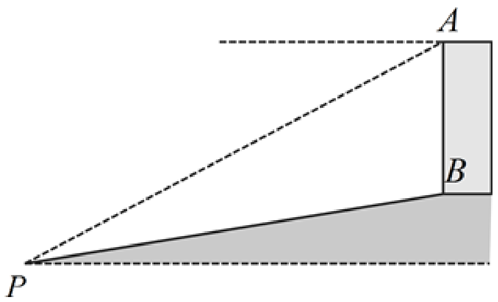
(iv) Ship *A* received a distress call from Ship *B* whose engine had failed. On what bearing will Ship *A* need to sail in order to rescue Ship *B*? Answer to the nearest degree. **3**

(c) The equations of two lines are $x + 2y - 6 = 0$ and $3x - 2y - 6 = 0$. Find the equation of the line that passes through the point of intersection of these lines and the point $(1, -1)$. **3**

End of Question 14

Question 15 (15 Marks) ***Start a new booklet*** **Marks**

(a) A point P is at the foot of the hill which is inclined at 5^{o} to the horizontal. The base B, of a tower AB, is situated 150 metres up the incline of the hill from P. From the top of the tower the angle of depression of the point P is 32°.



(i) Copy the diagram and mark on it the size of $\angle PAB$ and $\angle APB$. **2**

(ii) Find the height of the tower, AB, correct to the nearest metre. **2**

(b) Consider the parabola with equation $y = x - x^2$.

(i) Find the co-ordinates of the vertex. **2**

(ii) Sketch the parabola showing all important features. **2**

(c) A function is defined as

$$f(x) = \begin{cases} x - 5 & x \geq 0 \\ -2 & -3 < x < 0 \\ 2 + x & x \leq -3 \end{cases}$$

Find

(i) $f(-1) + f(-5)$ **1**

(ii) Sketch $f(x)$ **3**

(d) Find the values of A, B and C such that **3**

$$3x^2 + 4x + 1 \equiv Ax(x-2) + B(x+1) + C$$

End of Question 15

Question 16 (15 Marks) *Start a new booklet* **Marks**

(a) Solve the equation $4^x + 2^x - 6 = 0$. **3**

(b) If α and β are the roots of the equation $3x^2 + 5x - 4 = 0$, find

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) $\dfrac{3}{\alpha} + \dfrac{3}{\beta}$ **2**

(c) Given the equation $(k+1)x^2 + (k+2)x + k = 0$ find the value of k if the equation has equal roots. **3**

(d) The roots of the equation $x^2 - ax - 2b = 0$ differ by 4. **3**
Show that $a^2 + 8b - 16 = 0$.

End of Examination Paper

Section I Answer Sheet *Student Number:*

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use this multiple choice answer sheet for questions 1 – 10.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ● B ☒ C ○ D ○

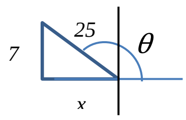
Completely fill the response oval representing the most correct answer.

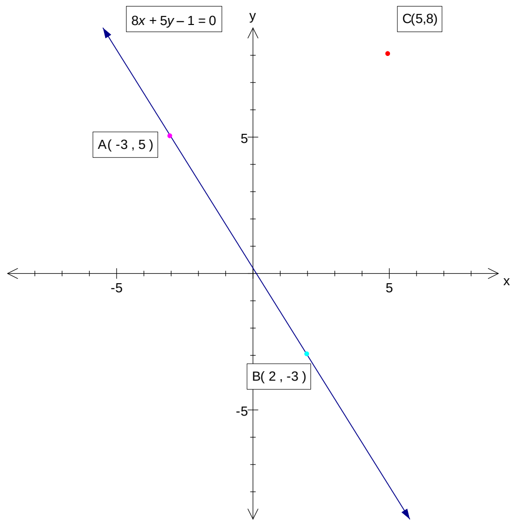
1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Solutions to Year 11 Mathematics 2012 Preliminary Examination

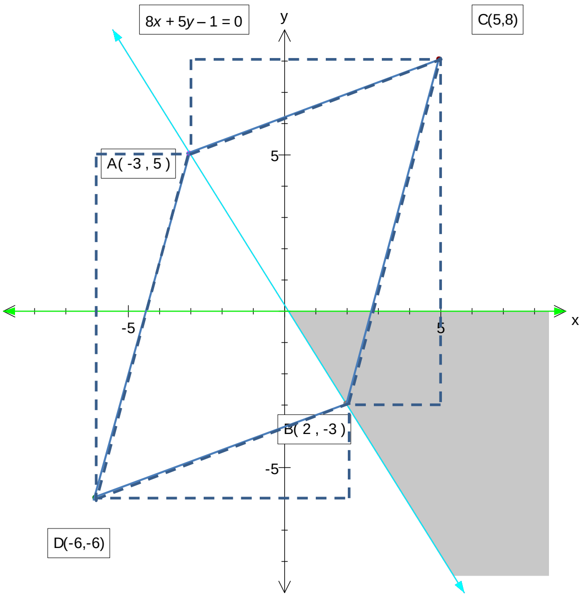
Multiple Choice	
1 B	2 D
3 C	4 D
5 D $\frac{2\sqrt{2}}{2\sqrt{2}-3}\times\frac{2\sqrt{2}+3}{2\sqrt{2}+3}$ $=\frac{2\sqrt{2}(2\sqrt{2}+3)}{(2\sqrt{2})^2-(3)^2}$ $=\frac{8+6\sqrt{2}}{-1}$ $=-8-\sqrt{72}$	6 B $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(-2)}}{2(-2)}$ $=\frac{7\pm\sqrt{49+16}}{-4}$ $=\frac{7\pm\sqrt{65}}{-4}$ $=\frac{-7\pm\sqrt{65}}{4}$
7 A $7-x\geq 0$ $x\leq 7$	8 B
9 A *For neg definite* $b^2-4ac<0$ *and* $a<0$ $b^2-4ac<0$ $(-1)^2-4(-\frac{1}{4})(-k)<0$ $1-k<0$ $k>1$	
10 D $\frac{\sin(180-\theta)}{\sin(90-\theta)}$ $=\frac{\sin\theta}{\cos\theta}$ $=\tan\theta$	

Question 11	Marking Criteria
a) Let $x = 2.77777...(1)$ then $10x = 27.7777...(2)$ $(2)-(1)$: $9x = 25$ $\therefore x = \frac{25}{9}$	2 - correct answer 1 – attempt at correct solution i.e correct x and $10x$
b) i) $\frac{5}{x+3} = \frac{3}{x}$ $5x = 3x+9$ $2x = 9$ $x = \frac{9}{2}$	2 – correct solution 1 – one error in working
b) ii) $\lvert 2x+1\rvert = 3x+2$ $2x+1 = \pm(3x+2)$ $2x+1 = 3x+2 \text{ or } 2x+1 = -3x-2$ $-x = 1 \quad \text{or} \quad 5x = -3$ $x = -1 \quad \text{or} \quad x = \frac{-3}{5}$ But $3x+2 \geq 0$ i.e $x \geq \frac{-2}{3}$ $\therefore x = \frac{-3}{5}$ is the only solution	3 – correct solution 2 – both values correct, no testing 1 –one value correct, no test
c) Let $\text{LHS} = 2x+3y-14$ $= 2(1)+3(4)-14$ $= 0$ $= \text{RHS}$ $\therefore (1,4)$ lies on the line $2x+3y-14=0$	1 – correct method or use of substitution

d) Compare the gradients of 2 intervals: m_{AB} , m_{BC} *or* m_{AC} *This solution looks at* m_{AB} *and* m_{BC} $m_{AB} = \frac{5--1}{5-3} = \frac{6}{2} = 3 = \frac{6}{2} = 3$ $\quad m_{BC} = \frac{5--4}{5-2} = \frac{9}{3} = 3$ Since $m_{AB} = m_{BC}$, A, B and C are collinear	2 – correct answer showing 2 equal gradients 1 – incorrect but equal gradients *or* error in solution *or* other method with error
e) $\sin\theta = \frac{7}{25}$ implies θ in 1st & 2nd quadrant $\cos\theta < 0$ implies θ in 2nd & 3rd quadrant $\therefore \theta$ lies in 2nd quadrant $\therefore \cot\theta < 0$  *By Pythagoras* $25^2 = x^2 + 7^2$ $625 = x^2 + 49$ $x^2 = 576$ $x = 24$ $\therefore \cot\theta = -\frac{24}{7}$	2 – correct answer 1 – answer $+\frac{24}{7}$ or correct negative sign for 2nd quad
f) $A = \frac{1}{2}bh$ $\frac{q}{2}(q+2) = 7.5$ $q(q+2) = 15$ $q^2 + 2q - 15 = 0$ $(q-3)(q+5) = 0$ $\therefore q = 3, -5$ Since q>0, the shortest side is 3cm.	3 – correct solution plus reason $q \neq -5$ 2 – correct values without consideration of the validity of the answer 1 – correct initial equation
Communication: ***1 mark each*** (b) (ii) $3x + 2 \geq 0$ (c) clear reasoning eg LHS = ….=RHS (d) efficiency of soln eg 2 gradients only (e) stating 2nd quadrant (f) clear reasoning for one solution	

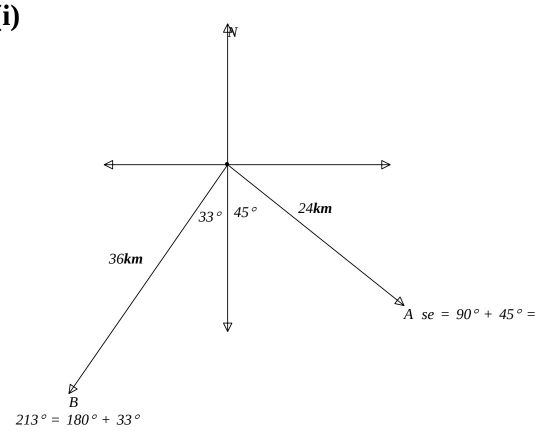
Question 12	
(a) 	
(i) $$m_{AB} = \frac{5 - -3}{-3 - 2} = -\frac{8}{5}$$	1 – correct solution
(ii) $$d_{AB} = \sqrt{(-3-2)^2 + (5 - -3)^2} = \sqrt{25+64} = \sqrt{89}$$	1 – correct solution
(iii) **Method 1:** $m_{AB} = -\frac{8}{5}$ from (i) Equation AB: $y - y_1 = m(x - x_1)$ $y - 5 = -\frac{8}{5}(x+3)$ $5y - 25 = -8x - 24$ $8x + 5y + 1 = 0$ **Method 2:** Equation AB is $8x + 5y - 1 = 0$ Subs $A(-3,5)$: $LHS = 8(-3) + 5(5) - 1$ $= -24 + 25 - 1$ $= 0$ $= RHS$ Subs $B(2,-3)$: $LHS = 8(2) + 5(-3) - 1$ $= 16 - 15 - 1$ $= 0$ $= RHS$ $\therefore$ A and Bsatisfy $8x + 5y - 1 = 0$ $\therefore$ the equation AB is $8x + 5y - 1 = 0$	1 – correct solution

(iv) gradient line perpendicular to AB $m_1 \times m_2 = -1$ $m_{AB} = \frac{-8}{5}$ $\therefore$ gradient line perpendicular to $AB = \frac{5}{8}$ Equat^n required with $m = \frac{5}{8}$ and point $(5,8)$ $y - y_1 = m(x - x_1)$ $y - 8 = \frac{5}{8}(x-5)$ $8y - 64 = 5x - 25$ $5x - 8y + 39 = 0$ *OR* Equat^n of a line perpendicular to $Ax + By + c = 0$ has form $Bx - Ay + k = 0$ $\therefore$ Req'd equation is $5x - 8y + k = 0$ subs $(5,8)$: $5(5) - 8(8) + k = 0$ $\therefore k = 39$ Req'd equation is $5x - 8y + 39 = 0$	2 – correct solution 1- correct with incorrect gradient
(v) Perp distance $= \left\| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\|$ $= \left\| \frac{5(8) + 8(5) - 1}{\sqrt{5^2 + 8^2}} \right\|$ $= \left\| \frac{40 + 40 - 1}{\sqrt{25 + 64}} \right\|$ $= \frac{79}{\sqrt{89}}$	1 – correct substitution and answer
(vi) $Area = \frac{1}{2}bh$ $= \frac{1}{2}\sqrt{89} \times \frac{79}{\sqrt{89}}$ $= \frac{79}{2}$ units^2	1 – correct solution

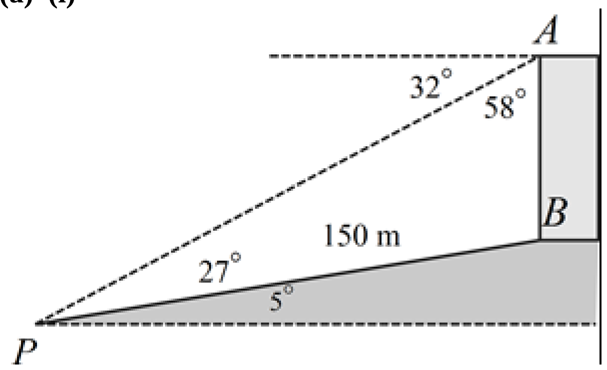
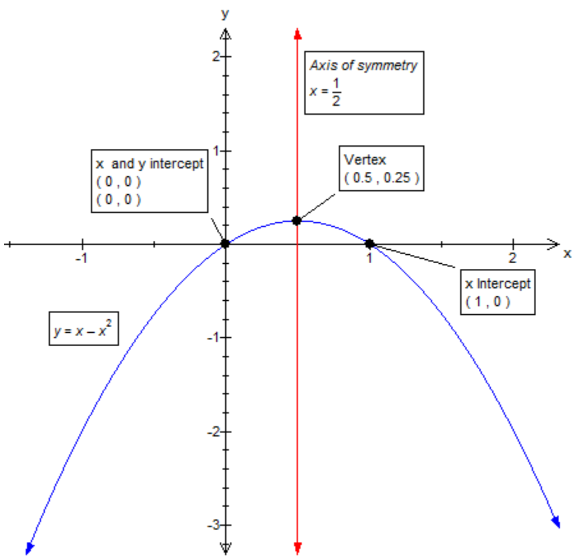
(vii) and (viii) 	vii) 2 – correct solution 1 – correct shading of one region viii) 1 – correct solution
(b) $\frac{1}{\mathrm{cosec}^2\theta}+\frac{1}{\sec^2\theta}+\frac{1}{\cot^2\theta}$ $=\sin^2\theta+\cos^2\theta+\tan^2\theta$ $=1+\tan^2\theta$ $=\sec^2\theta$	2 – correct substitution and simplification 1 – correct recognition of inverse ratios
(c) $\angle ABC=\angle DCB=54°$ (*equal alternate angles, $AB \parallel CD$*) $\angle ABC=\angle BCA=54°$ (*angles opposite equal sides in isosceles$\triangle ABC$*) $*\angle EAC=36°$ (*exterior $\angle BEA$ equals the sum of the two interior angles*) *OR* $*\angle EAC=36°$ (*angle sum of $\triangle EAC$ is 180°*)	3 – correct solution and clearly stated reasons 2 – one error in reasoning/calculation 1 – two errors in reasoning/calculation
Communication: *1 mark each* (a) (iv) inclusion $m_1m_2=-1$ (vii) extension of interval AB to a line (viii) explanation of how students arrived at a solution (diagram, words, algebra) (b) efficient use of identities (c) efficient solution	

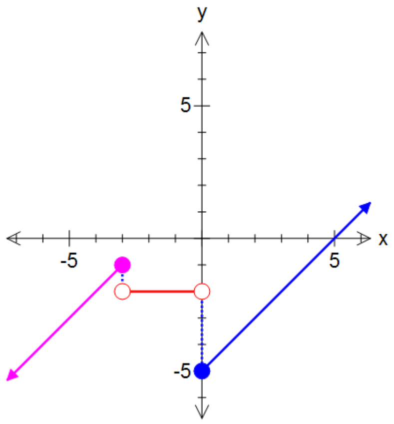
Question 13	
(a) $\tan 120^o = -\tan 60^o$ $= -\sqrt{3}$	1 – correct solution
(b) $4\cos^2\alpha = 3$ $\cos\alpha = \pm\frac{\sqrt{3}}{2}$ $\therefore \alpha$ in all quadrants α in 1^{st} quadrant = 30° $\therefore \alpha = 30°, 150°, 210°, 330°$	3 – correct solution 2 - no $\pm$ mentioned but correct answers 1 – no $\pm$ mentioned incorrect related angle but angles correct using incorrect related angle
(c) (i) $AB = AC = BC$ (*equal sides in equil* $\triangle ABC$) $AP = RC = BQ$ (*given*) $AR = AC - RC$ $= AB - AP$ $= BP$ $\therefore AR = BP$	2 – correct solution with reasoning 1 – correct working with partial or no reasoning
(c) (ii) *In* $\triangle APR$ *and* $\triangle BQP$ $AP = BQ$ (*given*) $AR = BP$ (*proven above*) $\angle PAR = \angle QBP = 60°$ (*vertex angles of equilateral* $\triangle ABC$) $\therefore \triangle APR \equiv \triangle BQP$ (*SAS*)	2 – correct proof with reasoning 1 – missing or incorrect reasons
(d) (i) $y = 2x - 2$ ......(1) $y = x^2 - 1$ ....(2) *Subs* (1) *in* (2) $x^2 - 1 = 2x - 2$ $x^2 - 2x + 1 = 0$ $(x-1)^2 = 0$ $\therefore x = 1$	2 – correct solution 1 – most working correct or not finding the value of *y*.
(ii) The line is a tangent to the parabola	1 – correct answer
(e) (i) In ΔPQR and ΔSTR $\angle R$ is common $\angle RPQ = \angle RST$ (equal corresponding angles on parallel lines, $PQ \parallel ST$) $\angle PQR = \angle STR$ (equal corresponding angles on parallel lines, $PQ \parallel ST$) $\therefore \Delta PQR \mid\mid\mid \Delta STR$ (equiangular)	2 – correct proof with reasoning 1 – proof with most of the reasoning correct.

(ii) Since $\Delta PQR \ \vert\vert\vert \ \Delta STR$, corr sides are in ratio $\frac{y}{y+2}=\frac{7}{11}$ $11y=7y+14$ $4y=14$ $y=3.5$ *OR* (ratio of intecepts made by parallel lines are equal) $\frac{y}{2}=\frac{7}{4}$ $y=3.5$	2 – correct solution with reasoning 1 – correct answer with no reasonng
Communication: ***1 mark each*** (b) diagram to show domain (c) (i) & (c) (ii) clear reasoning	(d) (ii) clear explanation (e) (i) efficiency of solution (e) (ii) efficiency of solution/ratio statement

Question 14	
(a) (i) $x^2-2x+y^2+12y-12=0$ $\left(x^2-2x+1\right)+\left(y^2+12y+36\right)=12+1+36$ $\left(x-1\right)^2+\left(y+6\right)^2=49$	2- correct solution showing completing the square 1- progress towards completing the square
(ii) Centre = $(1,-6)$	1- correct solution
(iii) Find the perpendicular distance of the line from the centre of the circle and then compare that length with the radius of the circle (7) • if d<7 the line cuts twice • d=7 line is a tangent to the circle • d> 7 the line does not cut the circle **Alternative solution** would be to solve the equations of the line and the circle simultaneously. You would get a quadratic equation to solve. Using the discriminate we could calculate the number of roots. $\Delta<0$ line does not cut circle $\Delta=0$ line is a tangent to the circle $\Delta>0$ line cuts the circle in 2 places	2 – correct explanation which includes how you got discriminant or perp dist of line to centre of circle 1 – explanation that was not clear
(b) (i) 	1 – diagram clearly indicating the length and bearing of each point

(ii) $\angle APB = 213^0 - 135^0$ $= 78^0$	1- correct show Alternate shows available
(iii) $AB^2 = 36^2 + 24^2 - 2(36)(24)\cos 78^0$ $= 1512.728598$ $AB = 38.8938..$ $= 39km$ (to nearest km)	2 – correct show which includes calculator readout 1 – correct substitution into cosine rule
(iv) $\frac{\sin A}{24} = \frac{\sin 78}{39}$ $\sin A = \frac{24 \sin 78}{39}$ $\angle A = 37^0 0'31.5''$ Bearing = $33^0 + 37^0$ $= 070^0$	3- correct 3 digit bearing 2- correct angle and attempt to use it to get bearing 1- correct attempt at sine rule
c) General form: $x + 2y - 6 + k(3x - 2y - 6) = 0$ Passes through $(1, -1)$ $\therefore 1 + 2(-1) - 6 + k(3(1) - 2(-1) - 6) = 0$ $-7 - k = 0$ $k = -7$ $\therefore x + 2y - 6 - 7(3x - 2y - 6) = 0$ $x + 2y - 6 - 21x + 14y + 42 = 0$ $-20x + 16y + 36 = 0$ $5x - 4y - 9 = 0$ Or Point of intersection of lines (3, 1.5) Gradient 5/4	3- correct solution 2- correct attempt at solution 1- correct attempt at using K or finding point of intersection of the 2 lines
Communication: ***1 mark each <u>except</u> 1(a) (iii)- 2 marks*** (a) (iii) clear reasoning (b) (i) diagram (b) (ii) clear reasoning	

Question 15	
(a) (i)  	2 – correct diagram 1 – 150 m in wrong position or angle APB labelled as 32°
(a) (ii) By the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{h}{\sin 27°} = \frac{150}{\sin 58°}$ $h = \frac{150 \sin 27°}{\sin 58°}$ $= 80.30....$ $= 80m\ (nearest\ metre)$	2- correct solution, correct solution from incorrect diagram in part i) 1 – correct attempt with 1 error
(b) (i) Axis of symmetry OR by roots $x = \frac{-b}{a}$ *solve:* $x(x-1) = 0$ $= \frac{1}{2}$ *x=0,1* $y = \frac{1}{4}$ *Vertex at* $x = \frac{1}{2}$ $\therefore$ *Vertex at* $\left(\frac{1}{2}, \frac{1}{4}\right)$	2 – correct coordinates 1 – correct *x* coordinate, or correct values for *x* and *y* not written as coordinates.
(ii)  	2 – correct sketch showing all intercepts and coordinates of vertex 1 – incorrect scale or missing points, Diagram must be neat or only 1 mark given

(c) (i) $f(-1)+f(-5)$ $=-2+(2-5)$ $=-5$	1 – correct answer
(c) (ii) 	3 – correct sketch 2 – correct diagram with incorrect circles, or dotted line drawn for y = -2 1 – correct lines ignoring domain
(d) $3x^2+4x+1 \equiv Ax(x-2)+B(x+1)+C$ $= Ax^2-2Ax+Bx+B+C$ $= Ax^2-x(2A+B)+B+C$ $\therefore A=3 \qquad -(2A+B)=4 \qquad B+C=1$ $-6-B=4 \qquad \therefore C=-9$ $\therefore B=10$	3 – correct solution 2 – correct attempt at solution with one error 1 – correct expansion of quadratic leading to correct factorising using x
Communication: ***1 mark each <u>except</u> 15(a) (i)- 2 marks*** (a) (i) all relevant information - 2 marks (b) (i) efficiency of solution (b) (ii) clarity of diagram (c) (ii) clarity of diagram	

Question 16	
(a) $4^x + 2^x - 6 = 0$ $(2^x)^2 + 2^x - 6 = 0$ $Let\ m = 2^x$ $m^2 + 2m - 6 = 0$ $(m+3)(m-2) = 0$ $m = -3, 2$ $ie\ 2^x = -3, 2$ $but\ 2^x > 0\ for\ all\ x$ $\therefore 2^x = 2$ $\therefore x = 1$	3 – correct solution including no solution for $2^x = -3$ 2 – two correct possible solutions 1 – correct values of *m*
(b) (i) $\alpha + \beta = \frac{-b}{a}$ $= \frac{-5}{3}$	1 – correct answer
(ii) $\alpha\beta = \frac{-4}{3}$	1 – correct answer
iii) $\alpha^2 + \beta^2$ $= (\alpha + \beta)^2 - 2\alpha\beta$ $= \frac{25}{9} - 2(\frac{-4}{3})$ $= \frac{49}{9}$	2 – correct solution or carried errors from parts (i) or (ii) 1 – one error
(iv) $\frac{3}{\alpha} + \frac{3}{\beta}$ $= \frac{3\beta + 3\alpha}{\alpha\beta}$ $= \frac{3(\beta + \alpha)}{\alpha\beta}$ $= -5 \div \frac{-4}{3}$ $= \frac{15}{4}$	2 – correct solution or carried errors from parts (i) or (ii) 1 – one error

(c) $\triangle = b^2 - 4ac = 0$ $(k+2)^2 - 4(k+1)k = 0$ $k^2 + 4k + 4 - 4k^2 - 4k = 0$ $-3k^2 + 4 = 0$ $k^2 = \frac{4}{3}$ $k = \pm \frac{2}{\sqrt{3}}$	3 – correct solution and answers 2 – one correct solution or $3k^2 = 4$ 1 – correct discriminant = 0
(d) $Let\ the\ roots\ be\ \alpha\ and\ \alpha + 4$ $then\ sum:$ $\alpha + \alpha + 4 = a$ $2\alpha + 4 = a$ $\alpha = \frac{a-4}{2}$ $\alpha = \frac{a}{2} - 2$.......(1) $product:$ $\alpha(\alpha + 4) = -2b$...........(2) $subs\ (1)\ in\ (2):$ $(\frac{a}{2} - 2)(\frac{a}{2} - 2 + 4) = -2b$ $(\frac{a}{2} - 2)(\frac{a}{2} + 2) = -2b$ $\frac{a^2}{4} - 4 = -2b$ $a^2 - 16 = -8b$ $a^2 + 8b - 16 = 0$	3 – correct solution and answers 2 – correct substitution for α 1 – correct equations for sum and product
Communication: ***1 mark each*** (a) appropriate substitution (b) (iii) use of identity (c) stated understanding of equal roots (d) efficiency of solution	