Question 1 *(12 marks)* **Marks**

(a) Evaluate $\sqrt[3]{\dfrac{4.37+0.8}{1.2-0.9}}$, correct to 2 decimal places. **1**

(b) Express $(4x)^{-\frac{1}{2}}$ without a fractional index. **1**

(c) Simplify completely:

(i) $(x + y) - (x - y)$ **1**

(ii) $\dfrac{m-1}{3} - \dfrac{2m+5}{2}$ **2**

(iii) $3\sqrt{54} + 2\sqrt{24}$ **2**

(d) Write $0.03\dot{4}$ as a rational number in the form of $\dfrac{p}{q}$, where p and q are integers. **2**

(e) Express $\dfrac{3\sqrt{5}}{\sqrt{5}-1}$ with a rational denominator. **2**

(f) Factorise completely $8a^3 - 27$ **1**

Question 2 *(12 Marks)* Start this question on a new page. **Marks**

(a) Show that $f(x) = -\frac{4}{x}$ is an odd function **2**

(b) Solve $|1 + 5x| \geq 4$ and graph your solution on the number line **3**

(c) State the domain and range of the function $y = -\sqrt{9 - x^2}$ **2**

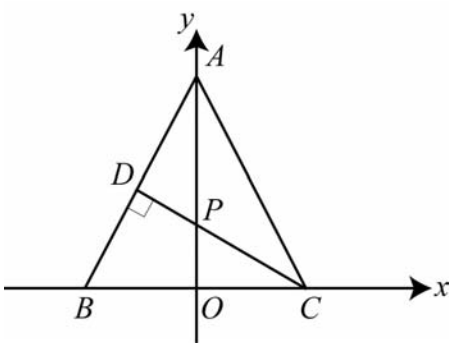
(d) Sketch the region where **3**

$$y > -3\,,\quad y - x - 1 \leq 0 \quad \textit{and} \quad y \leq -x + 1$$

hold simultaneously. Points of intersection are not required.

(e) Find the value(s) of x for which $f(x) = 0$ when $f(x) = x^2 - 3x - 1$ **2**
Give your answer in exact form.

Question 3 *(12 Marks)* Start this question on a new page. **Marks**



NOT TO SCALE

In the diagram, A is the point (0, 4) and B is (–3, 0). CD is perpendicular to AB.

(a) Find the length of AB. **1**

(b) If $AB = BC$, find the coordinates of C. **1**

(c) If $CD \perp AB$, show that the equation of CD is $3x + 4y = 6$. **3**

(d) CD intersects the y-axis in P. Show that $CP = \dfrac{5}{2}$. **2**

(e) Prove that $\Delta ADP \equiv \Delta COP$. **3**

(f) Calculate the area of the quadrilateral $DPOB$. **2**

Question 4 *(12 Marks)* *Start this question on a new page.* **Marks**

(a) Given $\cos\theta = \frac{12}{13}$, find the exact value of $\tan\theta$ if θ is an acute angle. **2**

(b) Solve for $0^0 \le \theta \le 360^0$ if $2\sin\theta + 1 = 0$ **2**

(c) Simplify $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$ **3**

(d) At 14:00 hours, a ship is at point *A*, which is due south of a small island, *I*. The ship is travelling on a bearing of 020° at 15 km/h. After 1 hour and 40 minutes, it is at a point *B*, where it has a bearing of 135° from the island.

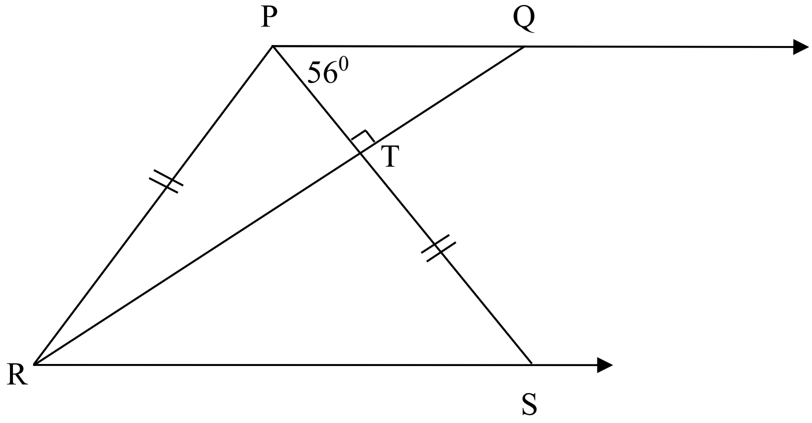
(i) Draw a neat diagram indicating all the relevant information. **1**

(ii) Calculate the distance from *A* to *I* to 2 significant figures. **2**

(iii) Find the time, to the nearest minute, at which the island will be due west of the ship. **2**

Question 5 *(8 Marks)* Start this question on a new page. **Marks**

(a)



In the diagram above, $PQ \parallel RS$, $PR = PS$,
$\angle QPS = 56^0$ *and* $PT \perp RQ$

(i) Find $\angle PRS$. No reasoning is required. **1**

(ii) Show that $\angle PRQ = 22^0$ **2**

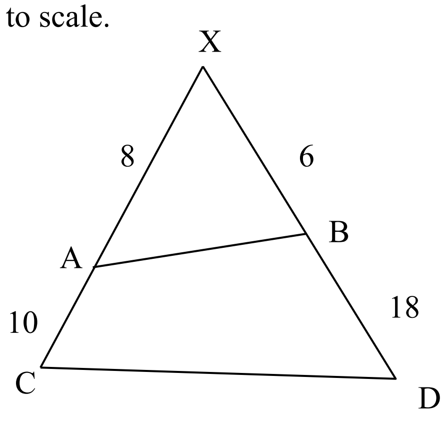
(b) For a regular octagon,

find (i) the sum of the interior angles **1**

(ii) the size of each interior angle. **1**

(c) In the figure below, prove $\Delta XAB \ ||| \ \Delta XDC$. **3**

The figure is not to scale.



Question 6 *(16 Marks)* Start this question on a new page. **Marks**

(a) Differentiate

(i) $y = 2x^3 + 5x - \frac{1}{x}$ **2**

(ii) $y = \left(1 - 4x^5\right)^3$ **2**

(iii) $y = 4x\sqrt{x}$ **2**

(iv) $y = \frac{6x + 5}{1 - 2x}$ **2**

(b) Find the value of $\lim_{x \to \infty} \frac{4x^2 + x}{1 + 2x - x^2}$ **2**

(c) If $V = 4t^2 - 12t + 2$,

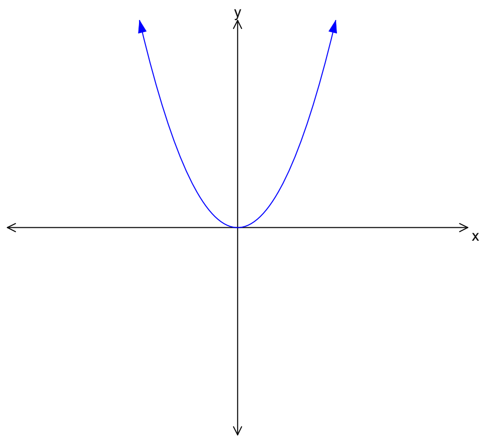
find (i) $\frac{dv}{dt}$ **1**

(ii) the value of t when $\frac{dv}{dt} = 0$ **1**

(d) Find the equation of the tangent to the curve $y = \frac{2}{x + 1}$ **3**

at the point where $x = 3$.

(e) Copy this graph onto your paper. On the same set of axes, draw a sketch of the gradient function. **1**



Question 7 *(8 Marks)* Start this question on a new page. **Marks**

(a) For the quadratic polynomial $y = -2x^2 + 7x + 4$,

find (i) the roots by factorisation **2**

(ii) the axis of symmetry **1**

(iii) the vertex **1**

(iv) Hence or otherwise, sketch the curve $y = -2x^2 + 7x + 4$, showing all the details above and the y intercept. **1**

(b) Find the value(s) of k such that $x^2 + (k+1)x + 4 = 0$ has no real roots. **3**

END OF PAPER

TOTAL : 80 MARKS

YR 11 MATHEMATICS PRELIMINARY FINAL 2008 — SOLUTIONS

QUESTION 1 (12 MARKS)

(a) 2.58 — 1

(b) $(4x)^{-\frac{1}{2}} = \frac{1}{\sqrt{4x}}$

$= \frac{1}{2\sqrt{x}}$ — 1

(c) (i) $x + y - x + y$

$= 2y$ — 1

(ii) $\frac{2(m-1) - 3(2m+5)}{6}$

$= \frac{2m - 2 - 6m - 15}{6}$ — 1

$= \frac{-4m - 17}{6}$ — 1

(iii) $3\sqrt{9}\sqrt{6} + 2\sqrt{4}\sqrt{6}$

$= 9\sqrt{6} + 4\sqrt{6}$ — 1

$= 13\sqrt{6}$ — 1

(d) $x = 0.0344\ldots$

$10x = 0.3444\ldots$

$9x = 0.31$ $\quad x = \frac{0.31}{9}$ — 1

$x = \frac{31}{900}$ — 1

(e) $(2a-3)(4a^2+6a+9)$ — 1

QUESTION 2 (12 MARKS)

(a) $f(-a) = -\,-\frac{4}{a} = \frac{4}{a}$ — 1

$-f(a) = -\,-\frac{4}{a} = \frac{4}{a}$ — 1

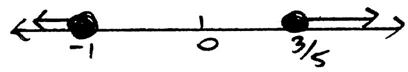
Since $f(-a) = -f(a)$, then function is ODD

(b) $1 + 5x \geq 4$ or $-(1+5x) \geq 4$

$5x \geq 3$ $\quad -1 - 5x \geq 4$

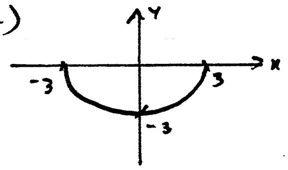
$x \geq \frac{3}{5}$ — 1 $\quad -5x \geq 5$

$x \leq -1$ — 1



— 1

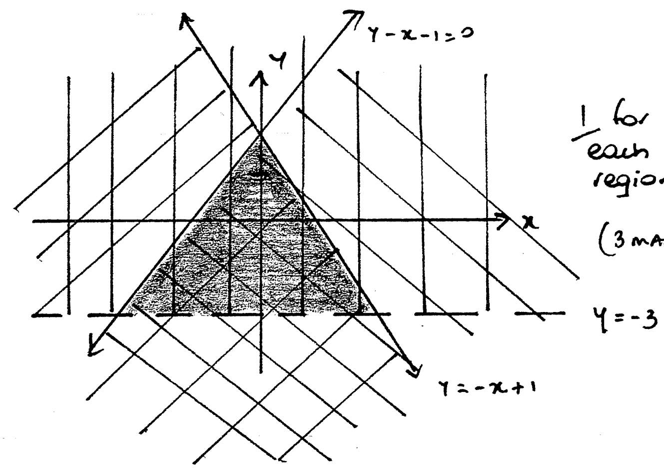
(c)



D : $-3 \leq x \leq 3$ — 1

R : $-3 \leq y \leq 0$ — 1

(d)

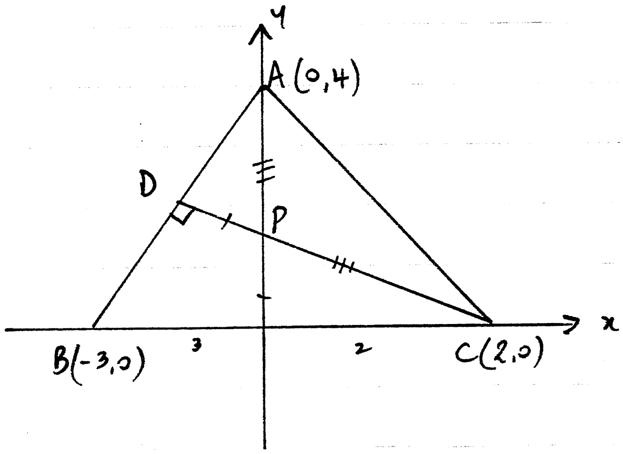


1 for each region (3 MARKS)

(e) $x = \frac{3 \pm \sqrt{9 - 4.1.-1}}{2}$ — 1

$= \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$ — 1

QUESTION 3 (12 MARKS)



(a) $AB = \sqrt{3^2+4^2}$

$= 5$ units 1

(b) $C(2,0)$ 1

(c) grad $AB = \frac{4}{3}$

grad $DC = -\frac{3}{4}$ 1

Eqn CD: $\frac{y}{x-2} = -\frac{3}{4}$ 1

$4y = -3x+6$

$3x+4y = 6$ 1

(d) coords of P:

Let $x=0$, $4y=6$

$y = 3/2$

$\therefore P(0, 3/2)$ 1

$CP = \sqrt{2^2+\left(\frac{3}{2}\right)^2}$

$= \sqrt{4+\frac{9}{4}}$

$= \sqrt{\frac{25}{4}}$ 1

$= 5/2$ units

(e) $\angle DPA = \angle OPC$ (vert. opp $\angle$s =) 1

$\angle ADP = \angle COP$ (given - both 90°) 1

$AP = OP = 3/2$ units 1

$\therefore \triangle ADP \equiv \triangle COP$ (AAS)

(f) Join B to P

$\therefore \triangle BDP \equiv \triangle BOP$ (RHS) 1

Area of Quad DPOB

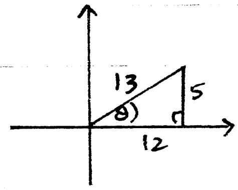
$= 2 \times \triangle POB$

$= 2 \times \frac{1}{2} \times 3 \times \frac{3}{2}$ 1

$= 4\frac{1}{2}$ square units

QUESTION 4 (12 MARKS)

(a)



Using pythagoras theorem

$h = \sqrt{13^2 - 12^2}$

$= \sqrt{25} = 5$ units

$\therefore \tan\theta = \frac{5}{12}$

(b) $2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

basic angle: $30°$

$\therefore$ Sin -ve in 3rd Quad:

$\theta = 210°$

sin -ve in 4th Quad:

$\theta = 330°$

(c) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$

$= \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

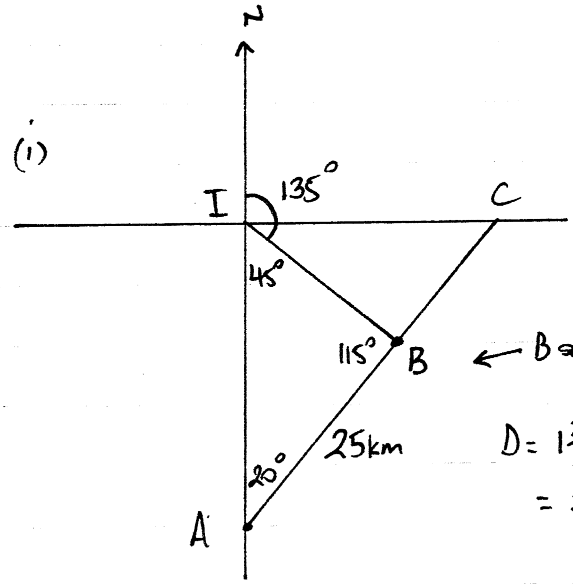
$= \frac{2 + 2\cos x}{\sin x(1+\cos x)}$

$= \frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$= \frac{2}{\sin x}$

$= 2\operatorname{cosec} x$

(d) (i)



← B at 15.40h

$D = 1\frac{2}{3} \times 15$

$= 25$ km

(ii) $\frac{IA}{\sin 115°} = \frac{25}{\sin 45°}$

$IA = \frac{25}{\sin 45°} \times \sin 115°$

$= 32.0428\ldots$

$\doteqdot 32$ km (to 2 sig.fig)

(iii) Find AC to find time from A to C

$\cos 20° = \frac{IA}{AC}$

$AC = \frac{32}{\cos 20°} = 34.1$ km

$\text{Time} = \frac{\text{Dist}}{\text{speed}} = \frac{34.1\text{ km}}{15\text{ km/h}}$

$= 2.27$ h

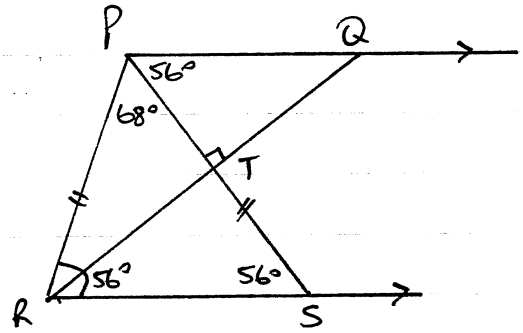
$= 2$ h 16 min

$\therefore$ Leaving at 14.00 h, ship will be east of island at 14h + 2h 16 min

= 16 h 16 min

QUESTION 5 (8 MARKS)

(a)



(i) $\angle PRS = 56°$ 1

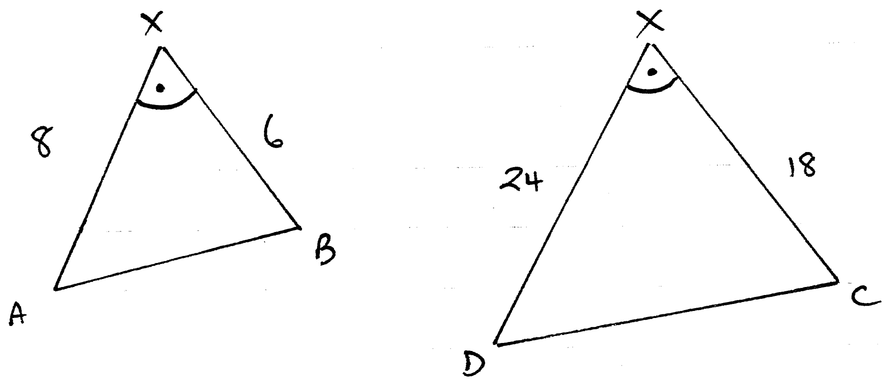
(ii) $\angle RPS = 68°$ ($\angle$ sum of isos. $\triangle PRS$) 1

$\therefore \angle PRQ = 22°$ ($\angle$ sum of $\triangle PRT$) 1

(b) (i) Angle sum $= 6 \times 180° = 1080°$ 1

(ii) Size of interior $\angle = \frac{1080°}{8} = 135°$ 1

(c)



$\angle X$ is the common angle 1

$\frac{XB}{XA} = \frac{6}{8} = \frac{3}{4}$ 1

$\frac{XC}{XD} = \frac{18}{24} = \frac{3}{4}$

$\therefore \triangle XAB \,|||\, \triangle XDC$ because the corresponding sides around the common angle are in the same ratio. 1

QUESTION 6 (16 MARKS)

(a) (i) $\frac{dy}{dx} = 6x^2 + 5 + x^{-2}$

$= 6x^2 + 5 + \frac{1}{x^2}$

(ii) $\frac{dy}{dx} = 3(1-4x^5)^2 . -20x^4$

$= -60x^4(1-4x^5)^2$

(iii) $\frac{dy}{dx} = 4x . \frac{1}{2}x^{-\frac{1}{2}} + \sqrt{x}.4$

$= \frac{2x}{\sqrt{x}} + 4\sqrt{x}$

$= 2\sqrt{x} + 4\sqrt{x}$

$= 6\sqrt{x}$

(iv) $\frac{dy}{dx} = \frac{(1-2x).6 - (6x+5).-2}{(1-2x)^2}$

$= \frac{6 - 12x + 12x + 10}{(1-2x)^2}$

$= \frac{16}{(1-2x)^2}$

(b) $\lim_{x \to \infty} \frac{\frac{4x^2}{x^2} + \frac{x}{x^2}}{1/x^2 + \frac{2x}{x^2} - \frac{x^2}{x^2}}$

$= \frac{4}{-1} = -4$

(c) (i) $\frac{dv}{dt} = 8t - 12$

(ii) $8t - 12 = 0$

$8t = 12$

$t = \frac{12}{8}$

$= 1\frac{1}{2}$

(d) when $x = 3$, $y = \frac{1}{2}$

$\therefore$ Point is $(3, \frac{1}{2})$

grad $= \frac{(x+1).0 - 2.1}{(x+1)^2}$

$= \frac{-2}{(x+1)^2}$

at $x = 3$, gradient $= \frac{-2}{(3+1)^2}$

$= \frac{-2}{16} = \frac{-1}{8}$

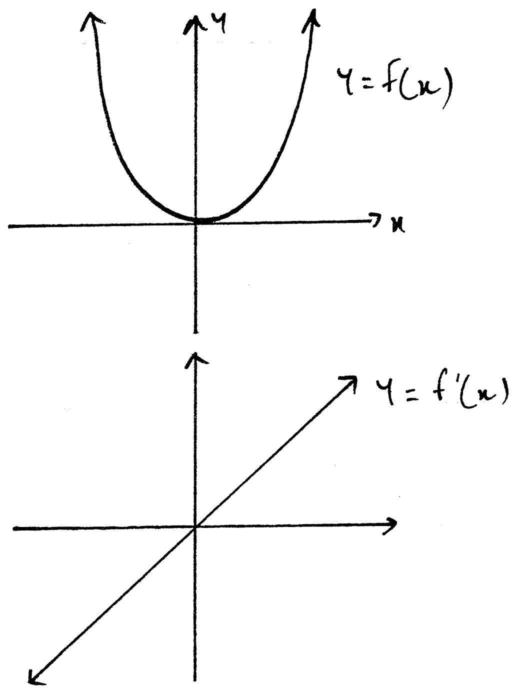
Eqn of tang: grad $= -\frac{1}{8}$, $(3, \frac{1}{2})$

$\frac{y - \frac{1}{2}}{x - 3} = -\frac{1}{8}$

$8y - 4 = -x + 3$

$x + 8y - 7 = 0$

(e)



QUESTION 7 (8 MARKS)

(a) $y = -2x^2 + 7x + 4$

(i) $-2x^2 + 7x + 4 = 0$

$2x^2 - 7x - 4 = 0$

$\dfrac{(2x-8)(2x+1)}{2} = 0$

$(x-4)(2x+1) = 0$ 1

$\therefore$ Roots are 4 and $-\frac{1}{2}$ 1

(ii) $x = \dfrac{-7}{2 \times -2} = \dfrac{-7}{-4}$

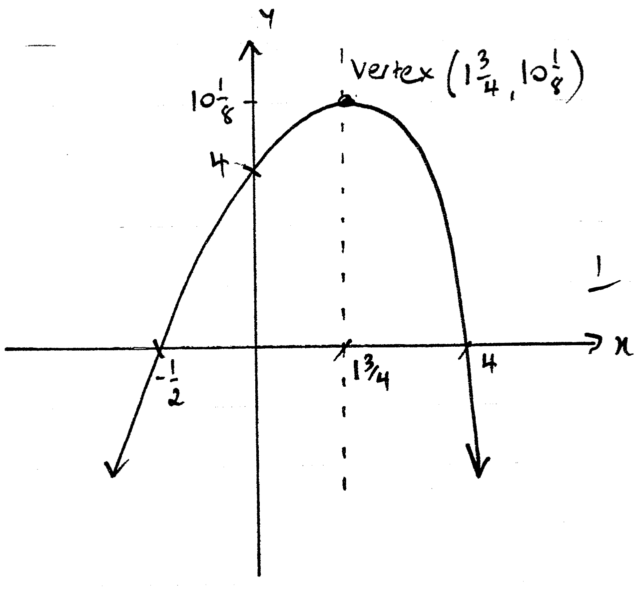
Axis of symmetry $x = 1\frac{3}{4}$ 1

(iii) $y = -2\left(1\frac{3}{4}\right)^2 + 7 \times 1\frac{3}{4} + 4$

$= 10\frac{1}{8}$ 1

$\therefore$ Vertex is $\left(1\frac{3}{4}, 10\frac{1}{8}\right)$

(iv)



1

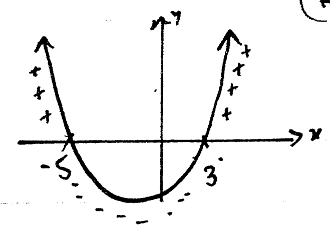
(b) No real roots $\Delta < 0$

$(k+1)^2 - 4.1.4 < 0$ 1

$k^2 + 2k + 1 - 16 < 0$

$k^2 + 2k - 15 < 0$ 1

$(k+5)(k-3) < 0$



$\therefore$ $x^2 + (k+1)x + 4 = 0$

has no real roots

when 1

$-5 \leq k \leq 3$