Question 1 *(14 marks)* **Marks**

(a) Evaluate $\dfrac{\sqrt{125-7.1^2}}{2009}$. Write your answer in scientific notation correct to two significant figures. **2**

(b) Write the recurring decimal $0.0\dot{8}\dot{7}$ in the form $\dfrac{a}{b}$ where a and b are integers. **2**

(c) Factorise $27-n^3$ **1**

(d) Express $\dfrac{3+\sqrt{2}}{3-\sqrt{2}}$ with a rational denominator in simplest form. **3**

(e) Solve $|2x+3|>21$ **2**

(f) Solve $2^{3x}=32$ **2**

(g) Find the exact solution(s) to $2x^2=-7x-3$ in simplest form. **2**

Question 2 *(14 Marks)* *Start this question on a new page.* **Marks**

(a) Consider the function $f(x)=x^2+6x+8$.

(i) Find the vertex. **2**

(ii) Find the y-intercept. **1**

(iii) Find the x-intercepts. **2**

(iv) Draw a neat sketch showing the vertex and any intercepts. **1**

(v) State the domain and range of the function. **2**

(b) Show that $f(x)=x^5+3x^3$ is an odd function. **2**

(c) Draw a neat sketch of $y=\dfrac{1}{x+1}$ **2**

(d) Draw the region defined by $x^2+y^2<4$ **2**

Question 3 *(14 Marks)* *Start this question on a new page.* **Marks**

(a) Find the exact value of $\cos 150°$ **1**

(b) Solve $2\sin\theta = 1$ for $0° \le \theta \le 360°$ **2**

(c) Prove that $\dfrac{1-\sin^2\theta}{1-\cos^2\theta} = \cot^2\theta$ **2**

(d) If $\cos\theta = \dfrac{5}{6}$ and $\sin\theta < 0$ then find the exact value of $\tan\theta$. **3**

(e) A hiker leaves base camp on a bearing of $48°$ and travels for 2.4 km until he hits the highway. The highway runs directly North-South, and he follows it South for 2.2 km.

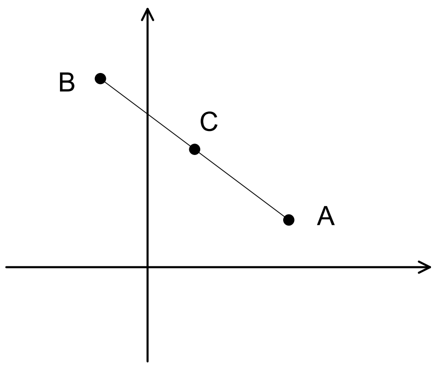
(i) Draw a neat diagram clearly indicating all relevant information. **1**

(ii) Show that the distance (as the crow flies) that the hiker is from the base camp is 1.88 km (to 2 decimal places). **2**

(iii) What bearing is the base camp from his current location? Give your answer to the nearest degree. **3**

Question 4 *(14 Marks)* *Start this question on a new page.* **Marks**

(a) In the diagram below A is the point $(3,1)$ and B is the point $(-1,7)$.



NOT TO SCALE

(i) Find the exact length of the interval AB. **2**

(ii) Find the gradient of the interval AB. **1**

(iii) Find the coordinates of C, the midpoint of the interval AB. **1**

(iv) Find the equation of the line that passes through C and is perpendicular to AB. Give your answer in general form. **3**

Question 4 continued.

(b) Find the equation of the line that makes an angle of 60° with the positive *x*-axis and has *y*-intercept of -3. Write your equation in the form $y = mx + b$ where *m* and *b* are exact values. **2**

(c) The two lines below intercept at the point P.

$$2x - y + 1 = 0$$
$$3x + y - 11 = 0$$

(i) Show that the coordinates of P is $(2,5)$ **2**

(ii) Find the equation of the line that passes through P and the point (3,1). Give your answer in general form. **3**

Question 5 *(14 Marks)* *Start this question on a new page.* **Marks**

(a) Differentiate:

(i) $y = 9x^5$ **1**

(ii) $f(x) = (2x+1)(x-2)^5$ **2**

(iii) $y = \dfrac{x^2}{x^2+1}$ **3**

(b) The derivative of a function is defined as $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ **3**

Use this equation to differentiate $f(x) = x^2 - 1$

(c) For the curve $y = x^3 - 3x^2 + 3x - 1$

(i) Find the gradient of the tangent to the curve at $x = 2$. **2**

(ii) Hence find the equation of the tangent to the curve at $x = 2$. Give your answer in general form. **3**

Question 6 *(14 Marks)* *Start this question on a new page.* **Marks**

(a) If the two roots of the equation $y = 2x^2 - 6x + 5$ are α and β then find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(b) Find the values of k for which $x^2 + 4kx + 16 = 0$ has equal roots. **3**

(c) Solve the inequation $q^2 \le 3q + 40$. **3**

(d) Find the equation of the parabola that passes through the points $(0,5)$, $(2,3)$ and $(4,9)$. **4**

END OF PAPER

Question 1

(a) 0.004298929

$= 4.3 \times 10^{-3}$

(b) let $x = 0.0878787\ldots$

$100x = 8.7878787\ldots$

$\therefore 99x = 8.7$

$\therefore x = 8.7/99$

$= 87/990$

$= 29/330$

(c) $27 - n^3 = (3-n)(9+3n+n^2)$

(d) $\frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{9+6\sqrt{2}+2}{9-2}$

$= \frac{11+6\sqrt{2}}{7}$

(e) if $|2x+3| > 0$ then:

$2x+3 > 21$

$2x > 18$

$x > 9$

if $|2x+3| < 0$ then:

$-2x-3 > 21$

$\therefore -2x > 24$

$x < -12$

Hence $x < -12$ or $x > 9$

(f) $2^{3x} = 2^5$

$3x = 5$

$x = 5/3$

(g) $2x^2 + 7x + 3 = 0$

$(2x+1)(x+3) = 0$

$x = -1/2$ or -3

/14

QUESTION 2

a) i) $x = \frac{-b}{2a}$ $\therefore y = (-3)^2 + 6(-3) + 8$ $\therefore$ Vertex $(-3, -1)$

$x = \frac{-6}{2}$ $y = 9 - 18 + 8$

$x = -3$ **(1)** $y = -1$ **(1)**

ii) y-int when $x = 0$

$\therefore y = (0)^2 + 6(0) + 8$

$y = 8$

$\therefore (0,8)$ **(1)**

iii) x-int when $y = 0$

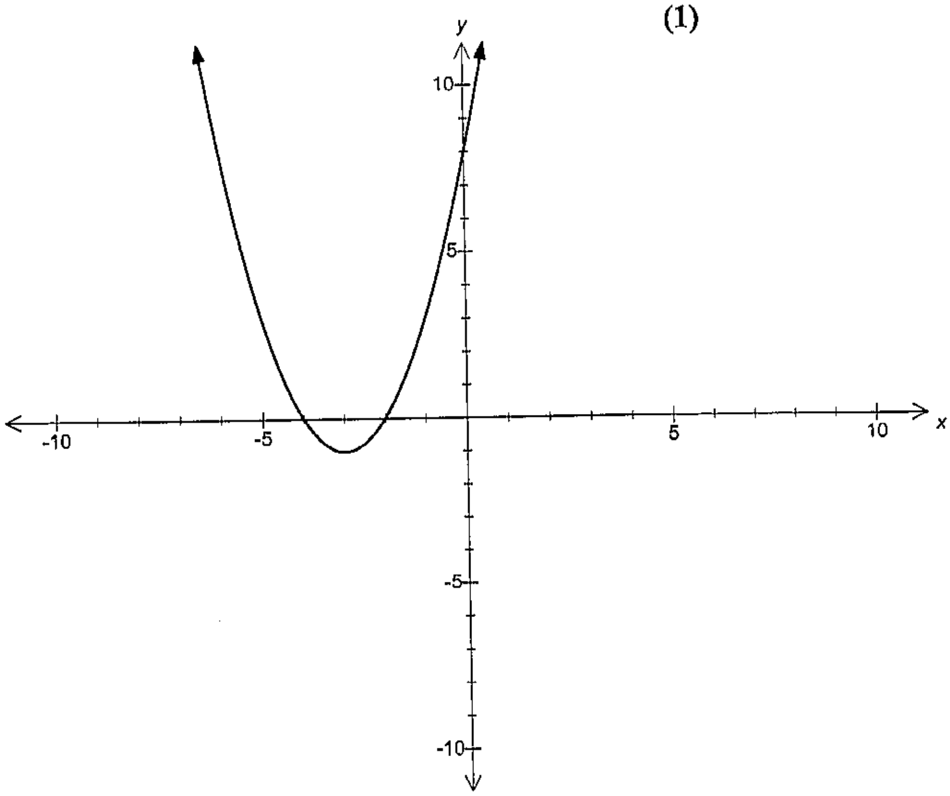
$x^2 + 6x + 8 = 0$

$(x+4)(x+2) = 0$

$x = -4$ *or* $x = -2$

$\therefore (-4,0)$ $(-2,0)$ **(2)**

iv) **(1)**



v) Domain: all real x-values **(1)**

Range: $y \geq -1$ **(1)**

b) For an odd function $f(-x) = -f(x)$ given $f(x) = x^5 + 3x^3$ OR $f(x) = -f(-x)$

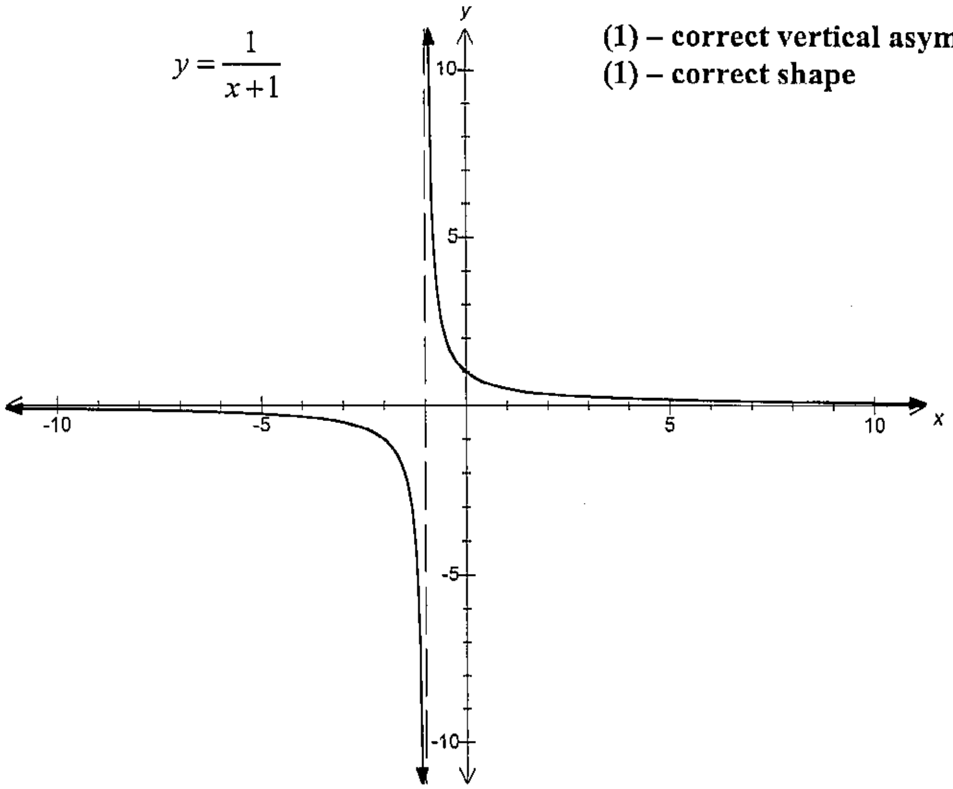
$f(-x) = (-x)^5 + (-x)^3$ and $-f(x) = -\left(x^5 + 3x^3\right)$

$= -x^5 - x^3$ **(1)** $= -x^5 - x^3$ **(1)**

$\therefore$ Function is an odd function.

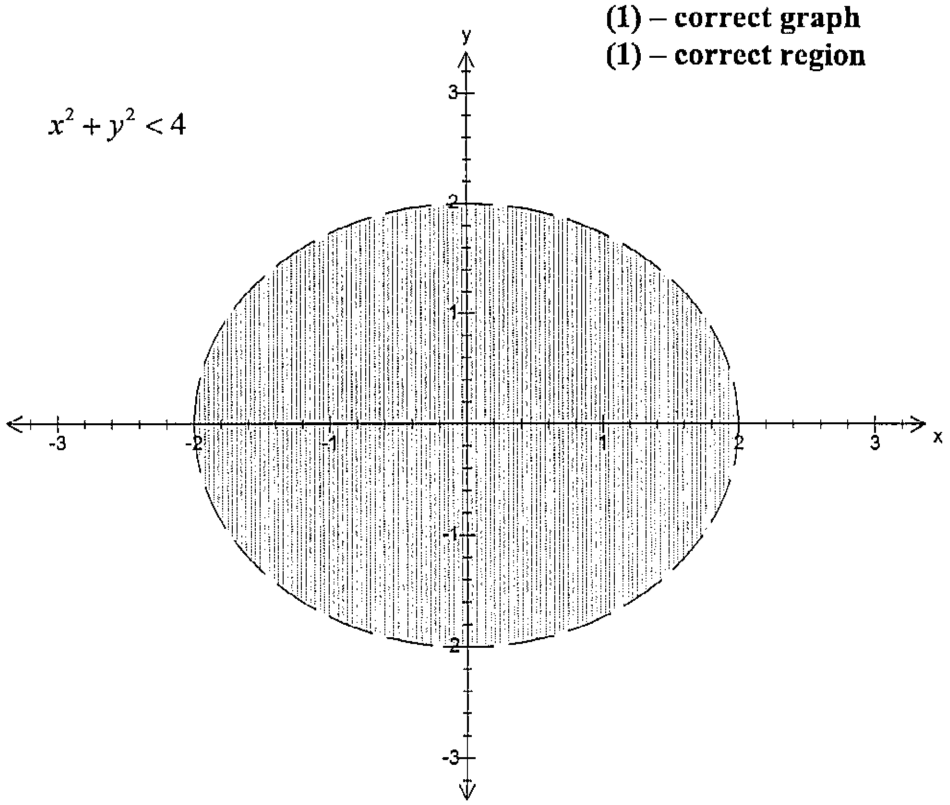
If show $f(x) \neq f(-x)$ award (1)

c)



(1) – correct vertical asymptote
(1) – correct shape

d)

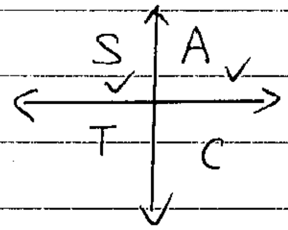


(1) – correct graph
(1) – correct region

Year 11 Math Preliminary 2009 Sample Answers Question 3

(a) $\cos(150^\circ) = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$ ①

(b) $2\sin\theta = 1$



$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$

$\sin 30^\circ = \sin(180^\circ - 150^\circ)$

$= \sin 150^\circ$

$= \frac{1}{2}$

$\therefore \theta = 30^\circ, 150^\circ$
① ①

①

(c) $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos^2 = 1 - \sin^2\theta$ and $\sin^2\theta = 1 - \cos^2\theta$

$\text{LHS} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

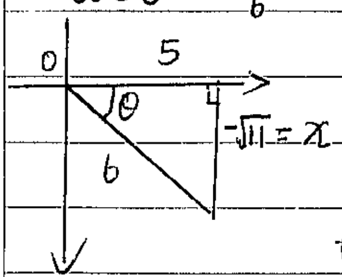
$= \left(\frac{\cos\theta}{\sin\theta}\right)^2$

$= (\cot\theta)^2$

$= \cot^2\theta$ ①

$= \text{RHS}$

(D) $\cos\theta = \frac{5}{6}$



$x^2 + 5^2 = 6^2$

$x^2 = 36 - 25$

$x^2 = 11$

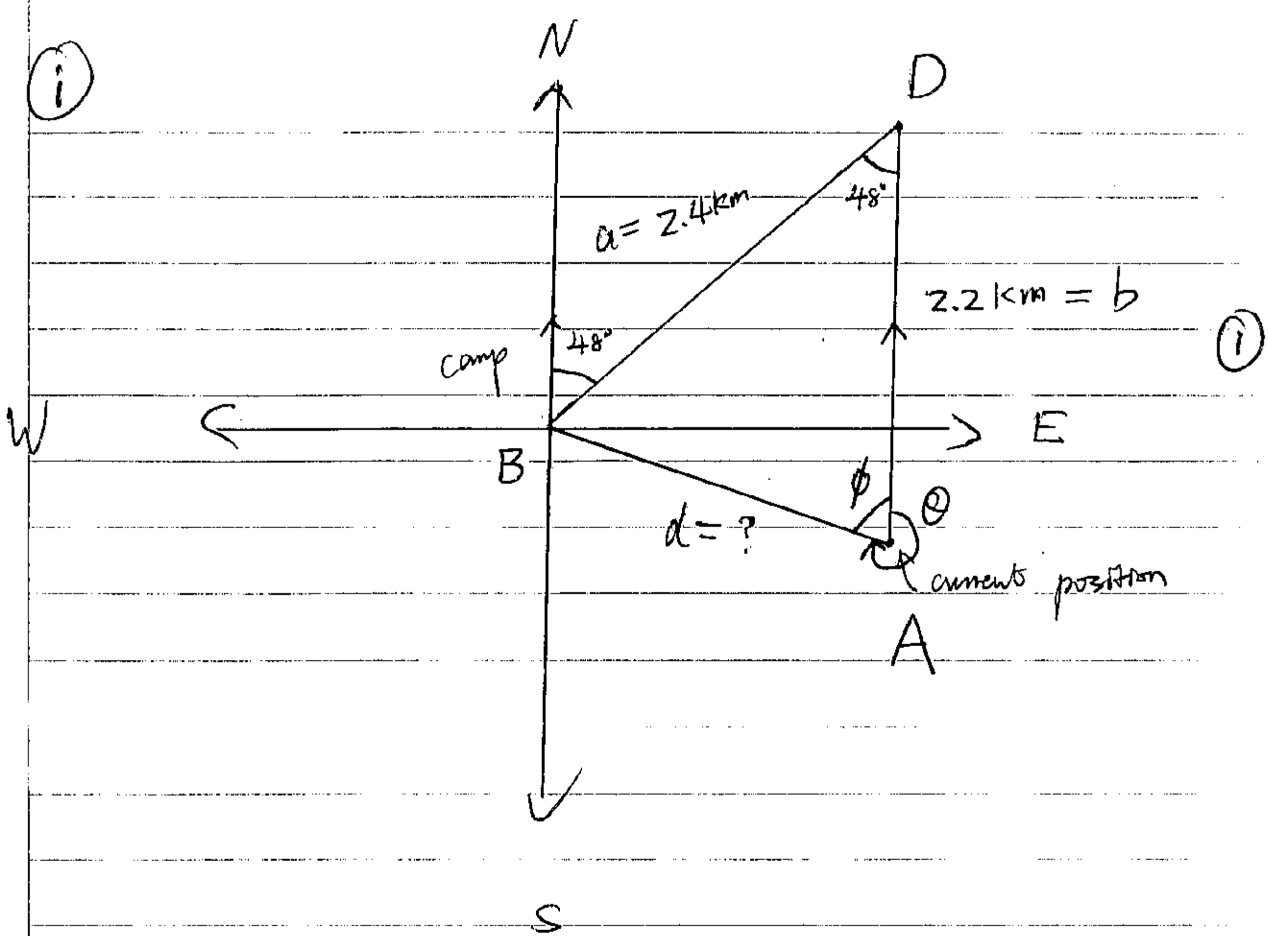
$x = \pm\sqrt{11}$ ①

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ given $\cos\theta > 0$ and $\sin\theta < 0$

$\therefore \tan\theta < 0 \Rightarrow x = -\sqrt{11}$

$\therefore \tan\theta = \frac{-\sqrt{11}}{5} = -\frac{\sqrt{11}}{5}$
① ①

(E) (i)



(ii) $\angle NBD = \angle BDA$ (Alternate angles, $BN \parallel AD$)

$d^2 = a^2 + b^2 - 2ab\cos(\angle BDA)$ (1) Cosine Rule

$= (2.4)^2 + (2.2)^2 - (2)(2.4)(2.2)(\cos 48^\circ)$

$= 3.53398$

$\sqrt{d^2} = d = 1.88$ km (1) to 2 decimal places

(iii) $\dfrac{\sin\phi}{a} = \dfrac{\sin\angle BDA}{d}$ (1) Sine Rule

$\dfrac{\sin\phi}{2.4} = \dfrac{\sin 48^\circ}{1.88}$

$\sin\phi = \dfrac{\sin 48^\circ \times 2.4}{1.88}$

$\sin^{-1}\left(\dfrac{\sin 48^\circ \times 2.4}{1.88}\right) = \phi$

$\phi = 71^\circ 34'$ (1)

$\theta = 360^\circ - 71^\circ 34'$

$= 288^\circ 26'$

$= 288^\circ$ (To nearest degree) (1)

$\therefore$ Bearing of camp from current position is 288°T

Question 4 Year 11 Mathematics Preliminary 2009.

(a)(i) $AB = \sqrt{(3+1)^2+(1-7)^2}$

$= \sqrt{16+36}$ ✓

$= \sqrt{52}$ ✓ (2)

$= 2\sqrt{13}$

[If they used the calculator a mark was deducted]

(ii) $m_{AB} = \frac{7-1}{-1-3}$

$= \frac{6}{-4}$

$= -\frac{3}{2}$ ✓ (1)

(iii) Midpoint AB $= \left(\frac{3-1}{2}, \frac{1+7}{2}\right)$

$\therefore \quad C = (1,4)$ ✓ (1)

[They need brackets for the point but a mark was not deducted]

(iv) Equation of line

$m = \frac{2}{3}$ ✓ $C = (1,4)$

$y-4 = \frac{2}{3}(x-1)$ ✓

$3y-12 = 2x-2$

$0 = 2x-3y+10$ ✓ (3)

(v) Equation of line through $(2,5)$ and $(3,1)$

$m = \frac{1-5}{3-2}$

$m = -4$ ✓

$y-1 = -4(x-3)$ ✓

$y-1 = -4x+12$

$\therefore 4x+y-13=0$ ✓ (3)

(b) $\tan 60° = m$

$\therefore m = \sqrt{3}$ ✓

$\therefore y = \sqrt{3}x - 3$ ✓ (2)

[1 mark for $\sqrt{3}$ or -3 correctly used.

1 mark for $y = mx+b$ in this order only]

(c) $2x-y+1=0$ — ①

$3x+y-11=0$ — ②

① + ② $\quad 5x-10=0$

$5x=10$

$\therefore x=2$ ✓

Sub in ① $\quad 4-y+1=0$

$5-y=0$

$y=5$ ✓ (2)

$\therefore P$ is $(2,5)$

[Both parts $x=2$, $y=5$ must be shown.
1 mark each part.]

Year 11 Mathematics Preliminary Examination

Question 5

a) i. $y = 9x^5$

$\frac{dy}{dx} = 45x^4$ (1)

(ii) $f(x) = (2x+1)(x-2)^5$

$= uv$

$u = 2x+1$ $\quad v = (x-2)^5$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = 5(x-2)^4$

(1)

$f'(x) = v\frac{du}{dx} + u\frac{dv}{dx}$

$= 2(x-2)^5 + (2x+1)5(x-2)^4$ (1) mark ignore subsequent errors

$= (x-2)^4\left[2(x-2) + 5(2x+1)\right]$

$= (x-2)^4(12x+1)$

(iii) $y = \frac{x^2}{x^2+1}$

$= \frac{u}{v}$

$u = x^2$ $\quad v = x^2+1$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 2x$

(1)

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{2x(x^2+1) - 2x \times x^2}{(x^2+1)^2}$ (1)

$= \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2}$

$= \frac{2x}{(x^2+1)^2}$ (1)

* many students have not learnt the product or quotient rules

(b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 1 - (x^2 - 1)}{h}$ (1)

$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 1 - x^2 + 1}{h}$ (1)

$= \lim_{h \to 0} \frac{h(2x+h)}{h}$

$= 2x$ (1)

no marks if they just wrote $f'(x) = 2x$

c) (i) $y = x^3 - 3x^2 + 3x - 1$

$\frac{dy}{dx} = 3x^2 - 6x + 3$ at $x = 2$ (1)

$= 3(2)^2 - 6(2) + 3$

$= 12 - 12 + 3$

$= 3$ (1)

$\therefore$ gradient of the tangent at $x = 2$ is 3

(ii) when $x = 2$, $y = x^3 - 3x^2 + 3x - 1$

$= 2^3 - 3(2)^2 + 3(2) - 1$

$= 1$ (1)

some students have done this in part (i).

$y - y_1 = m(x - x_1)$ $m = 3$, $(2,1)$

$y - 1 = 3(x - 2)$ (1)

$y - 1 = 3x - 6$

$3x - y - 5 = 0$ (1)

Maths Solutions

Q6 (a) $y = 2x^2 - 6x + 5$

(i) $\alpha + \beta = -\frac{b}{a} = -\frac{(-6)}{2} = 3.$ (1)

(ii) $\alpha\beta = \frac{c}{a} = \frac{5}{2}.$ (1)

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta.$ 1 mark identity (1) ②

$= 3^2 - 2(5/2).$

$= 4.$ 1 mark (1)

(b) $x^2 + 4kx + 16 = 0$ has equal roots if $\Delta = 0$ (1)

$\therefore (4k)^2 - 4(1)(16) = 0$ 1 mark equating to zero.

$\therefore 16k^2 - 64 = 0$ ③

$\therefore \cancel{4}k^2 - \cancel{4} = 0$

$\therefore (k-2)(k+2) = 0$ (1 mark factorisation)

$\therefore k = 2$ or $-2.$ (1)

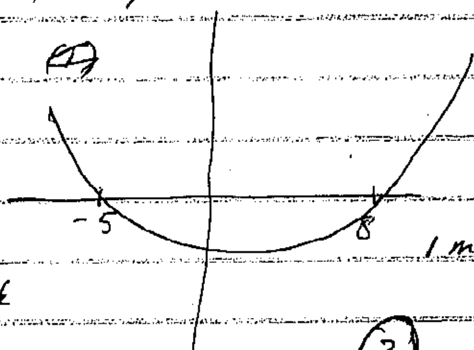
1 mark solution

(c) $q^2 \le 3q + 40$

$q^2 - 3q - 40 \le 0$

$\therefore (q-8)(q+5) \le 0$ 1 mark

$\therefore -5 \le q \le 8.$ 1 mark.



Test = 1 mark. ③

(d) $y = ax^2 + bx + c$ if $x = 0$, $y = 5$

$\therefore c = 5$ 1 mark.

$\therefore y = ax^2 + bx + 5$ if $x = 2$, $y = 3.$

$\therefore 3 = 4a + 2b + 5$

$\therefore 4a + 2b = -2$

$\therefore 2a + b = -1 \quad \ldots (1)$ 1 mark

if $x = 4$, $y = 9$

$\therefore 9 = 16a + 4b + 5$

$\therefore 16a + 4b = 4$

$\therefore 4a + b = 1. \quad \ldots (2).$ 1 mark

$\therefore\ 2a = 2$

$\therefore\ a = 1.$

$\therefore\ b = -3$

ie. $y = x^2 - 3x + 5.$ 1 mark.