Section I

10 Marks

Use the multiple-choice answer sheet for Questions 1 - 10

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1) $\dfrac{8.964}{\sqrt{61.328}}$ expressed to 3 significant figures is:

A 1.145 **B** 1.14 **C** 1.144 **D**7.83

2) $p^3 - 8$ factorises to give:

A $(p - 2)^3$

B $(p - 2)(p^2 + 2p + 4)$

C $(p - 2)(p^2 - 2p + 4)$

D $(p - 2)(p^2 + 4p + 4)$

3) If $f(x) = 5 - 3x - 2x^2$ then $f(-2)$ is equal to:

A 3 **B** -11 **C** 5 **D** -7

4) The largest possible domain of $y = \sqrt{4 - x^2}$ is

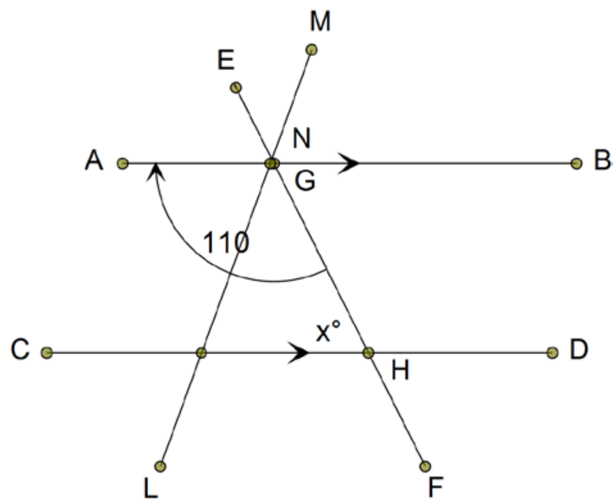
A All real numbers

B $x \le 2$

C $-2 \le x \le 2$

D $x > -2$ and $x < 2$

5) The value of x is



A 60° **B** 70° **C** 50° **D** 65°

6) Which of the following has solutions 4 and -3 ?

A $x^2+x-12=0$ **B** $x^2-x-12=0$

C $x^2-7x+12=0$ **D** $x^2+7x-12=0$

7) The solution for $x^2-49\geq 0$ is

A $x\leq -7$ and $x\geq 7$ **B** $x\geq -7$ and $x\geq 7$

C $x\leq -7$ and $x\leq 7$ **D** $-7\leq x\leq 7$

8) $(2\sqrt{5}-1)^2$ is equal to

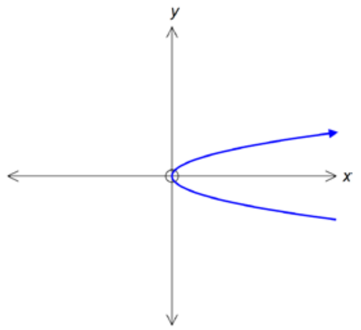
A 19 **B** $11-2\sqrt{5}$ **C** $19-2\sqrt{5}$ **D** $21-4\sqrt{5}$

9) The perpendicular distance from the line $3x-4y+6=0$ to the point (–1,–3) is

A $\frac{3}{5}$ **B** 3 **C** $\frac{4}{3}$ **D** 15

10) Which of the following graphs represent a function ?

A



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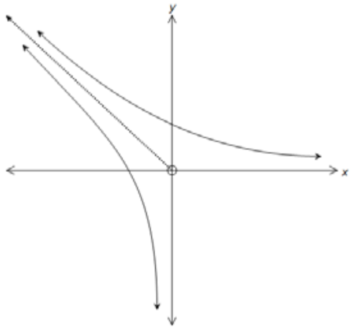
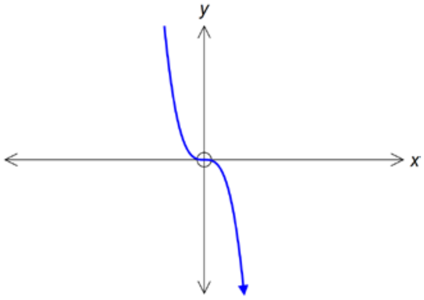
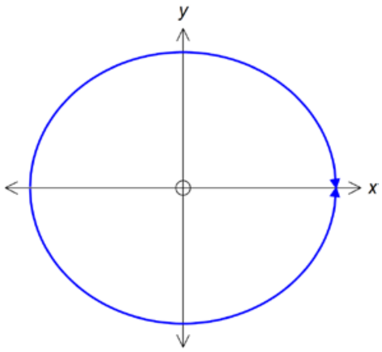


C



D



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Section II
Attempt Questions 11 to 16
Answer each question in separate writing booklets.
All necessary working should be shown in every question.

Question 11 *(14 Marks)* **Marks**

a) Express $0.02\dot{4}$ as a fraction in its simplest form. **2**

b) Solve simultaneously **2**

$$3x - 2y = 29$$
$$4x + 5y = 8$$

c) Solve $|3x - 8| < 1$ **2**

d) Solve $\dfrac{x+1}{3} + \dfrac{x}{4} = 5$ **2**

e) Simplify $\dfrac{x^2 - 144}{x^2 + 15x + 36} \div \dfrac{2x^2 - 24x}{(x+3)^2}$ **3**

f) Find the exact solution(s) to $4x^2 + 12x + 1 = 0$ in simplest form. **3**

<u>Question 12</u> *(13 Marks)* <u>Start a new booklet.</u> **Marks**

a) Sketch the following showing all essential features

i) $x^2 + y^2 = 9$ **2**

ii) $y = \dfrac{1}{x+3}$ **2**

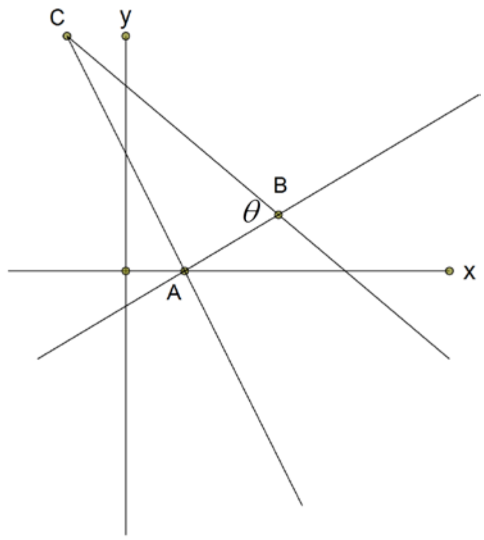
iii) $y = |x| + 3$ **2**

iv) $y = 3^{-x}$ **2**

b) Sketch the parabola $y = x^2 - 3x - 18$. Clearly showing the coordinates of the x and y intercepts and the vertex. **3**

c) Show that $f(x) = x^5 - 16x$ is an odd function. **2**

Question 13 *(13 Marks)* Start a new booklet. **Marks**



The diagram shows the points A(1,0) B(4,1) and C(–1, 6) in the Cartesian plane.
Angle ABC is θ.

Copy and trace the diagram into your writing booklet.

a) Show that C lies on the line $3x + y = 3$. **1**

b) Find the gradient of AB. **1**

c) Find the equation of AB. **1**

d) Find the length of AB. **1**

e) Show that AB and AC are perpendicular. **2**

f) Find the area of ΔABC. **2**

g) Find $\tan\theta$. **1**

h) Find the equation of the circle with centre A the passes through B. **1**

i) The point D is not shown on the diagram. The point D lies on the line $3x + y = 3$ between A and C, AD = AB. Find the coordinates of D. **2**

j) On your diagram shade the region satisfying the inequality $3x + y \le 3$. **1**

Question 14 *(17 Marks)* Start a new booklet. **Marks**

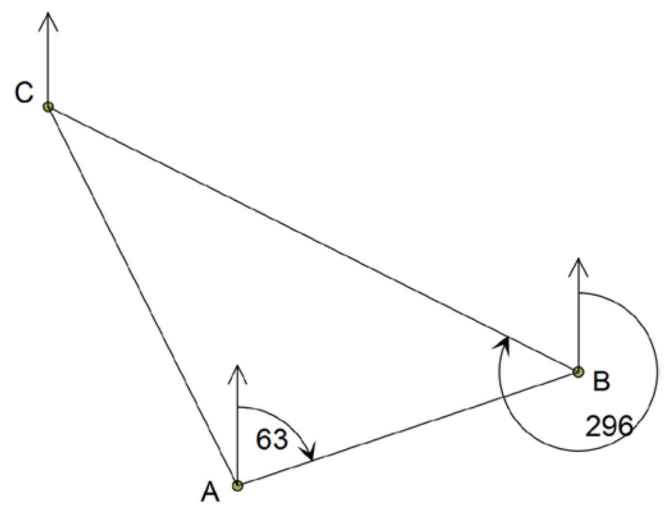
a) Find the exact value of $\cos 150^0$ **1**

b) Solve each equation for $0^0 \le \theta \le 360^0$

i) $\cos^2\theta = \dfrac{1}{2}$ **2**

ii) $\tan 2x = \sqrt{3}$ **2**

c) A ship sales 50km from port A to port B on a bearing of 063°T then sails 130km from port B to port C on a bearing of 296°T.



i) Explain why $\angle ABC = 53^0$. **2**

ii) Find, correct to the nearest km, the distance of port A from port C. **2**

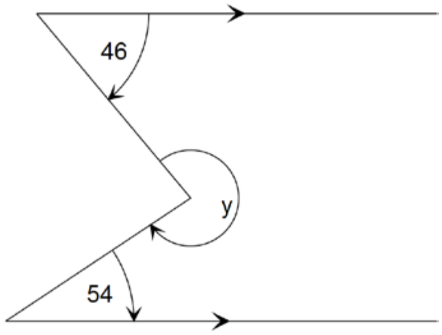
iii) Find the bearing of port A from port C. Answer correct to the nearest minute. **3**

d) Prove that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$. **3**

e) If $\cos\alpha = \dfrac{4}{5}$ and $\sin\alpha < 0$, find $\tan\alpha$. **2**

Question 15 *(12 Marks)* Start a new booklet. **Marks**

a) Find the value of y, giving reasons. **2**

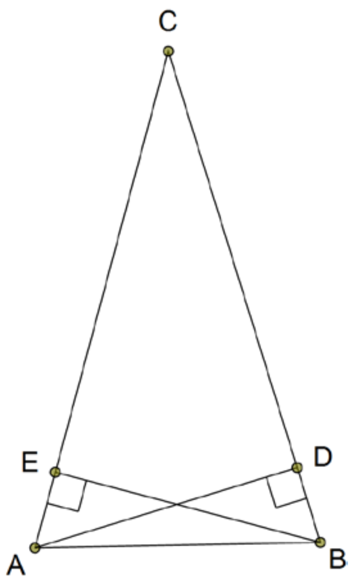


b) Find the size of each interior angle of a regular octagon. **2**

c) Let AD and BE be the two altitudes of triangle ABC. Suppose that AD=BE **4**

i) Prove that $\Delta ABE \equiv \Delta BAD$.

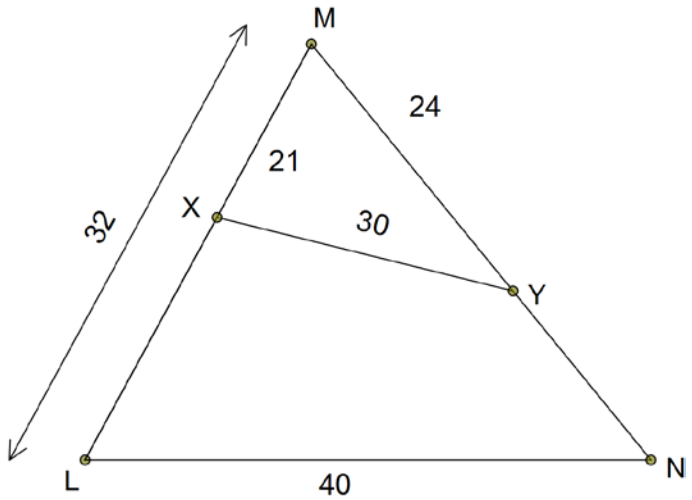
ii) Hence prove that ΔABC is isosceles.



Question 15 continued. **Marks**

d) i) Prove that ΔLMN and ΔYMX are similar. **4**

ii) Hence find YN.



Question 16 (21 Marks) <u>Start a new booklet.</u>

a) Differentiate the following with respect to x

i) $y = 5x^4 - 7x^2 + 8$ **1**

ii) $y = \dfrac{3x^4 - 5x^2}{x}$ **2**

iii) $y = \dfrac{1}{x\sqrt{x}}$ **2**

v) $y = \dfrac{6}{x^4}$ **1**

vi) $y = (3x+4)^5$ **2**

vii) $y = x(3-2x)^4$ **2**

viii) $y = \dfrac{x^2+5}{x-2}$ **2**

ix) $y = x\sqrt{1-x^2}$, answer without fractional or negative indices **3**

Question 16 Continued. **Marks**

b) A function $f(x) = x^2 + 4x - 12$ has a tangent with a gradient of –6 at the point P on the curve. Find the coordinates of P. **3**

c) Find the equation of the normal to the curve $y = x^2 - 8x + 7$ at the point (3,–8). **3**

End of paper

Year 11 Mathematics Preliminary Exam 2012

Section 1

1. $\dfrac{8.964}{\sqrt{61.328}} \doteqdot 1.1446493$

$= 1.14$ to 3 sig fig (B)

2. $p^3 - 8 = p^3 - 2^3$

$= (p-2)(p^2+2p+4)$ (B)

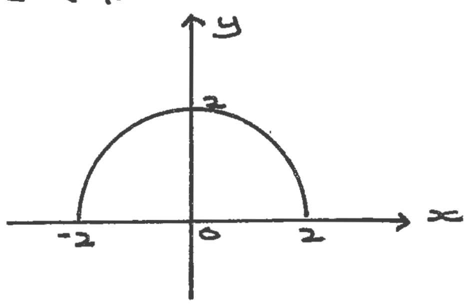
3. $f(x) = 5 - 3x - 2x^2$

$f(-2) = 5 - 3(-2) - 2(-2)^2$

$= 5 + 6 - 8$

$= 3$ (A)

4. $y = \sqrt{4 - x^2}$



$\therefore$ Domain $-2 \leq x \leq 2$ (C)

5. $x = 70$ (B)

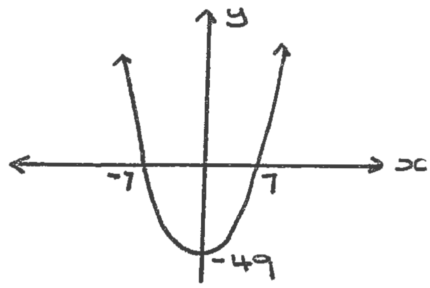
cointerior angles add up to 180° AB ∥ CD

6. $x = 4, -3$

$\therefore (x-4)(x+3) = 0$

$x^2 - x - 12 = 0$ (B)

7. $x^2 - 49 \geq 0$



$\therefore x \leq -7,\ x \geq 7$ (A)

8. $(2\sqrt{5}-1)^2 = (2\sqrt{5}-1)(2\sqrt{5}-1)$

$= 4 \times 5 - 2\sqrt{5} - 2\sqrt{5} + 1$

$= 21 - 4\sqrt{5}$ (D)

9. $3x - 4y + 6 = 0 \quad (-1, -3)$

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|3 \times -1 - 4 \times -3 + 6|}{\sqrt{3^2+4^2}}$

$= \dfrac{|-3+12+6|}{5}$

$= \dfrac{15}{5}$

$= 3$ (B)

(10) A graph represents a function if a vertical line cuts the graph once

(C)

Section 2

Question 11

a) Let $x = 0.02\dot{4}$

$= 0.024444\ldots$

$1000x = 24.444\ldots$

$100x = 2.444\ldots$

$900x = 22$

$x = \frac{22}{900}$

$= \frac{11}{450}$

$\therefore 0.02\dot{4} = \frac{11}{450}$

b)

$3x - 2y = 29$ —— ①

$4x + 5y = 8$ —— ②

① × 4 $\quad 12x - 8y = 116$ —— ③

② × 3 $\quad 12x + 15y = 24$ —— ④

③ − ④ $\quad -23y = 92$

$y = -4$

$3x - 2y = 29$

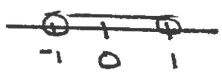
$3x - 2(-4) = 29$

$3x = 21$

$x = 7$

$\therefore x = 7,\ y = -4$

c) $|3x - 8| < 1$



$-1 < 3x - 8 < 1$

$7 < 3x < 9$

$\frac{7}{3} < x < 3$

d) $\frac{x+1}{3} + \frac{x}{4} = 5$

$4(x+1) + 3x = 60$

$4x + 4 + 3x = 60$

$7x = 56$

$x = 8$

e) $\frac{x^2 - 144}{x^2 + 15x + 36} \div \frac{2x^2 - 24x}{(x+3)^2}$

$= \frac{(x-12)(x+12)}{(x+12)(x+3)} \times \frac{(x+3)(x+3)}{2x(x-12)}$

$= \frac{x+3}{2x}$

f) $4x^2 + 12x + 1 = 0$

$a = 4,\ b = 12,\ c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-12 \pm \sqrt{12^2 - 4 \times 4 \times 1}}{2(4)}$

$= \frac{-12 \pm \sqrt{128}}{8}$

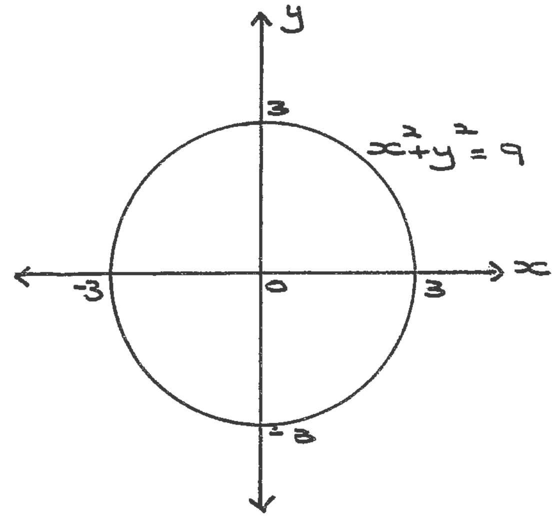
$= \frac{-12 \pm 8\sqrt{2}}{8}$

$= \frac{4(-3 \pm 2\sqrt{2})}{8}$

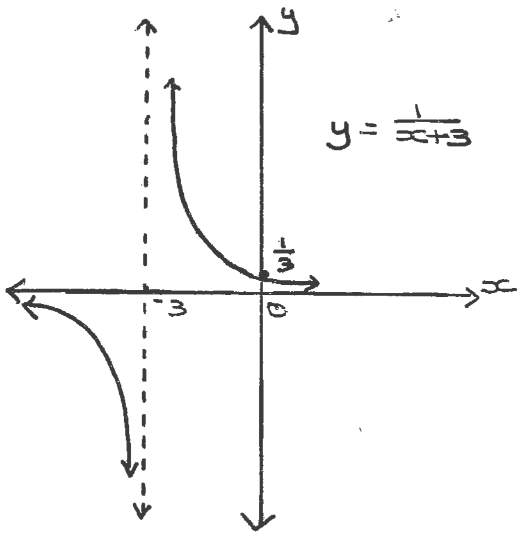
$= \frac{-3 \pm 2\sqrt{2}}{2}$

Question 12

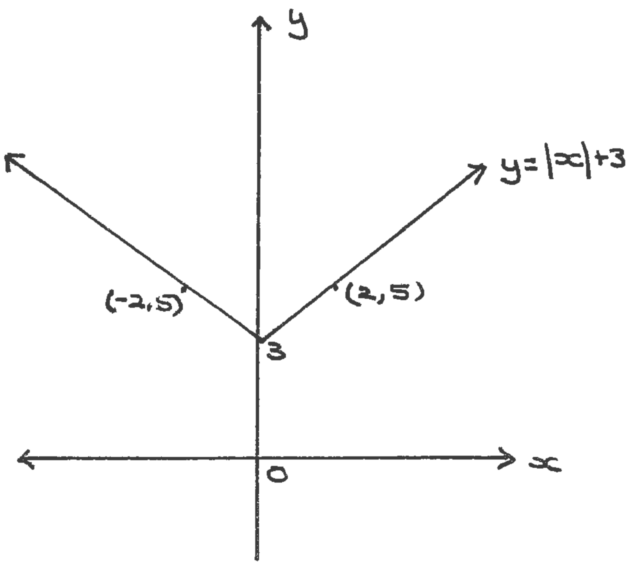
a) (i)

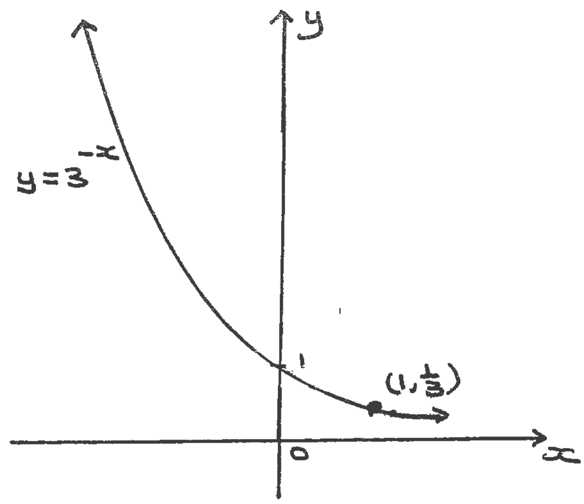


(ii)



(iii)





b) $y=x^2-3x-18$

x - intercepts $\Rightarrow y=0$

$x^2-3x-18=0$

$(x-6)(x+3)=0$

$x=6,-3$

$\therefore$ x intercepts are (6,0) (-3,0)

y - intercept $\Rightarrow x=0$

$y=x^2-3x-18$

$=-18$

$\therefore$ y-intercept is (0,-18)

equation of the axis of symmetry

$x=\frac{-b}{2a}$

$x=\frac{--3}{2}$

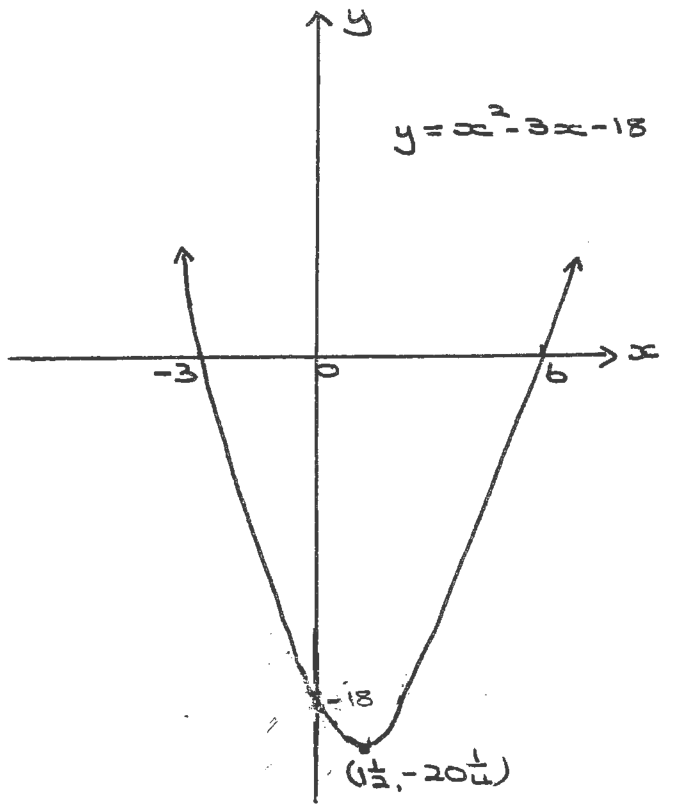
$x=\frac{3}{2}$

$y=x^2-3x-18$ when $x=\frac{3}{2}$

$=\left(\frac{3}{2}\right)^2-3\left(\frac{3}{2}\right)-18$

$=-20\frac{1}{4}$

$\therefore$ vertex is $(1\frac{1}{2},-20\frac{1}{4})$



c) $f(x) = x^5 - 16x$

$f(-x) = (-x)^5 - 16(-x)$

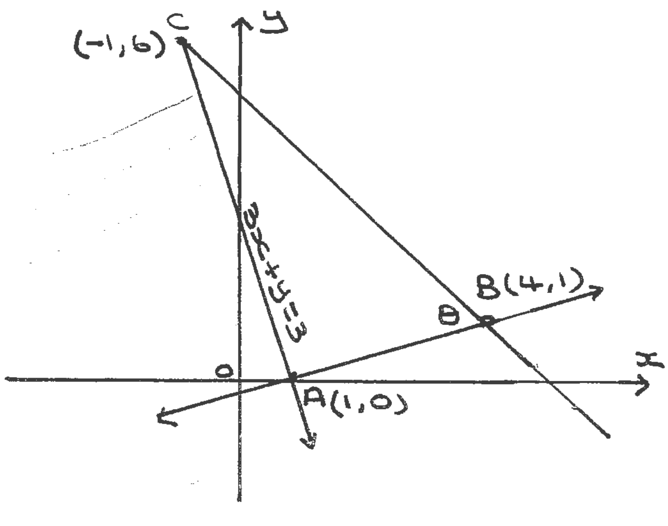
$= -x^5 + 16x$

$-f(-x) = -(-x^5 + 16x)$

$= x^5 - 16x$

As $f(x) = -f(-x)$ the function is odd.

Question 13



a) $3x+y=3$ C is $(-1,6)$

LHS $= 3x+y$

$= 3(-1)+6$

$= 3$

$=$ RHS

$\therefore$ C lies on the line $3x+y=3$

b) $m_{AB} = \dfrac{y_2-y_1}{x_2-x_1}$ A(1,0) B(4,1)

$= \dfrac{1-0}{4-1}$

$= \frac{1}{3}$

$\therefore$ gradient AB is $\frac{1}{3}$

c) $y-y_1 = m(x-x_1)$ $m=\frac{1}{3}$ A(1,0)

$y-0 = \frac{1}{3}(x-1)$

$y = \frac{1}{3}x - \frac{1}{3}$

d) $d_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-1)^2+(1-0)^2}$

$= \sqrt{10}$

$\therefore$ length AB is $\sqrt{10}$ units

e) $m_{AB} = \frac{1}{3}$ from part (b)

$m_{AC} = \dfrac{y_2-y_1}{x_2-x_1}$ A(1,0) C(-1,6)

$= \dfrac{6-0}{-1-1}$

$= -3$

$m_{AB} \times m_{AC} = \frac{1}{3} \times -3$

$= -1$

$\therefore AB \perp AC$

f) $d_{AC} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-1-1)^2+(6-0)^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times AC \times AB$

$= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$

$= 10$

$\therefore$ Area $\triangle ABC$ is 10 units^2

g) $\tan\theta = \frac{AC}{AB}$

$= \frac{2\sqrt{10}}{\sqrt{10}}$

$= 2$

h) $(x-h)^2+(y-k)^2 = r^2$

centre A(1,0) $r = \sqrt{10}$

$\therefore (x-1)^2 + y^2 = 10$

i) Length AD = length AB

$= \sqrt{10}$

length AC $= 2\sqrt{10}$

$\therefore$ D is the midpoint of AC

$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{1-1}{2}, \frac{0+6}{2}\right)$

$= (0,3)$

j) test (0,0) in $3x+y \leq 3$

$0 \leq 3$ true

$\therefore$ shade side of line containing (0,0)

see previous page

Question 14

a) $\cos 150° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

b) i) $\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

S | A / T | C

Related angle $\cos 45° = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45°, 180-45°, 180+45°, 360°-45°$

$\theta = 45°, 135°, 225°, 315°$

ii) $\tan 2\theta = \sqrt{3}$ $\quad 0 \leq \theta \leq 360$

$0 \leq 2\theta \leq 720$

Related angle $\tan 60° = \sqrt{3}$

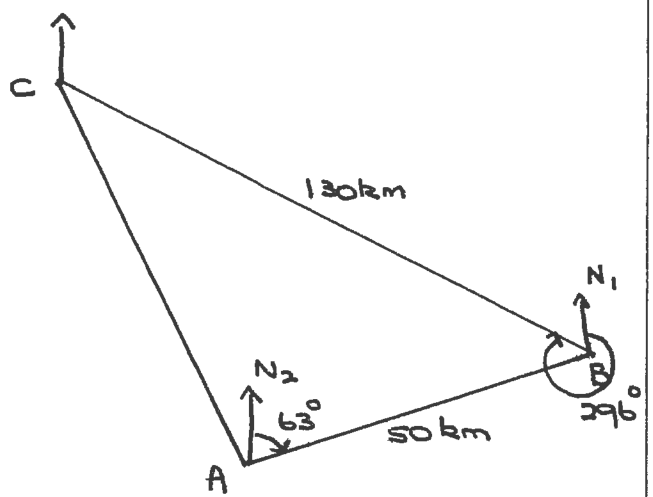
S | A / T | C

$\therefore 2\theta = 60°, 180°+60°, 360°+60°, 360°+180°+60°$

$2\theta = 60°, 240°, 420°, 600°$

$\theta = 30°, 120°, 210°, 300°$

c)



$N_1BC = 64°$ angles about a point add up to 360°

$N_1BA = 117°$ cointerior angles add up to 180° $N_1B \parallel N_2A$

$\therefore ABC = 53°$ by subtraction

ii) $b^2 = a^2 + c^2 - 2ac\cos B$

$= 130^2 + 50^2 - 2 \times 130 \times 50 \cos 53^\circ$

$\doteqdot 11576 \cdot 4047$

$b \doteqdot 107 \cdot 5937019$

$\therefore$ distance is 108 km to nearest km

iii) $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\dfrac{\sin C}{50} = \dfrac{\sin 53}{b}$

$\sin C = \dfrac{50 \sin 53^\circ}{b}$

$C = 21^\circ 47'$ to nearest min

$\therefore$ Bearing is $(116 + 21^\circ 47')$ T

$= 137^\circ 41'$ T

d) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

L.HS $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$

$= \dfrac{1}{\cos\theta \sin\theta}$

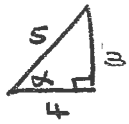
$= \sec\theta \operatorname{cosec}\theta$

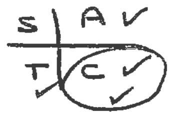
$=$ R.HS

$\therefore \tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

e) $\cos\alpha = \frac{4}{5}$

$\sin\alpha < 0$

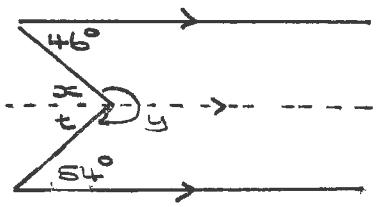




$\therefore \tan\alpha = -\frac{3}{4}$

Question 15

a)



draw a line parallel to the other 2

$x = 46$ alternate angles are equal if the lines are parallel

$t = 54$ alternate angles are equal if the lines are parallel

$y = 360 - (46 + 54)$ angles about a point add up to 360°

$= 260$

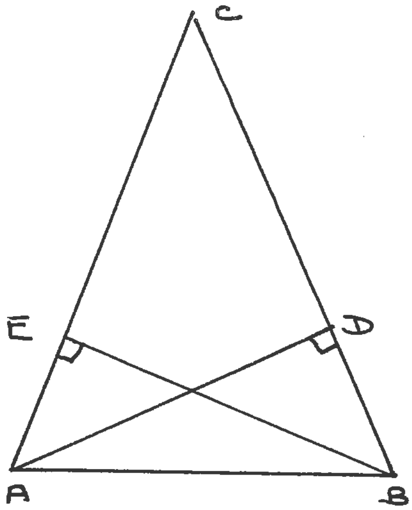
b) Interior angle $= \dfrac{(n-2) \times 180}{n}$ $n = 8$

$= \dfrac{(8-2) \times 180}{8}$

$= 135^\circ$

$\therefore$ Each interior angle is 135°

c)



In $\triangle ABE$ and $\triangle BAD$

AB is common

BE = AD given

$A\hat{E}B = A\hat{D}B$ given

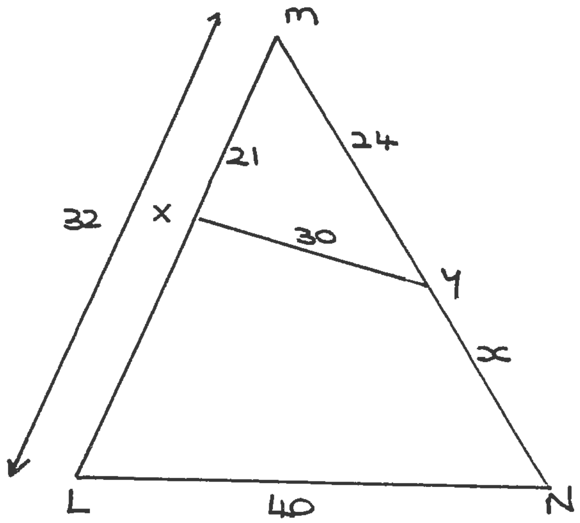
$\therefore \triangle ABE \equiv \triangle BAD$ RHS

$E\hat{A}B = D\hat{B}A$

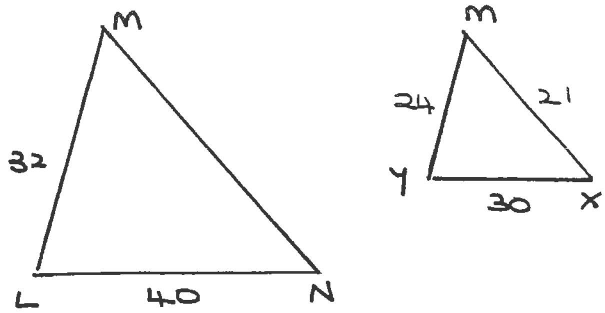
corresponding angles of congruent triangles are equal

$\therefore \triangle ABC$ is isosceles as it has 2 equal angles

d)



i)



In $\triangle LMN$ and $\triangle YMX$

M is common

$$\frac{LM}{MY} = \frac{32}{24} = \frac{4}{3}$$

$$\frac{LN}{YX} = \frac{40}{30} = \frac{4}{3}$$

$\therefore \triangle LMN \parallel\!\mid \triangle YMX$

two pairs of corresponding angles are in proportion and the included angles are equal.

(ii) $\frac{LN}{YX} = \frac{MN}{MX}$ corresponding sides of similar triangles are in proportion

$$\frac{40}{30} = \frac{24+x}{21}$$

$$30(24+x) = 40 \times 21$$

$$24 + x = \frac{40 \times 21}{30}$$

$$24 + x = 28$$

$$x = 4$$

$\therefore YN = 4$

Question 16

a)

(i) $y = 5x^4 - 7x^2 + 8$

$\frac{dy}{dx} = 20x^3 - 14x$

(ii) $y = \frac{3x^4 - 5x^2}{x}$

$= \frac{x(3x^3 - 5x)}{x}$

$= 3x^3 - 5x$

$\frac{dy}{dx} = 9x^2 - 5$

(iii) $y = \frac{1}{x\sqrt{x}}$

$= \frac{1}{x^{3/2}}$

$= x^{-3/2}$

$\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{5}{2}}$

$= \frac{-3}{2\sqrt{x^5}}$

$= \frac{-3}{2x^2\sqrt{x}}$

(iv) $y = \frac{6}{x^4}$

$= 6x^{-4}$

$\frac{dy}{dx} = -24x^{-5}$

$= \frac{-24}{x^5}$

v) $y = (3x+4)^5$

$\frac{dy}{dx} = 5(3x+4)^4 \times 3$

$= 15(3x+4)^4$

vi) $y = x(3-2x)^4$

$= u\ v$

where

$u = x \qquad v = (3-2x)^4$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = -8(3-2x)^3$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (3-2x)^4 - 8x(3-2x)^3$

$= (3-2x)^3[3-2x-8x]$

$= (3-2x)^3(3-10x)$

vii) $y = \frac{x^2+5}{x-2}$

$= \frac{u}{v}$

where

$u = x^2+5 \qquad v = x-2$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{2x(x-2) - (x^2+5)}{(x-2)^2}$

$= \frac{2x^2 - 4x - x^2 - 5}{(x-2)^2}$

$= \frac{x^2 - 4x - 5}{(x-2)^2}$

$= \frac{(x-5)(x+1)}{(x-2)^2}$

viii) $y = x\sqrt{1-x^2}$

$= u\ v$

where $u = x$ $\quad v = (1-x^2)^{\frac{1}{2}}$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times -2x$

$= -x(1-x^2)^{-\frac{1}{2}}$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (1-x^2)^{\frac{1}{2}} - x^2(1-x^2)^{-\frac{1}{2}}$

$= (1-x^2)^{-\frac{1}{2}}\left[1-x^2-x^2\right]$

$= (1-x^2)^{-\frac{1}{2}}(1-2x^2)$

$= \frac{1-2x^2}{\sqrt{1-x^2}}$

b) $f(x) = x^2+4x-12$ $\quad f'(x) = -6$

$f'(x) = 2x+4$

But $f'(x) = -6$

$\therefore 2x+4 = -6$

$2x = -10$

$x = -5$

when $x = -5$

$f(x) = x^2+4x-12$

$f(-5) = (-5)^2+4(-5)-12$

$= 25-20-12$

$= -7$

$\therefore$ P is the point $(-5,-7)$

c) $y = x^2-8x+7$

$\frac{dy}{dx} = 2x-8$ at $x=3$

$= 2(3)-8$

$= -2$

$\therefore$ gradient of tangent is -2

For perpendicular lines

$m_1 m_2 = -1$

$\therefore$ gradient of normal is $\frac{1}{2}$

$y - y_1 = m(x-x_1)$ $\quad (3,-8)$

$y+8 = \frac{1}{2}(x-3)$

$2y+16 = x-3$

$x-2y-19 = 0$