## Section I

## Multiple Choice 10 Marks

**Use the multiple choice answer sheet for Questions 1 – 10**

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**1.** The solution of the inequation $1-3x \le -x$ is

**A.** $x < \frac{1}{2}$ **B.** $x \ge -\frac{1}{2}$ **C.** $x \ge \frac{1}{2}$ **D.** $x < -\frac{1}{2}$

**2.** $\sqrt{x^5}$ can be written as

**A.** $x^{\frac{2}{5}}$ **B.** $x^{\frac{5}{2}}$ **C.** $\sqrt[5]{x^2}$ **D.** $x^{5.2}$

**3.** 0.000006053 in scientific notation is

**A.** $6.053 \times 10^{-6}$ **B.** $6053 \times 10^{4}$ **C.** $6.053 \times 10^{6}$ **D.** $6053 \times 10^{-6}$

**4.** Factorising $x^3 - 27y^3$ becomes

**A.** $(x-3y)(x^2+3xy+9y^2)$ **B.** $(x-3y)^3$

**B. C.** $(x^2-3)(x+9y^2)$ **D.** $(x^2-3y^2)(x+9y)$

**5.** $(3\sqrt{7}+2\sqrt{3})(3\sqrt{7}-2\sqrt{3})$ is equal to

**A.** 51 **B.** 75 **C.** $51+4\sqrt{3}$ **D.** 63

**6.** The gradient function of $f\left(x\right) = \sqrt{x}$ is

**A.** $\frac{1}{2}x$ **B.** $\frac{2}{\sqrt{x}}$ **C.** $\frac{\sqrt{x}}{2}$ **D.** $\frac{1}{2\sqrt{x}}$

**7.** $1 + \dfrac{2}{a+b} =$

**A.** $\frac{a+b+2}{a+b}$ **B.** $\frac{3}{a+b}$ **C.** 3 **D.** $\frac{2}{a+b}$

**8.** $\dfrac{2 - \tan 60^0}{\sec^2 45^0}$ in exact form is

**A.** $2\sqrt{3}$ **B.** $1 - \frac{\sqrt{3}}{2}$ **C.** $-\frac{1}{\sqrt{3}}$ **D.** $\frac{2-\sqrt{3}}{\sqrt{2}}$

**9.** The number of sides of a regular polygon with exterior angles of $30^0$ is

**A.** 6 **B.** 8 **C.** 10 **D.** 12

**10.** The equation of the line that passes through ( 2, -5 ) and is parallel to the line $2x - 3y + 1 = 0$ can be expressed as

**A.** $2x + 3y - 19 = 0$ **B.** $2x - 3y - 19 = 0$

**C.** $3x + 2y + 4 = 0$ **D.** $3x + 2y + 4 = 0$

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**Section II**

**Attempt Questions 11 to 16**

**Answer each question in separate writing booklets.**

**All necessary working should be shown in every question**

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**Question 11** ***14 Marks*** **Start a new booklet** **Marks**

(a) Simplify $2\sqrt{75} - 3\sqrt{48}$ **2**

(b) Solve $x^2 = 5x$ **2**

(c) Evaluate, correct to 3 significant figures **2**

$$\frac{6.2^5 - 5.2^4}{\sqrt{18} - 4 \times 6^{\frac{1}{3}}}$$

(d) Solve $|2x + 1| \geq 5$ and graph your solution on the number line **3**

(e) Simplify $\dfrac{2}{m^2 - 4} - \dfrac{3}{m + 2}$ **3**

(f) Solve simultaneously **2**

$$5p - t = -28$$
$$16p - 5t = -14$$

**Question 12** ***13 Marks*** **Start a new booklet** **Marks**

(a) Sketch the following, showing all essential features **8**

$(i)$ $2x - 4y - 1 = 0$

$(ii)$ $y = \sqrt{16 - x^2}$

$(iii)$ $y = -\dfrac{4}{1 + x}$

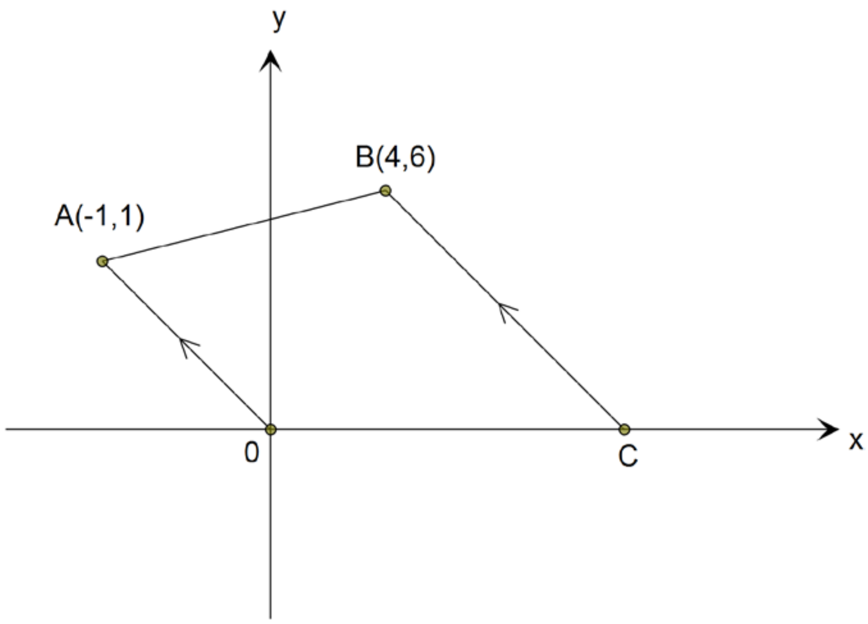
$(iv)$ $y = x^3 + 8$

(b) Shade the region in the Cartesian plane for which **3**

$y < x - 2$ , $y \geq 0$ *and* $x \geq 6$ hold simultaneously.

(c) Find the centre and radius of the circle **2**
$x^2 - 2x + y^2 + 10y + 22 = 0$ .

**Question 13** ***13 Marks*** **Start a new booklet** **Marks**




In the diagram, OABC is a trapezium with OA || CB. The coordinates of O, A and B are (0 , 0) , (–1 , 1) and (4 , 6) respectively.

(i) Calculate the length of OA. **2**

(ii) Write down the gradient of the line OA. **1**

(iii) What is the size of $\angle AOC$ ? **1**

(iv) Find the equation of the line BC, and hence find the coordinates of C. **3**

(v) Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$. **2**

(vi) Hence or otherwise, calculate the area of the trapezium OABC. **2**

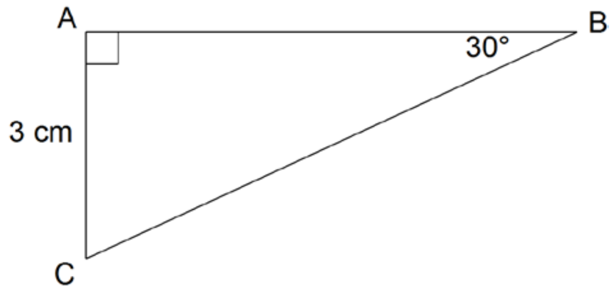
(vii) Find the equation of the line that passes through O and is perpendicular to the line BC. **2**

**Question 14** **16 *Marks*** **Start a new booklet** **Marks**

(a) Find $\cos\theta$ if $\text{cosec}\,\theta = 4$ *for* $90^0 \le \theta \le 180^0$ **2**

(b) Sketch $y = \sin x$ in the domain $0^0 \le x \le 360^0$. Show all essential features. **1**

(c) Find the exact length of AB. **2**




(d) Solve each equation for $0^0 \le \theta \le 360^0$. Correct answers to the nearest minute.

(i) $\cos\theta = -\dfrac{3}{4}$ **2**

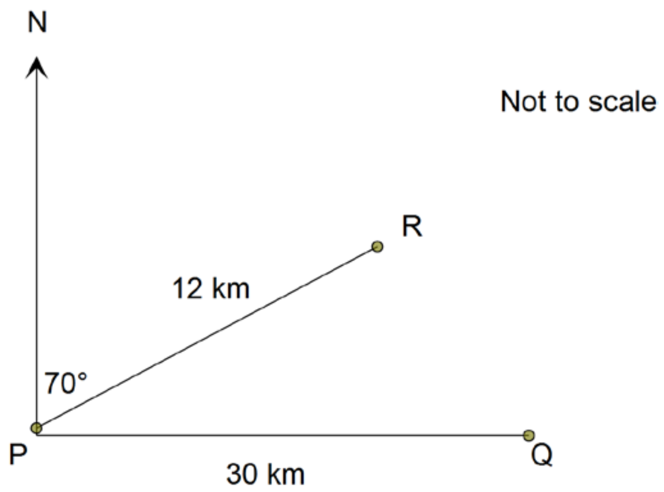
(ii) $\cot\theta = 4$ **2**

(e) Find the exact value of $\sin 225^0$ **1**

*Question 14 continues on next page.*

**Question 14 continued** **Marks**

(f)




The diagram shows a point P which is 30 km due west of the point Q. The point R is 12 km from P and has a bearing from P of $070^0$.

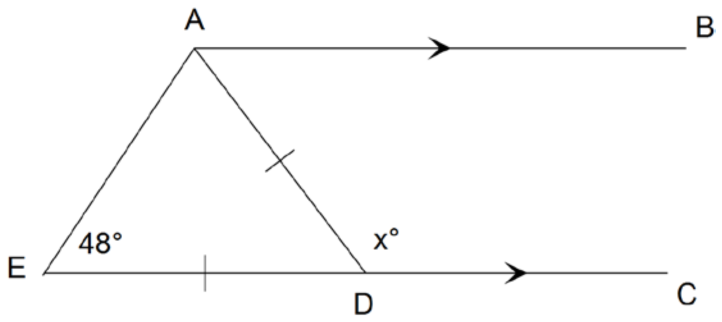
(i) Find the distance of R from Q , correct to 1 decimal place. **2**

(ii) Find the bearing of R from Q, correcting your answer to the nearest degree. **2**

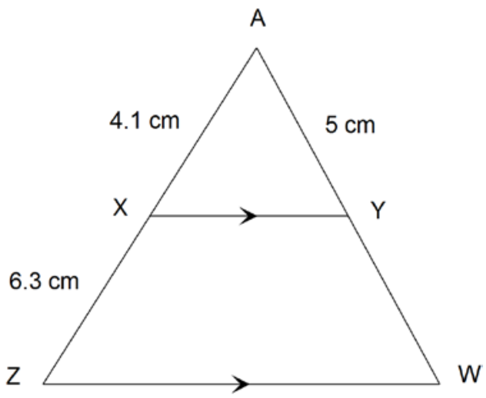
(g) Prove $\sec\theta + \tan\theta = \dfrac{1 + \sin\theta}{\cos\theta}$ **2**

**Question 15** ***13 Marks*** **Start a new booklet** **Marks**

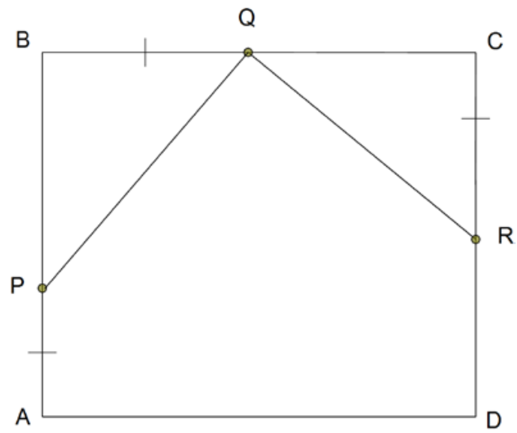
(a) Find the value of $x$, giving reasons. **2**




(b) Find the length of YW, correct to 1 decimal place. **2**




(c)




Not to scale

In the diagram, ABCD is a square. The points P, Q and R lie on AB, BC and CD respectively, such that AP = BQ = CR.

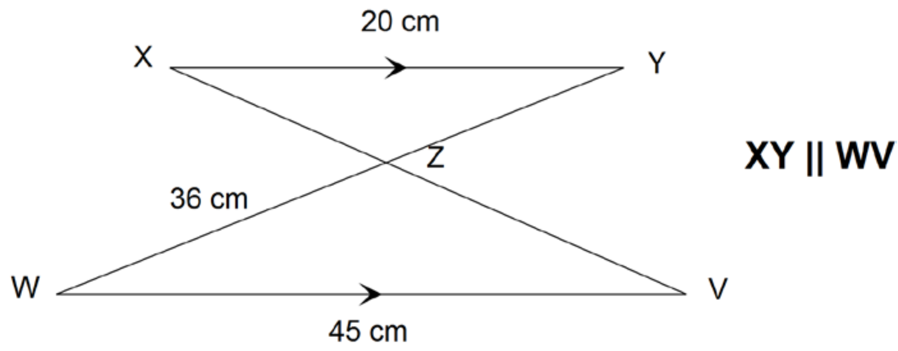
(i) Prove the triangles PBQ and QCR are congruent **3**

(ii) Prove $\angle PQR$ is a right angle **2**

*Question 15 continues on next page.*

**Question 15 continued** **Marks**

(d) Given the following diagram,




(i) Prove $\Delta XYZ \;|||\; \Delta VWZ$ **3**

(ii) Hence find the length of $ZY$ **1**

**QUESTION 16** ***21 Marks*** **Start a new booklet** **Marks**

(a) Differentiate the following with respect to $x$

(i) $y = 3 - 2x^2 - 7x^4$ **2**

(ii) $y = 5\sqrt{x} - \dfrac{x^2}{2}$ **2**

(iii) $y = (1 - 2x)(1 + 3x)$ **2**

(iv) $y = \dfrac{4x - 1}{1 - x}$ **2**

(v) $y = \left(8x^2 + 4x\right)^3$ **2**

(vi) $y = 2x\sqrt[3]{x - 5}$ *leave your answer without a fractional or negative index* **2**

(vii) $y = (3x-1)(2-3x)^4$ **2**

*Question 16 continues on next page.*

**QUESTION 16 continued** **Marks**

(b) Differentiate $f\left(x\right) = x^2 - 2x$ from 1st principles and find the gradient of the tangent to the curve at $x = 1$. **3**

(c) Differentiate $y = x^2 + bx + c$ and hence find $b$ and $c$ given that the line $3x + y - 5 = 0$ is a normal to the curve at the point ( 3 , -1 ) **3**

# END OF PAPER

YEAR 11 MATHEMATICS PRELIMINARY FINAL EXAMINATION

SOLUTIONS 2013

## SECTION I (10 MARKS)

1. $x \geq \frac{1}{2}$ C

2. $x^{\frac{5}{2}}$ B

3. $6.053 \times 10^{-6}$ A

4. $(x-3y)(x^2+3xy+9y^2$ A

5. 51 A

6. $\frac{1}{2\sqrt{x}}$ D

7. $\frac{a+b+2}{a+b}$ A

8. $1 - \frac{\sqrt{3}}{2}$ B

9. $n = 12$ D

10. $2x - 3y - 19 = 0$ B.

## SECTION II

### QUESTION 11. (14 MARKS)

(a) $2\sqrt{25}\sqrt{3} - 3\sqrt{16}\sqrt{3}$ 1

$= 10\sqrt{3} - 12\sqrt{3}$

$= -2\sqrt{3}$ 1

(b) $x^2 - 5x = 0$

$x(x-5) = 0$

$\therefore x = 0$ or $x = 5$

1 1

(c) $-2786.056774\ldots$ 1

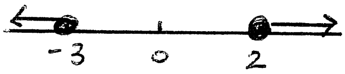
$= -2790$ 1

(d) $2x+1 \geq 5$ or $-(2x+1) \geq 5$

$2x \geq 4$ $-2x-1 \geq 5$

$x \geq 2$ 1 $-2x \geq 6$

$x \leq -3$ 1



1

(e) $\frac{2}{(m+2)(m-2)} - \frac{3(m-2)}{(m+2)(m-2)}$ 1

$= \frac{2 - 3m + 6}{(m+2)(m-2)}$ 1

$= \frac{8-3m}{(m+2)(m-2)}$ 1

(f) $t = 5p + 28$ — ①

$16p - 5t = -14$ — ②

Sub ① into ②

$16p - 5(5p+28) = -14$

$16p - 25p - 140 = -14$

$-9p = 126$

$p = -14$ 1

$\rightarrow t = 5 \times -14 + 28$

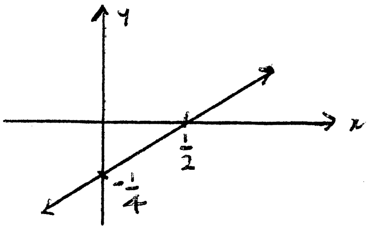
$= -42$

1

## QUESTION 12 (13 MARKS)

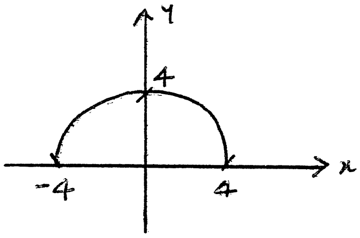
(a)

(i)



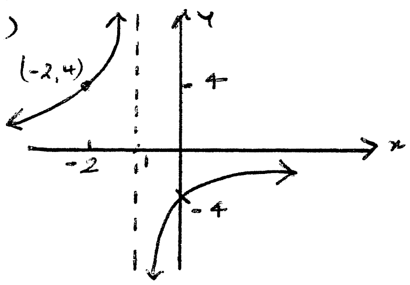

2

(ii)



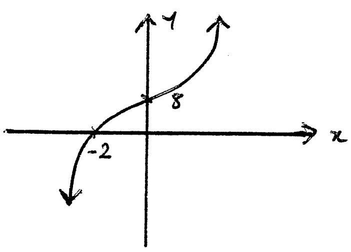

2

(iii)



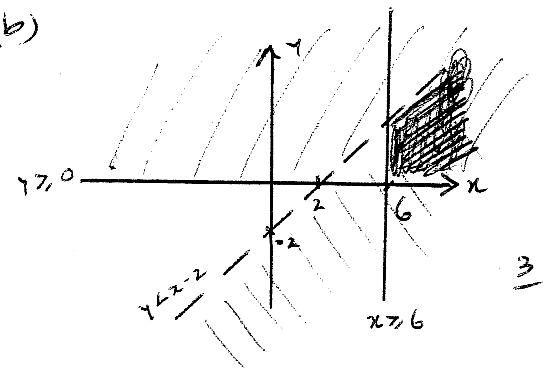

2

(iv)




2

(b)




3

(c) $x^2 - 2x + y^2 + 10y = -22$

$x^2 - 2x + 1 + y^2 + 10y + 25 = -22 + 1 + 25$

$(x-1)^2 + (y+5)^2 = 4$ 1

Centre $(1,-5)$ Radius $= 2$ 1

## QUESTION 13 (13 MARKS)

(i) $d_{OA} = \sqrt{1^2+1^2}$ 1

$= \sqrt{2}$ 1

$= 1.41$ (2 dp)

(ii) $m = -1$ 1

(iii) $\tan\theta = -1$

$\theta = 135°$ 1

$\therefore \angle AOC = 135°$

(iv) $m = -1$ $(4,6)$

$y - 6 = -1(x-4)$

$y - 6 = -x + 4$ 2

$y = -x + 10$

Let $y = 0$ $x = 10$

$\therefore$ coords of C are $(10,0)$ 1

(v) $x + y - 10 = 0$ $(0,0)$

$d_\perp = \frac{|1\times 0 + 1\times 0 - 10|}{\sqrt{1^2+1^2}}$ 1

$= \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ units 1

(vi) $A = \frac{1}{2} \times 5\sqrt{2}\,(\sqrt{2} + 6\sqrt{2})$

$= \frac{1}{2} \times 5\sqrt{2} \times 7\sqrt{2}$  $BC = \sqrt{6^2+6^2}$

$= \frac{1}{2} \times 35 \times 2$ 1  $= \sqrt{72}$ $= 6\sqrt{2}$

$= 35$ sq. units 1

(vii) grad $BC = -\frac{6}{6} = -1$

$\therefore$ Grad of line $\perp BC$ is 1 1

Eqn through $(0,0)$ grad 1

$y - 0 = 1(x - 0)$

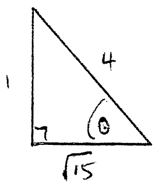
$y = x$ 1

## QUESTION 14 (16 MARKS)

(a) $\operatorname{cosec}\theta = 4$

$\frac{1}{\sin\theta} = 4$

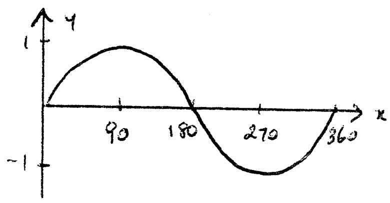
$\sin\theta = \frac{1}{4}$ 1




$\therefore \cos\theta = -\frac{\sqrt{15}}{4}$ 1

(2nd quad)

(b)




2

(c) $\tan 30^\circ = \frac{3}{AB}$ 1

$AB = 3 \div \tan 30^\circ$

$= 3 \div \frac{1}{\sqrt{3}}$

$= 3\sqrt{3}$ cm 1

(d) (i) $\cos\theta = -\frac{3}{4}$

Basic angle: $\theta = 41^\circ 25'$

2nd Quad: $138^\circ 35'$ 1

3rd Quad: $221^\circ 25'$ 1

(ii) $\frac{1}{\tan\theta} = 4$

$\tan\theta = \frac{1}{4}$

1ST QUAD: $\theta = 14^\circ 2'$ 1

3RD QUAD: $\theta = 194^\circ 2'$ 1

(e) $\sin 225^\circ = \sin(180 + 45^\circ)$

$= -\sin 45^\circ$

$= -\frac{1}{\sqrt{2}}$

$= -\frac{\sqrt{2}}{2}$ 1

(f) (i) $RQ^2 = 12^2 + 30^2 - 2.12.30.\cos 20^\circ$

$RQ = \sqrt{12^2 + 30^2 - 2.12.30.\cos 20^\circ}$

$= 19.168$

$\doteqdot 19.2$ cm 1

(ii) $\frac{\sin\theta}{12} = \frac{\sin 20^\circ}{19.2}$

$\sin\theta = \frac{12 \sin 20^\circ}{19.2}$ 1

$= 0.213$

$\theta = 12^\circ 21'$

Bearing is $270^\circ + 12^\circ 21'$

$= 282^\circ 21'$ 1

(OR $\frac{\sin\theta}{12} = \frac{\sin 20}{RQ}$

$\sin\theta = 0.21$

$\theta = 12^\circ 22'$ $\therefore$ Bearing $282^\circ 22'$)

(g) LHS $= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$ 1

$= \frac{1 + \sin\theta}{\cos\theta}$ 1

$=$ RHS.

QUESTION 15 (13 MARKS)

(a) $\angle ADE = 48^\circ$ (base $\angle$'s isos $\triangle$ =) 1

$\therefore x = 132^\circ$ (suppl. $\angle EDC$) 1

(b) $\frac{YW}{5} = \frac{6.3}{4.1}$ 1

$YW = \frac{5 \times 6.3}{4.1}$

$= 7.6829...$

$\doteqdot 7.7$ cm 1

(c) (i) $\angle PBQ = \angle RCQ$ (right $\angle$'s in square)

$BQ = CR$ (given) 1

$BP = QC$ (since $AB = BC$, $BQ = PA$) 1

$\therefore \triangle PBQ \equiv \triangle QCR$ (SAS) 1

(ii) $\angle BPQ + \angle BQP = 90^\circ$ ($\angle$ sum $\triangle = 180^\circ$)

$\angle RQC + \angle CQR = 90^\circ$ ($\angle$ sum $\triangle = 180^\circ$) 1

$\angle BQP + \angle CQR + \angle PQR = 180^\circ$

$90^\circ + \angle PQR = 180^\circ$

$\therefore \angle PQR = 90^\circ$ 1

(d) (i) $\angle YXZ = \angle WVZ$ (alt $\angle$'s; $XY \parallel WV$)

$\angle ZWV = \angle XYZ$ (alt $\angle$'s; $XY \parallel WV$) 1

$\angle XZY = \angle VZW$ (vert. opp $\angle$'s) 1

$\therefore \triangle XYZ \parallel\!| \triangle VWZ$ (equiangular) 1

(ii) $\frac{ZY}{36} = \frac{20}{45}$

$ZY = \frac{20}{45} \times 36$

$= 16$ cm 1

QUESTION 16 (21 MARKS)

(a) (i) $y' = -4x - 28x^3$ 1

(ii) $y = 5x^{\frac{1}{2}} - \frac{x^2}{2}$

$y' = \frac{5}{2}x^{-\frac{1}{2}} - x$ 1

$= \frac{5}{2\sqrt{x}} - x$ 1

(iii) $y' = (1-2x).3 + (1+3x).-2$ 1

$= 3 - 6x - 2 - 6x$

$= 1 - 12x$ 1

(iv) $y' = \frac{(1-x).4 - (4x-1).-1}{(1-x)^2}$ 1

$= \frac{4 - 4x + 4x - 1}{(1-x)^2}$

$= \frac{3}{(1-x)^2}$ 1

(v) $y' = 3(8x^2+4x)^2(16x+4)$

$= 3(16x+4)(8x^2+4x)^2$ 2

(vi) $y = 2x(x-5)^{\frac{1}{3}}$

$y' = 2x\left[\frac{1}{3}(x-5)^{-\frac{2}{3}}.1\right] + (x-5)^{\frac{1}{3}}.2$ 1

$= \frac{2x}{3}(x-5)^{-2/3} + 2(x-5)^{\frac{1}{3}}$

$= 2(x-5)^{\frac{1}{3}}\left(\frac{x(x-5)^{-1}}{3} + 1\right)$ 1

$= 2\sqrt[3]{x-5}\left(\frac{x}{3(x-5)} + 1\right)$ 1

(accept $y' = \frac{2x}{3\sqrt[3]{(x-5)^2}} + 2\sqrt[3]{x-5}$)

(vii) $y' = (3x-1)\left[4(2-3x)^3 \cdot -3\right] + (2-3x)^4 \cdot 3$

$= -12(3x-1)(2-3x)^3 + 3(2-3x)^4$ 1

$= -3(2-3x)^3\left(4(3x-1) + 2-3x\right)$

$= -3(2-3x)^3(12x-4+2-3x)$

$= -3(2-3x)^3(9x-2)$ 1

(b) $f(x) = x^2 - 2x$

$f(x+h) = (x+h)^2 - 2(x+h)$

$= x^2 + 2xh + h^2 - 2x - 2h$

$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - (x^2 - 2x)}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - 2h}{h}$

$= \lim_{h \to 0} 2x + h - 2$

$= 2x - 2$ 2

Gradient at $x = 1$ is $y' = 2 \times 1 - 2$
$= 0$.

(c) $y = x^2 + bx + c$

$\frac{dy}{dx} = 2x + b$ 1

Line $y = -3x + 5$ has grad $-3$

∴ Grad of normal is $\frac{1}{3}$

use $(3,-1)$ and $\frac{dy}{dx} = \frac{1}{3}$

(sub into $y' = 2x + b$) $\frac{1}{3} = 2 \times 3 + b$

$b = -5\frac{2}{3}$ 1

Sub into original $-1 = 3^2 - 5\frac{2}{3} \times 3 + c$

∴ $c = 7$ 1