NORTH SYDNEY BOYS HIGH SCHOOL

2009

Preliminary Examination

Mathematics

Teacher: Mr Berry

Instructions

- Working time – **2 hours 30 minutes**
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new **Question** is to be started on a **new page.**
- Attempt all questions

Student Name:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	Total
Mark	— 21	— 20	— 12	— 12	— 15	— 13	— 12	— 105

Question 1 **(21 Marks)** Use a Separate Sheet of paper **Marks**

a) Find the value of $\frac{1}{7.38}+\frac{1}{9.85}$, correct to 3 significant figures. **2**

b) Express the decimal $0.\dot{4}\dot{8}$ as a fraction in simplest form. **2**

c) If $\sqrt{56}+\sqrt{14}=\sqrt{A}$, find A. **2**

d) Express $\frac{4\sqrt{3}+3}{3\sqrt{3}-1}$ with a rational denominator. Simplify your answer. **2**

e) Factorise the following expressions fully:

i) $x^2-5x-14$ **1**

ii) $ax+ba+by+xy$ **2**

f) Simplify: $(x-1)^2-(x-2)^2$ **2**

g) Simplify $\frac{10x-15}{6}\times\frac{1}{8x-12}$ as a single fraction in simplest form. **2**

h) Solve:

i) $6(y-1)=3(y+8)$ **2**

ii) $\frac{a+2}{3}=\frac{a}{2}-2$ **2**

i) Solve for x: $|2x+2|<8$. **2**

End of Question 1.

Question 2 **(20 Marks)** Use a Separate Sheet of paper **Marks**

a) If $f\left(x\right)=2x^2-x$ is this an odd function, even function or neither? **1**

b) A function is defined by the rule $g(x)=\begin{cases} x+1, & \text{if } x\geq 1 \\ -1, & \text{if } -2<x<1 \\ 1-x, & \text{if } x\leq -2 \end{cases}$ **4**

Find if they exist,

i) $g(1)$

ii) $g(-1)$

iii) $g(0)$

iv) $g(2)+g(-2)$

c) Sketch the graphs of the following, showing the x and y **intercepts**, stating the **domain** and **range** of each.

i) $y=2^x$ **4**

ii) $x^2+(y+3)^2=36$ **4**

iii) $0=3x-y-5$ **4**

d) Show the region of the number plane where the following hold simultaneously: **3**

$y\leq x+1$

and $xy>4$

End of Question 2.

Marks

Question 3 **(12 Marks)** Use a Separate Sheet of paper

a) Find the derivative of the following: (You do not need to simplify your answers after finding the derivative.)

i) $2x^7 - 3x^5 + 5x^3 - 17$ **2**

ii) $\dfrac{1}{\sqrt[3]{x^4}}$ **2**

iii) $(x-2)(6x+7)$ **2**

iv) $\dfrac{2x^2+1}{5-3x^2}$ **2**

b) Find $g'(-1)$ for $g(x) = \left(-x^4+3\right)^5$. **2**

c) Given $f(x) = (x+1)\sqrt{x}$ find $f'(x)$. **2**

End of Question 3.

Marks

Question 4 **(12 Marks)** Use a Separate Sheet of paper

The points A(2,0), B(8,4) ,C(4,6) and D(x_1, y_1) form the 4 vertices of a parallelogram.

a) Draw a number plane and mark A, B & C on it. **1**

b) Find the gradient of line AB **1**

c) Show that the equation of the line l parallel to AB and going through C is $2x-3y+10=0$ **2**

d) If the equation of the line k through A parallel to BC is $x+2y-2=0$. Find the point $D(x_1, y_1)$ the intersection of the lines l and k. Mark this point on your diagram. **2**

e) Find the angle θ to the nearest degree that the line AB makes with the positive x-axis **2**

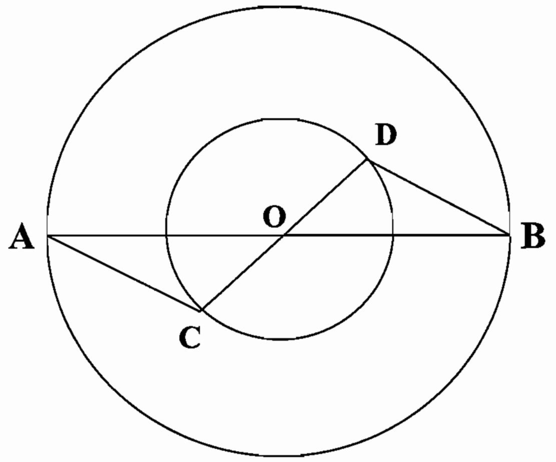
f) Find the perpendicular distance between the line l and A. **2**

g) Find the exact area of $ABCD$ **2**

End of Question 4.

Question 5 **(15 Marks)** Use a Separate Sheet of paper **Marks**

a) In the diagram below O is the centre of the two circles. AB is the diameter of the larger circle and CD is the diameter of the smaller circle.



i) Prove that $\Delta AOC \equiv \Delta ODB$ **3**

Hence, or otherwise, prove:

ii) $AC = DB$ **1**

ii) $AC \parallel DB$ **1**

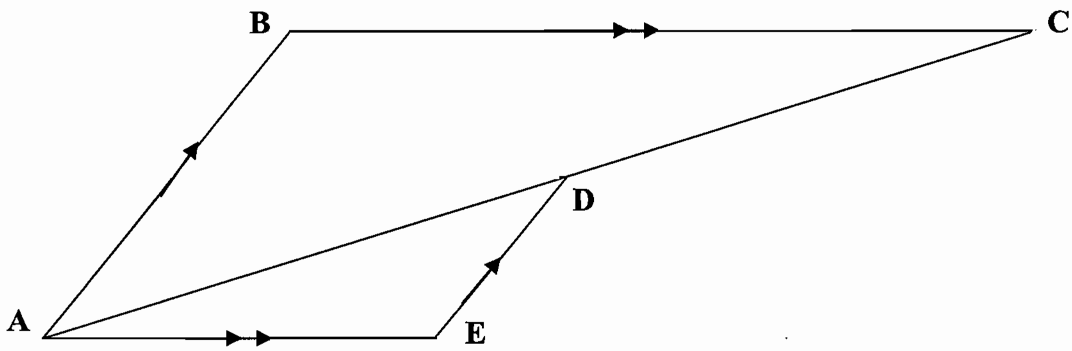
b) The sum of the interior angles of a regular polygon is $2700°$

i) How many sides has the polygon? **1**
ii) Find the size of each interior angle to the nearest minute. **1**
iii) Hence find the size of each exterior angle. **1**

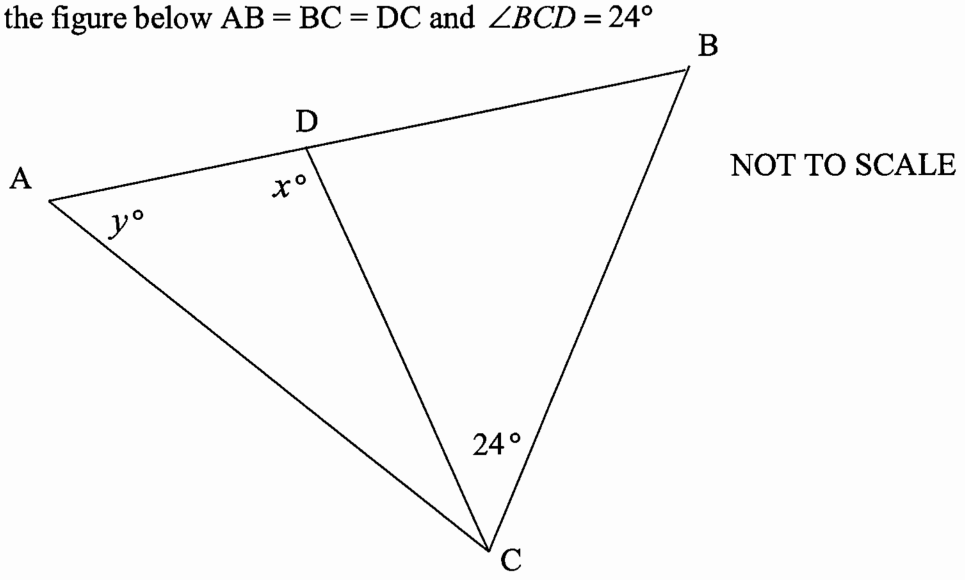
Question 5 continues on page 7

Marks

c) Prove that ΔABC is similar to ΔDEA **3**



d) In the figure below AB = BC = DC and $\angle BCD = 24°$ **4**



Find the values of x and y, giving reasons for each step.

End of Question 5.

Question 6 **(13 Marks)** Use a Separate Sheet of paper **Marks**

a) From a point 5m above the ground, the angle of depression of the bottom of a wall is $21°$ and the angle of elevation of the top of the wall is $32°$.

i) Draw a diagram to represent this information. **1**

ii) Find the distance from the point of observation to the bottom of the wall. (correct to 2 decimal places) **2**

iii) Using your answer from part (i) and the Sine Rule. Find the height of the wall. (correct to 2 decimal places) **2**

b) Zoe and Kobi set out on a bike ride from point P at the same time. One travels at 20km/h along a straight road in the direction $032°$T. The other travels at 25km/h along another straight road in the direction $132°$T.

ii) Draw a diagram to represent this information. **1**

iii) Find the distance Zoe and Kobi are apart to the nearest kilometre after 3 hours. **2**

c) Find the exact value of $\sec(60°)$. **1**

d) Solve $\sin\theta = \dfrac{-1}{\sqrt{2}}$ *for* $0^0 \le \theta \le 360^0$. **2**

e) Prove $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$. **2**

End of Question 6.

Question 7 **(12 Marks)** *Start a new page* **Marks**

(a) Solve for x: $(x + 1)^2 = 6$, leaving your answer in exact form. **2**

(b) Simplify $(4-\sqrt{3})^3-(4+\sqrt{3})^3$ **3**

(c) Find the gradient of the curve $y = 2x^3 - 4x^2$ at the point (1, -2) and hence find the equation of the normal to this curve at the point (1, -2). **3**

(d) (i) Show that: $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$ **1**

(ii) Hence or otherwise, solve: **3**

$$\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta+\cot\theta} = -1,\ 0^\circ \le \theta \le 360^\circ$$

End of Examination.

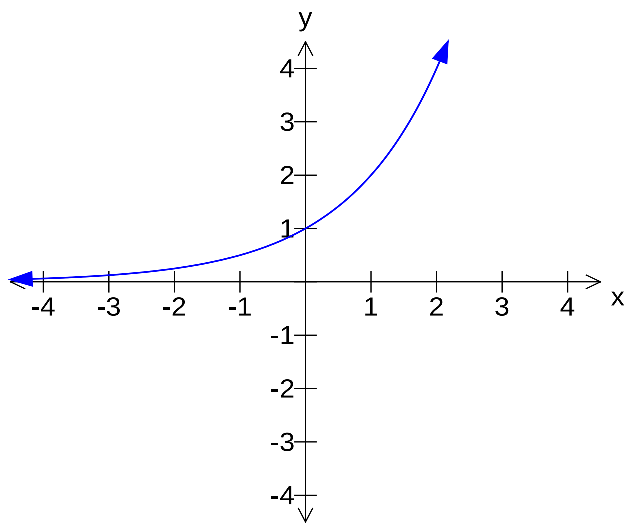
QUESTION 1

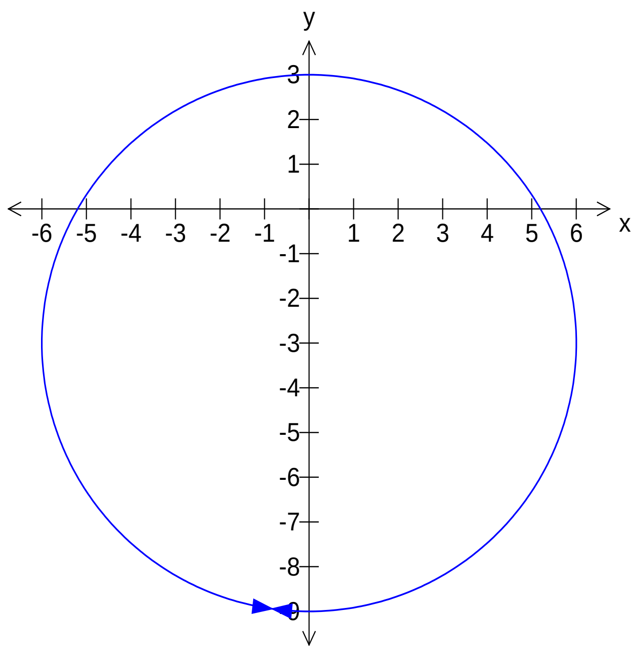
Solution	Marking Scheme
a) $\frac{1}{7.38} + \frac{1}{9.85} = 0.237\ (3\ s.f.)$ b) $x = 0.\overline{48}$ $100x = 48.\overline{48}$ $100x - x = 48.\overline{48} - 0.\overline{48}$ $99x = 48$ $x = \frac{48}{99}$ $x = \frac{16}{33}$ c) $\sqrt{56} + \sqrt{14} = \sqrt{4 \times 14} + \sqrt{14}$ $= 2\sqrt{14} + \sqrt{14}$ $= 3\sqrt{14}$ $= \sqrt{9 \times 14}$ $= \sqrt{126}$ $\therefore A = 126$ d) $\frac{4\sqrt{3} + 3}{3\sqrt{3} - 1} \times \frac{3\sqrt{3} + 1}{3\sqrt{3} + 1}$ $= \frac{(4\sqrt{3} + 3)(3\sqrt{3} + 1)}{(3\sqrt{3})^2 - 1^2}$ $= \frac{12 \times 3 + 4\sqrt{3} + 9\sqrt{3} + 3}{27 - 1}$ $= \frac{39 + 13\sqrt{3}}{26}$ $= \frac{13(3 + \sqrt{3})}{2 \times 13}$ $= \frac{3 + \sqrt{3}}{2}$	

e) i) $x^2 - 5x - 14 = (x-7)(x+2)$ ii) $ax + ba + by + xy = a(x+b) + y(x+b)$ $= (a+y)(x+b)$ f) $(x-1)^2 - (x-2)^2$ $= ((x-1) - (x-2))((x-1) + (x-2))$ $= (1)((2x-3)$ $= 2x-3$ g) $\frac{10x-15}{6} \times \frac{1}{8x-12} = \frac{10x-15}{6(8x-12)}$ $= \frac{5(2x-3)}{24(2x-3)}$ $= \frac{5}{24}$ h) i) $6(y-1) = 3(y+8)$ $6y - 6 = 3y + 24$ $6y - 3y = 24 + 6$ $3y = 30$ $\therefore \quad y = 10$ ii) $\frac{a+2}{3} = \frac{a}{2} - 2$ $\frac{a+2}{3} = \frac{a-4}{2}$ $2(a+2) = 3(a-4)$ $2a + 4 = 3a - 12$ $3a - 2a = 4 + 12$ $\therefore \quad a = 16$	

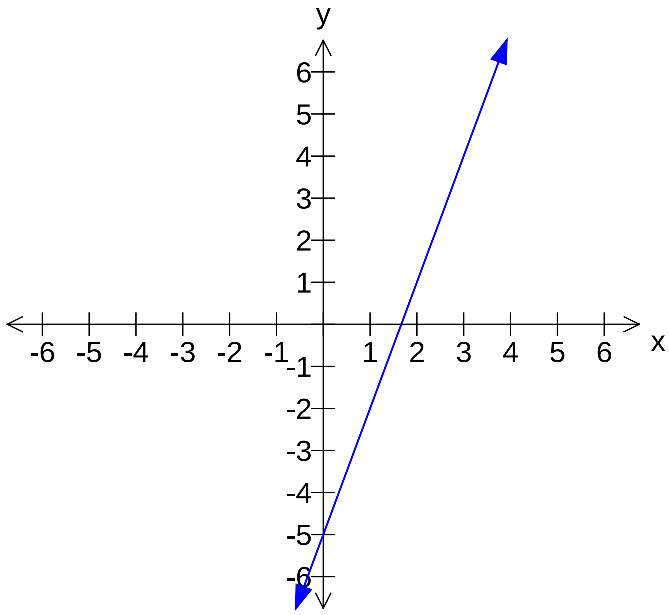
i) $\|2x + 2\| < 8$ <u>Case 1:</u> $(2x + 2) < 8$ $2x < 6$ $x < 3$ <u>Case 2:</u> $-(2x + 2) < 8$ $2x + 2 > -8$ $2x > -10$ $x > -5$	

QUESTION 2

Solution	Marking Scheme
a) $f(x) = 2x^2 - x$ $f(-x) = 2(-x)^2 - (-x)$ $= 2x^2 + x$ $\neq \pm f(x)$ $\therefore$ *neither even or odd* b) i) $g(1) = 1 + 1$ $= 2$ ii) $g(-1) = -1$ iii) $g(0) = -1$ iv) $g(2) + g(-2) = (2 + 1) + (1 - (-2))$ $= 3 + 3$ $= 6$ c) 	

Domain: All real *x* Range: $y > 0$ ii)  Domain: $-6 \le x \le 6$ Range : $-9 \le y \le 3$	

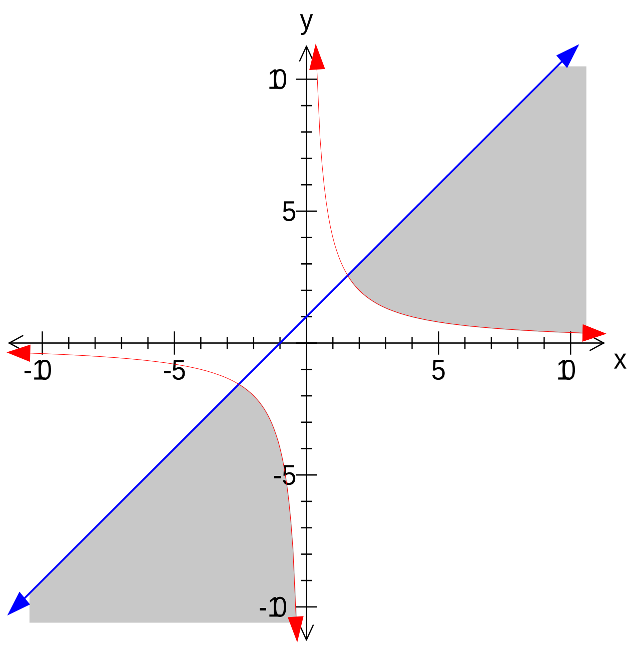
iii)



Domain: All real *x*

Range: All real *y*

d)



QUESTION 3

Solutions	Marking Scheme

a)

i)

$$\frac{d}{dx}\, 2x^7 - 3x^5 + 5x^3 - 17$$

$$= 14x^6 - 15x^4 + 15x^2$$

ii)

$$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x^4}}\right) = \frac{d}{dx}\left(x^{-\frac{4}{3}}\right)$$

$$= -\frac{4}{3}x^{-\frac{7}{3}}$$

$$= -\frac{4}{3\left(\sqrt[3]{x^7}\right)}$$

iii)

$$\frac{d}{dx}\,((x-2)(6x+7))$$

$$= \left(\frac{d}{dx}(x-2)\right)(6x+7) + \left(\frac{d}{dx}(6x+7)\right)(x-2)$$

$$= (1)(6x+7) + (6)(x-2)$$

$$= 6x + 7 + 6x - 12$$

$$= 12x - 5$$

iv)

$$\frac{d}{dx}\left(\frac{2x^2+1}{5-3x^2}\right)$$

$$= \frac{\left(\frac{d}{dx}\, 2x^2+1\right)(5-3x^2) - \left(\frac{d}{dx}\, 5-3x^2\right)(2x^2+1)}{(5-3x^2)^2}$$

$$= \frac{(4x)(5-3x^2) - (-6x)(2x^2+1)}{(5-3x^2)^2}$$

$$= \frac{20x - 12x^3 + 12x^3 + 6x}{(5-3x^2)^2}$$

$$= \frac{26x}{(5-3x^2)^2}$$

b)

$$g(x) = (-x^4 + 3)^5$$

$$\therefore \ g'(x) = 5 \times (-x + 3)^4 \times -4x^3$$

$$= -20x^3(-x^4 + 3)^4$$

$$\therefore g'(-1) = -20(-1)^3(-(-1)^4 + 3)^4$$

$$= -20(4)^4$$

$$= -20 \times 64$$

$$= -1280$$

c)

$$f(x) = (x + 1)\sqrt{x}$$

$$= x\sqrt{x} + \sqrt{x}$$

$$= x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

$$f'(x) = \frac{d}{dx}x^{\frac{3}{2}} + \frac{d}{dx}x^{\frac{1}{2}}$$

$$= \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}$$

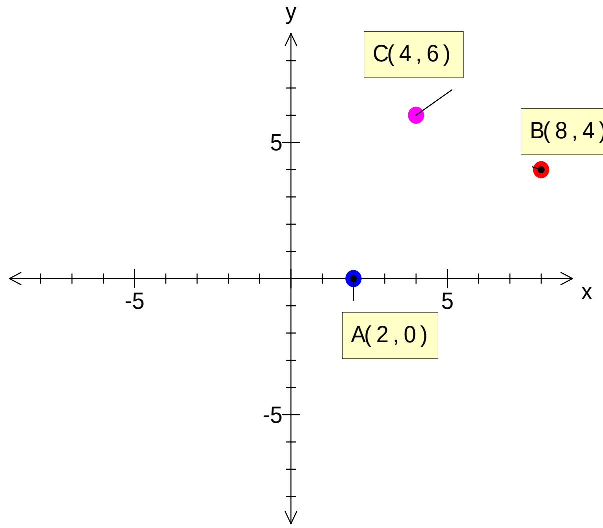
$$= \frac{3\left(\sqrt{x}\right)^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}$$

$$= \frac{3x + 1}{2\sqrt{x}}$$

$$= \frac{3x + 1}{2\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{\sqrt{x}\,(3x + 2)}{2x}$$

QUESTION 4

Solutions	Marking Scheme
a) 	
b) $m_{AB} = \frac{y_B - y_A}{x_B - x_A}$ $= \frac{4-0}{8-2}$ $= \frac{4}{6}$ $= \frac{2}{3}$	
c) $y - y_1 = m(x - x_1)$ $y - 6 = \frac{2}{3}(x-4)$ $3y - 18 = 2x - 8$ $3y - 2x - 18 + 8 = 0$ $\therefore \quad 2x - 3y + 10 = 0$ **as required**	

d)

$$2x - 3y + 10 = 0 \quad ①$$
$$x + 2y - 2 = 0 \quad ②$$

$2 \times ②$

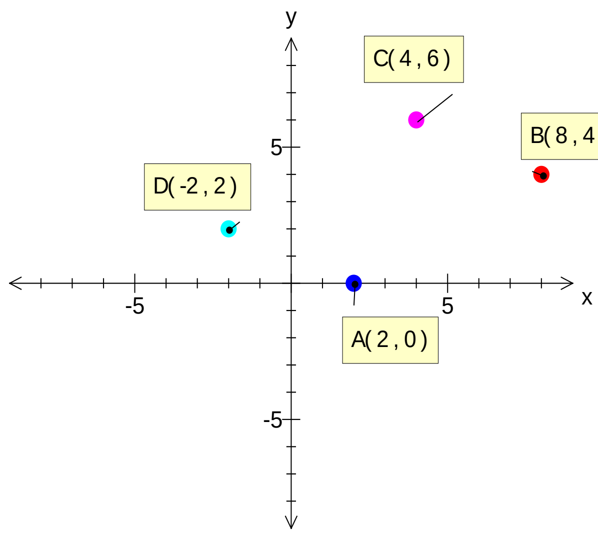
$$2x + 4y - 4 = 0 \quad ③$$

$① - ③$

$$-7y + 14 = 0$$
$$7y = 14$$
$$\therefore \quad y = 2$$

substitite into ②

$$x + (2 \times 2) - 2 = 0$$
$$x + 2 = 0$$
$$x = -2$$
$$\therefore \quad D = (-2, 2)$$



e)

$$\tan\theta = \frac{rise}{run}$$
$$= \frac{4 - 0}{8 - 2}$$
$$\therefore \quad \theta = \tan^{-1}\left(\frac{4}{6}\right)$$
$$= 34^{\circ} \text{ (to the nearest degree)}$$

f) $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}}$ $= \frac{\lvert (2)(2) + (-3)(0) + (10) \rvert}{\sqrt{2^2 + (-3)^2}}$ $= \frac{\lvert 4 + 10 \rvert}{\sqrt{13}}$ $= \frac{14}{\sqrt{13}}$ *units* g) $D_{AB} = \sqrt{((8 - 2)^2 + (4 - 0)^2}$ $= \sqrt{52}$ $= \sqrt{4 \times 13}$ $= 2\sqrt{13}$ $A = bh$ $= 2\sqrt{13} \times \frac{14}{\sqrt{13}}$ $= 28$ $units^2$	

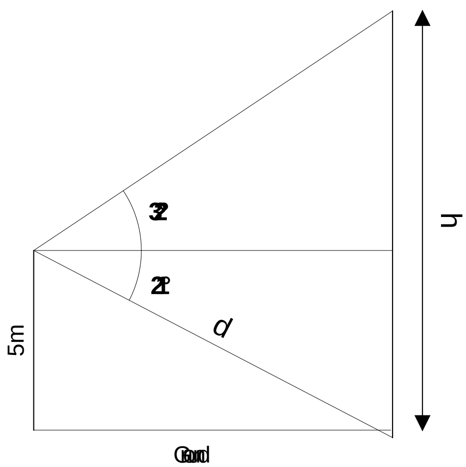
QUESTION 5

Solutions	Marking Scheme
a) i) *In* Δ*s AOC & ODB* $AO = BO$ *(radii of large circle)* $CO = DO$ *(radii of small circle)* $\angle AOC = \angle BOD$ *(vertically opposite)* $\therefore \Delta AOC = \Delta ODB$ (*SAS*) ii) AC = DB (matching sides in congruent triangles AOC and ODB) iii) $\angle ACO = \angle BDO$ *(matching angles in congrunt trianges)* *AC* ‖ *DB (alternate angles ACO & BDO are equal)* b) i) $(n - 2) \times 180^\circ = 2700$ $n - 2 = 15$ $n = 17$ $\therefore$ 17 *sides* ii) $\angle = 2700^\circ \div 17$ $= 158^\circ 49^\circ$ *(nearest minute)* iii) $\angle = 180^\circ - 158^\circ 49^\circ$ $= 21^\circ 11^\circ$ *(nearest minute)* c) *In* Δ*s ABC & DEA* $\angle DAE = \angle ACB$ *(alternate angles, AE* ‖ *BC)* $\angle BAC = \angle ADE$ *(alternate angles, AB* ‖ *DE)* $\therefore \Delta ABC \parallel\!| \Delta DEA$ *(equi-angular)*	

d) $2\angle CDE + 24^\circ = 180^\circ$ *(angle sum of ΔBCD)* $2\angle CDE = 156^\circ$ $\therefore \angle CDE = 78^\circ$ $\therefore x = 102^\circ$ *(on a straight line with $\angle CDE$)* $2y^\circ + 78^\circ = 180^\circ$ $2y^\circ = 102^\circ$ $\therefore y = 51^\circ$	

QUESTION 6

a) i)



ii)

$$\sin(21^\circ) = \frac{5}{d}$$

$$d = \frac{5}{\sin(21^\circ)}$$

$$\therefore \quad d = 13.95m \ (2\ d.p)$$

iii)

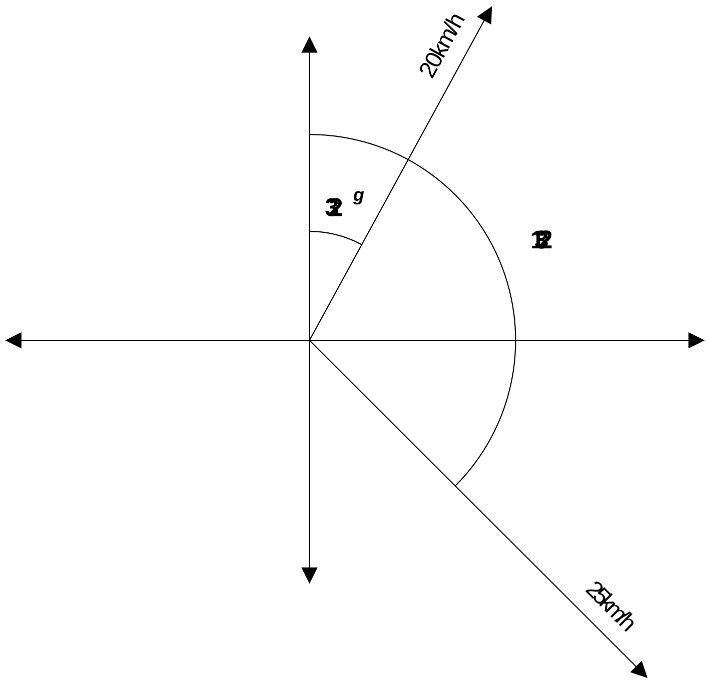
$$\frac{h}{\sin(53^\circ)} = \frac{d}{\sin(90^\circ - 32^\circ)}$$

$$h\sin(58^\circ) = d\sin(53^\circ)$$

$$h = \frac{d \times \sin(53^\circ)}{\sin(58^\circ)}$$

$$\therefore \quad h = 13.14m \ (2\ d.p.)$$

b)



let the distance be d

$$d^2 = (20 \times 3)^2 + (25 \times 3)^2 - 2(20 \times 3)(25 \times 3)\cos(132 - 32)$$

$$d = \sqrt{60^2 + 75^2 - (2 \times 60 \times 75 \times \cos(100)}$$

$$\therefore \; d = 104 \; km \; (nearest \; km)$$

c)

$$\sec(60^\circ) = \frac{1}{\cos(60^\circ)}$$

$$= \frac{1}{\left(\frac{1}{2}\right)}$$

$$= 2$$

d)

$$\sin\theta = \frac{-1}{\sqrt{2}}$$

$$\theta = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$$

$$\therefore \quad \theta = 225^\circ \; or \; 315^\circ$$

e)

$$\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

as required

QUESTION 7

Solutions

a)

$$(x+1)^2 = 6$$
$$x + 1 = \pm\sqrt{6}$$
$$x = -1 \pm \sqrt{6}$$

b)

$$\left(4-\sqrt{3}\right)^3 - \left(4+\sqrt{3}\right)^3$$
$$= \left(\left(4-\sqrt{3}\right) - \left(4+\sqrt{3}\right)\right)\left(\left(4-\sqrt{3}\right)^2 + \left(4+\sqrt{3}\right)\left(4-\sqrt{3}\right) + \left(4+\sqrt{3}\right)^2\right)$$
$$= \left(-2\sqrt{3}\right)\left(\left(16 - 8\sqrt{3} + 3\right) + (16-3) + \left(16 + 8\sqrt{3} + 3\right)\right)$$
$$= \left(-2\sqrt{3}\right)(51)$$
$$= -102\sqrt{3}$$

c)

$$\frac{dy}{dx} = 6x^2 - 4x$$
$$at\ x = 1$$
$$m_{TANGENT} = 6(1)^2 - 4(1)$$
$$= 2$$
$$\therefore \quad m_{NORMAL} = -\frac{1}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$-2(y + 2) = x - 1$$
$$-2y - 4 = x - 1$$
$$\therefore x + 2y + 3 = 0$$

d) i)

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\tan\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta\sin\theta + \cos\theta\cos\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

as required

ii)

$$\frac{1 + \cot\theta}{\text{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta} = -1$$

$$\frac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta\cos\theta}} = -1$$

$$\sin\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right) - \sin\theta\cos\theta\left(\frac{1}{\cos\theta}\right) = -1$$

$$\sin\theta + \cos\theta - \sin\theta = -1$$

$$\cos\theta = -1$$

$$\theta = \cos^{-1}(-1)$$

$$\therefore \quad \theta = 180^\circ$$