NORTH SYDNEY BOYS HIGH SCHOOL

2011 YEAR 11 YEARLY EXAM

Mathematics

General Instructions

- Working time – 2.5 hours
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:

O Mr Lin

Student Number:______________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	— 10	— 10	— 14	— 12	— 10	— 10	— 11	— 11	— 13	— 8	— 109	— 100

Question 1 (10 Marks) Commence a NEW page. **Marks**

(a) Find the value of

$$\frac{13.81^2}{8.09+\sqrt{4.62}}$$

Correct to (i) 2 decimal places **1**

(ii) 1 significant figure **1**

(b) Fully factorise $8p^3 + 64$ **2**

(c) Solve the following pair of simultaneous equations **3**

$$2x - y - 7 = 0$$

$$3x + 2y - 14 = 0$$

(d) Express the following surd with a rational denominator **3**

$$\frac{2\sqrt{2}-3}{2\sqrt{2}+3}.$$

Question 2 (10 Marks) Commence a NEW page. **Marks**

(a) Simplify the following expression, giving the answer in index form:

$2x^2y^{\frac{1}{3}} \times 3x^{-1}y^{\frac{5}{3}}$ **2**

(b) Given that $\log_2 3 = 1.58$ and $\log_2 7 = 2.81$ find the value correct to 2 decimal places of:

(i) $\log_2 49$ **1**

(ii) $\log_2 42$ **2**

(c) Solve $3^x = 8$ **2**

(d) Sketch the following graph clearly indicating the **asymptote** and the **intercepts**: **3**

$y = -\log_{10} x + 2$

Question 3 (14 Marks) Commence a NEW page. **Marks**

$P(-3, 2)$, $Q(2, 1)$ and $R(-1, -2)$ are three points on the number plane.

(a)	Draw a neat sketch of the points on the number plane.	1
(b)	Show that the gradient of PQ is $-\frac{1}{5}$.	2
(c)	Show that the equation of PQ is $x + 5y - 7 = 0$.	2
(d)	Derive the equation of the line perpendicular to PQ passing through R.	2
(e)	Find the midpoint of PR.	2
(f)	Find the length of PQ.	2
(g)	Find the perpendicular distance of R from PQ.	2
(h)	Find the area of the triangle PQR	1

Question 4 (12 Marks) Commence a NEW page. **Marks**

(a) Show whether the following functions are even, odd, or neither. Justify answers.

(i) $f(x) = \sqrt{9 - x^2}$ **2**

(ii) $f(x) = x^3 - 4x + 5$ **2**

(iii) $f(x) = \dfrac{x^2}{x^2 + 4}$ **2**

(b) If $f(x) = 4x - 5$, Find

$\dfrac{f(a) + f(b)}{a - b}$ in its simplest form. **2**

(c) (i) Sketch the graph of $y = x^2 + x - 12$ **2**

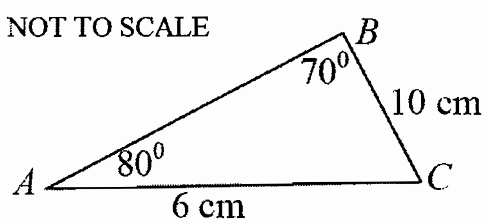
(ii) Hence, or otherwise sketch the graph of $y = |x^2 + x - 12|$ **1**

(iii) Hence, or otherwise state the **domain** and **range** of $y = |x^2 + x - 12|$ **1**

Question 5 (10 Marks) Commence a NEW page. **Marks**

(a) Write down the exact value of $\tan 300°$. **2**

(b) Solve $2\sin x + 1 = 0$ for $0° \leq x \leq 360°$. **2**

(c) 

Find the area of the triangle ABC. **2**

(d) If $\sin\theta = -\frac{5}{13}$ and $\tan\theta < 0$, find $\cos\theta$ **2**

(e) Prove the identity **2**

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

Question 6 (10 Marks) Commence a NEW page. **Marks**

Differentiate the following with respect to x:

(a) $y = 2x^4 - 8$ **1**

(b) $y = (2x-1)^3$ **1**

(c) $y = x^2\sqrt{x}$ **2**

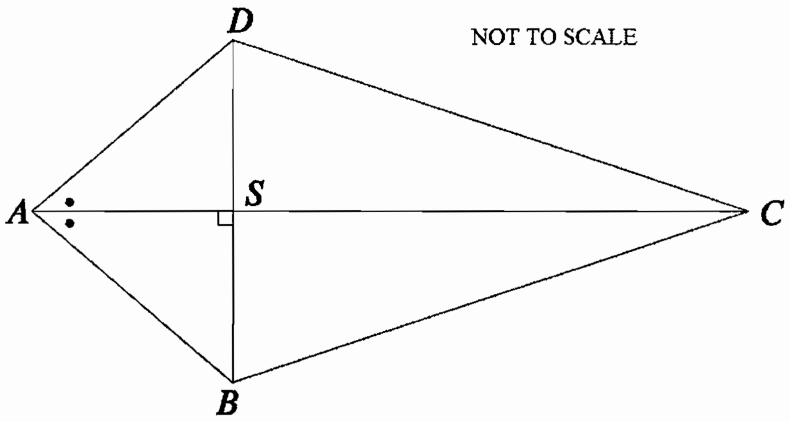
(d) $y = \dfrac{3}{2x^3}$ **1**

(e) $y = 5x^3(4x-3)^5$ **2**

(f) $y = \dfrac{x^2-2}{x+1}$ **3**

Question 7 (11 Marks) Commence a NEW page. **Marks**

(a)

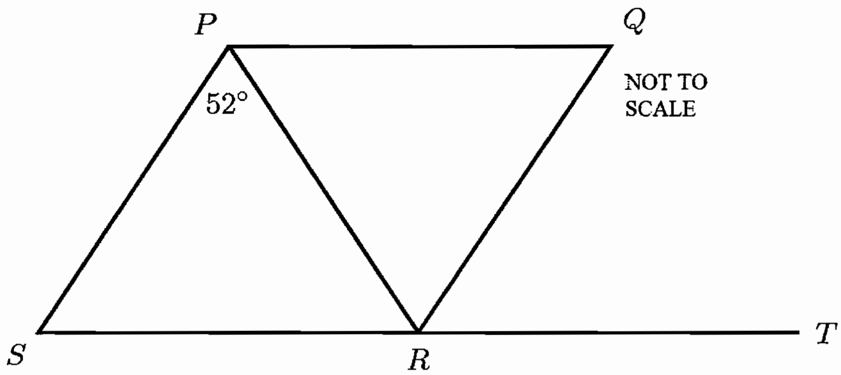


In the above diagram, $ABCD$ is a quadrilateral. The diagonals AC and DB intersect at right angles at point S. If $\angle DAS = \angle BAS$

(i) Prove that $\Delta ASB \equiv \Delta ASD$ **3**

(ii) Hence prove that $DA = BA$ **1**

(b) $PQRS$ is a rhombus where $\angle SPR = 52°$ and SR is produced to T.



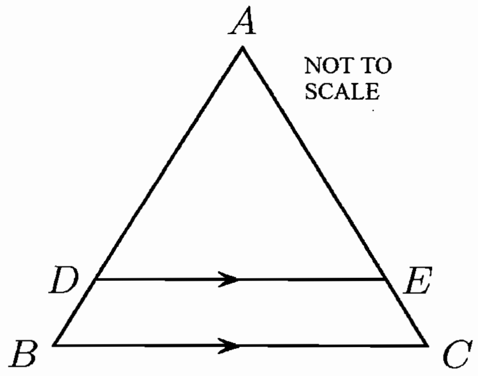
(i) Find the size of $\angle SPQ$. **1**

(ii) What is the size of $\angle QRT$? Give reasons **2**

Question 7 continued on next page...

...Question 7 continued

(c) In the diagram below $DE \parallel BC$. AB =16cm, $AE = 18$cm and EC=6cm.



(i) Prove that $\Delta ADE ||| \Delta ABC$. **2**

(ii) Find the length of DB **2**

Question 8 (11 Marks) Commence a NEW page. **Marks**

(a) Find all values of x for which the tangent at x has positive gradient.

$$y = x^3 - 3x$$ **3**

(b) Find the equation of the normal to $y = x^2 + 2x - 4$ at the point $(0, -4)$ **3**

(c) From a point O, an observer can see a lighthouse L on a bearing of $285°T$. An oil rig R can also been seen by the observer on a bearing of $215°T$. The lighthouse is 12km from the oil rig and on a bearing of $012°T$ from the oil rig.

(i) Draw a neat diagram showing the above information. **1**

(ii) Find the distance, to the nearest kilometre, of the oil if from the observer. **3**

(iii) Find the bearing of the oil rig from the lighthouse. **1**

Question 9 (13 Marks) Commence a NEW page. **Marks**

If $f(x) = 6x^3 + 9x^2 - 3$,

(a) (i) Show that $6x^3 + 9x^2 - 3 = 3(x+1)^2(2x-1)$ **2**

(ii) Hence find the x-intercepts **2**

(b) Determine the y-intercept **1**

(c) Find the stationary point(s) and determine their nature **4**

(d) Find the point(s) of inflexion **2**

(e) On the number plane sketch the curve **2**

$f(x) = 6x^3 + 9x^2 - 3$

showing all of the above features

Question 10 (8 Marks) Commence a NEW page. **Marks**

A closed cylindrical can is made from 100π square centimetres of metal. If h is the height and r is the radius,

(i) show that $h = \frac{50}{r} - r$. **2**

(ii) Hence, show that the volume can be expressed as $V = 50\pi r - \pi r^3$ **2**

(iii) Find the maximum possible volume of the can and show why it is a maximum. **4**

Answer to the nearest centimetres

END OF EXAMINATION

Question 1

a) i) 18.63

(ii) 20

b) $8(p^3+8) = 8(p+2)(p^2-2p+4)$

c) $2x - y - 7 = 0 \quad -①$

$3x + 2y - 14 = 0 \quad -②$

$y = 2x - 7 \quad -③$

sub ③ into ②

$3x + 2(2x-7) - 14 = 0$

$7x - 28 = 0$

$7x = 28$

$x = 4$

sub into ①

$y = 1$

d) $\dfrac{2\sqrt{2}-3}{2\sqrt{2}+3} \times \dfrac{2\sqrt{2}-3}{2\sqrt{2}-3} = \dfrac{8 - 6\sqrt{2} - 6\sqrt{2} + 9}{8-9}$

$= \dfrac{17 - 12\sqrt{2}}{-1}$

$= 12\sqrt{2} - 17$

Question 2.

a) $2x^2y^{\frac{1}{3}} \times 3x^{-1}y^{\frac{5}{3}} = 6xy^2$

b) i) $\log_2 3 = 1.58$, $\log_2 7 = 2.81$

i) $\log_2 7^2$

$= 2\log_2 7$

$= 2 \times 2.81$

$= 5.62$

ii) $\log_2 42 = \log_2(6\times7)$

$= \log_2 3 + \log_2 2 + \log_2$

$= 1.58 + 1 + 2.81$

$= 5.39.$

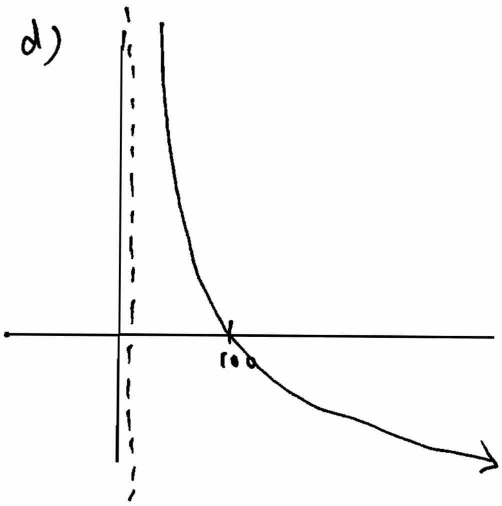
c) $3^x = 8$

$\log_3 8 = x$

$x = \dfrac{\log 8}{\log 3}$

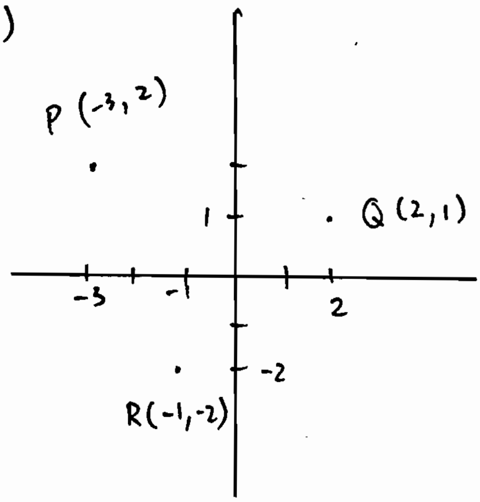
$= 1.89$ to 2dp.

d)



Question 3

a)



b) $m_{PQ} = \frac{2-1}{-3-2} = \frac{1}{-5}$

$= -\frac{1}{5}$

c) $PQ \Rightarrow$

$y - 2 = -\frac{1}{5}(x+3)$

$5y - 10 = -x - 3$

$x + 5y - 7 = 0$

d) gradient of normal

$= -\frac{1}{-\frac{1}{5}}$

$= 5$

$\therefore\ y + 2 = 5(x+1)$

$5x - y + 3 = 0$

e). $\left(\frac{-3-1}{2}, \frac{2-2}{2}\right)$

$M(-2, 0)$

f) $\sqrt{(2-1)^2 + (-3-2)^2}$

$= \sqrt{1^2 + 25}$

$= \sqrt{26}$

g) $\frac{|(-1\times 1)+(5\times -2)-7|}{\sqrt{26}}$

$= \frac{|-18|}{\sqrt{26}}$

$= \frac{18}{\sqrt{26}}$

h) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times \sqrt{26} \times \frac{18}{\sqrt{26}}$

$= 9$

Question 4

a) i) $f(x) = \sqrt{9-x^2}$

$f(-x) = \sqrt{9-(-x)^2}$

$= \sqrt{9-x^2}$

$f(x) = f(-x)$

$\therefore$ even

(ii) $f(x) = x^3 - 4x + 5$

$f(-x) = (-x)^3 - 4(-x) + 5$

$= -x^3 + 4x + 5$

$\therefore$ not even

$-f(x) = -x^3 + 4x - 5$

$\neq f(x)$ $\therefore$ not odd

$\therefore$ neither

(iii) $f(x) = \frac{x^2}{x^2+4}$

$f(-x) = \frac{(-x)^2}{(-x)^2+4}$

$= \frac{x^2}{x^2+4}$

$= f(x)$

$\therefore$ even

4 (b) $f(a) = 4a - 5$

$f(b) = 4b - 5$

$\therefore \dfrac{f(a) + f(b)}{a - b}$

$\Rightarrow \dfrac{4a - 5 + 4b - 5}{a - b}$

$= \dfrac{4a + 4b - 10}{a - b}$

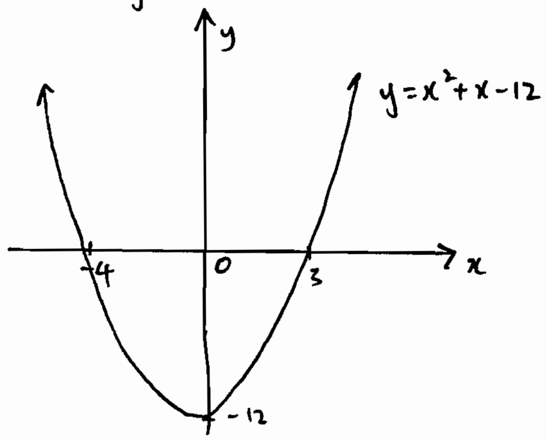
$= \dfrac{2(2a + 2b - 5)}{a - b}$

(c) i) $y = x^2 + x - 12 \Rightarrow y = (x+4)(x-3)$

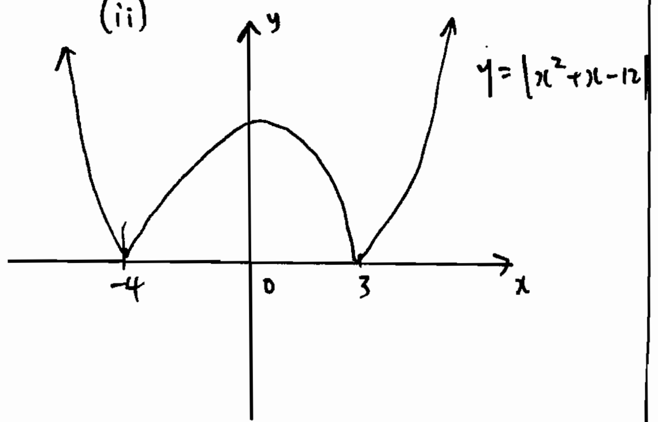
concave up

x int: $-4, 3$

y int.. -12



(ii)



(iii) D: $x \in \mathbb{R}$

R: $y \geq 0$

Question 5

a) $\tan 300 = -\tan 60$

$= -\sqrt{3}$

b) $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$\therefore 180^\circ \leq x \leq 360^\circ$

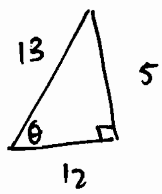
$x = 210^\circ$ or 330°

c) $\angle BCA = 30^\circ$

$\therefore A = \frac{1}{2} \times 10 \times 6 \times \sin 30$

$= 15 \text{ cm}^2$

d)



$\sin\theta = -\frac{5}{13}$ $\tan\theta < 0$

$\therefore$ θ is in 4th quad

$\therefore$ $\cos\theta$ is positive

$\therefore$ $\cos\theta = \frac{12}{13}$

e) $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$

LHS $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta\sin\theta}$

$=$ RHS.

Question 6

a) $y = 2x^4 - 8$

$y' = 8x^3$

b) $y = (2x-1)^3$

$y' = 3(2x-1)^2 \times 2$

$= 6(2x-1)^2$

c) $y = x^2\sqrt{x}$

$= x^2 \cdot x^{\frac{1}{2}}$

$= x^{\frac{5}{2}}$

$\therefore y' = \frac{5}{2}x^{\frac{3}{2}}$

d) $y = \frac{3}{2x^3}$

$= \frac{3}{2}x^{-3}$

$\therefore y' = -\frac{9}{2}x^{-4}$

e) $y = 5x^3(4x-3)^5$

$u = 5x^3 \qquad v = (4x-3)^5$

$u' = 15x^2 \qquad v' = 5(4x-3)^4 \times 4$

$= 20(4x-3)^4$

$\therefore y' = 5x^3 \times 20(4x-3)^4 + 15x^2(4x-3)^5$

$= 100x^3(4x-3)^4 + 15x^2(4x-3)^5$

$= 5x^2(4x-3)^4[20x + 3(4x-3)]$

$= 5x^2(4x-3)^4[20x + 12x - 9]$

$= 5x^2(4x-3)^4[32x-9]$

f) $y = \frac{x^2-2}{x+1}$

$u = x^2 - 2 \qquad v = x+1$

$u' = 2x \qquad v' = 1$

$y' = \frac{(x+1)\cdot 2x - (x^2-2)}{(x+1)^2}$

$= \frac{2x^2 + 2x - x^2 + 2}{(x+1)^2}$

$= \frac{x^2 + 2x + 2}{(x+1)^2}$

Question 7

(a) (i) In Δ's ASB and ASD

AS is common

$\angle ASB = \angle ASD$ (right $\angle$'s, given)

$\angle BAS = \angle DAS$ (given)

$\therefore \Delta ASB \equiv \Delta ASD$ (AAS)

(ii) $DA = BA$ (corresponding sides of congruent Δ's are equal)

(b) (i) $\angle SPQ = 104°$

(ii) $\angle QRS = 104°$ (opposite $\angle$'s of rhombus equal)

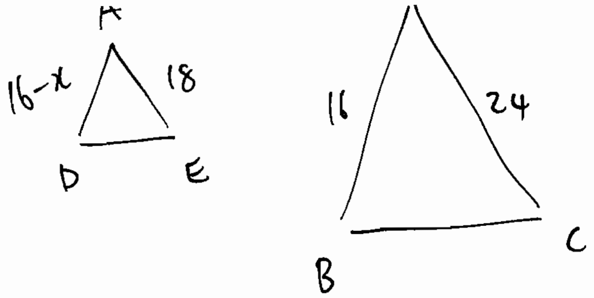
$\therefore \angle QRT = 76°$ (straight angle)

(c) In Δ's ADE and ABC

$\angle A$ is common

$\angle ADE = \angle ABC$ (corresponding $\angle$'s equal, $DE \parallel BC$)

$\therefore \Delta ADE \parallel\!| \Delta ABC$ (equiangular



$$\frac{16-x}{16} = \frac{18}{24}$$

$$16-x = \frac{18\times 16}{24}$$

$$x = 16-12$$
$$= 4$$

8. a) $y = x^3 - 3x$

$$\frac{dy}{dx} = 3x^2 - 3$$

positive gradient when

$$\frac{dy}{dx} > 0$$

$$\therefore\ 3x^2 - 3 > 0$$
$$x^2 - 1 > 0$$
$$x > 1 \text{ or } x < -1$$

b) $y = x^2 + 2x - 4$

$$\frac{dy}{dx} = 2x + 2$$

$$m_T = 2\times 0 + 2$$
$$= 2$$

$$\therefore m_N = -\frac{1}{2}$$

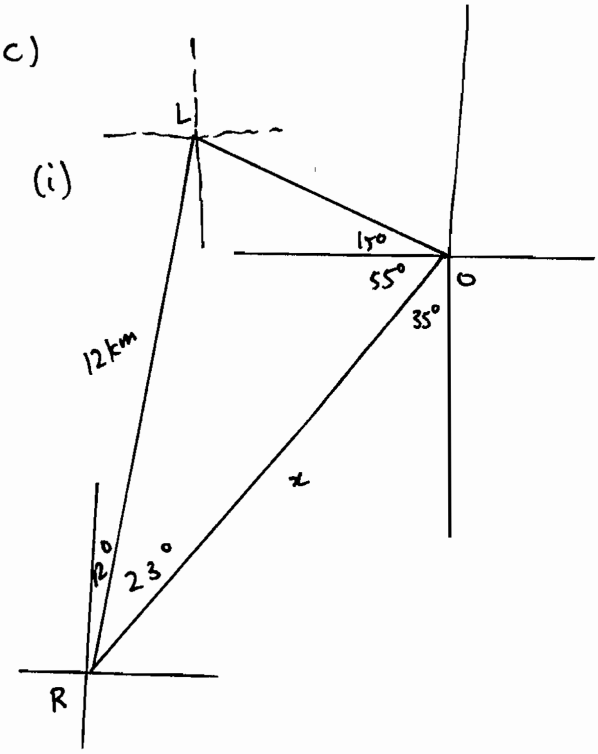
$$\therefore\ y + 4 = -\frac{1}{2}(x - 0)$$

$$-2y - 8 = x$$

$$\therefore\ x + 2y + 8 = 0$$

c)

(i)



(ii) $\angle RLO = 180 - 23 - 70°$
$$= 87°$$

$$\frac{x}{\sin 87°} = \frac{12}{\sin 70°}$$

$$x = \frac{12 \sin 87°}{\sin 70°}$$

$$\doteqdot 13 \text{ km}$$

(iii) $180 + 12°$
$$= 192° T$$

<u>Question 9</u>

a) i) $6x^3 + 9x^2 - 3 = 3(x+1)^2(2x-1)$

$$RHS = 3(x+1)^2(2x-1)$$
$$= 3(x^2+2x+1)(2x-1)$$
$$= 3(2x^3 - x^2 + 4x^2 - 2x + 2x - 1)$$
$$= 3(2x^3 + 3x^2 - 1)$$
$$= 6x^3 + 9x^2 - 3$$
$$= LHS$$

(ii) x int: $-1,\ \frac{1}{2}$

b) $x = 0$

$\therefore f(0) = -3$

$\therefore$ y int: -3.

c) $f(x) = 6x^3 + 9x^2 - 3$

$f'(x) = 18x^2 + 18x$

Stationary point(s) when $f'(x) = 0$

$18x^2 + 18x = 0$

$18x(x+1) = 0$

$\therefore x = 0$ or -1

$\therefore y = -3$ or 0

$\therefore (0,-3)$ $(-1,0)$

Testing nature:

$f''(x) = 36x + 18$

$f''(0) = 18 > 0$ $\therefore$ concave up

$\therefore$ min at $(0,-3)$

$f''(-1) = -36 + 18$

$= -18 < 0$ $\therefore$ concave down

$\therefore$ max at $(-1,0)$

$\therefore$ minimum at $(0,-3)$

$\therefore$ maximum at $(-1,0)$

d) point of inflexion occurs

$f''(x) = 0$

$\Rightarrow 36x + 18 = 0$

$36x = -18$

$x = -\frac{1}{2}$

$y = -\frac{3}{2}$

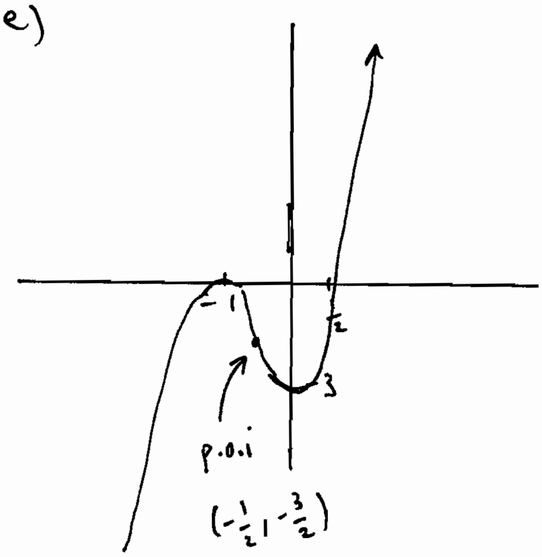
testing

x	-1	$-\frac{1}{2}$	0
$f''(x)$	-18	0	18
	$\frown$	$\cdot$	$\smile$

$\therefore$ change in concavity

$\therefore$ $(-\frac{1}{2}, -\frac{3}{2})$ is poi

e)



Question 10.

(i). $SA = 2\pi r^2 + 2\pi r h$

$SA = 100\pi$

$\therefore \quad 2\pi r^2 + 2\pi r h = 100\pi$

$r^2 + rh = 50$

$rh = 50 - r^2$

$h = \frac{50}{r} - r$

ii) $V = \pi r^2 h$

$= \pi r^2 \left(\frac{50}{r} - r\right)$

$= 50\pi r - \pi r^3$

iii) $\frac{dV}{dr} = 50\pi - 3\pi r^2$

$\frac{dV}{dr} = 0 \Rightarrow$

$50\pi - 3\pi r^2 = 0$

$3\pi r^2 = 50\pi$

$r^2 = \frac{50}{3}$

$r = \pm\sqrt{\frac{50}{3}}$

$r > 0$

$\therefore \quad r = \sqrt{\frac{50}{3}}$

$= \frac{5\sqrt{2}}{\sqrt{3}}$

testing

$\frac{d^2V}{dr^2} = -6\pi r$

$= -6\pi \times \frac{5\sqrt{2}}{\sqrt{3}} < 0 \quad \therefore$ concave down

$\therefore$ max occur at $r = \frac{5\sqrt{2}}{3}$

$\therefore \quad V = 50\pi \times \left(\frac{5\sqrt{2}}{\sqrt{3}}\right) - \pi\left(\frac{5\sqrt{2}}{\sqrt{3}}\right)^3$

$\approx 428\ \text{cm}^3$