NORTH SYDNEY GIRLS HIGH SCHOOL

2006

YEAR 11 EXAMINATION

Preliminary Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

NAME:_____ **TEACHER:**_____

STUDENT NUMBER:_____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Begin each question on a NEW PAGE.

Question 1: (12 marks) **Marks**

a) Find the value of $\dfrac{4 \cdot 23}{\sqrt{6 \cdot 14 - 1 \cdot 78}}$ correct to 3 significant figures. **2**

b) Express $0 \cdot \dot{2}\dot{1}$ as a fraction in its simplest form. **1**

c) Show that $\dfrac{1}{3-\sqrt{2}} + \dfrac{1}{3+\sqrt{2}}$ is a rational number. **2**

d) Simplify $\dfrac{2}{3} - \dfrac{x-1}{4}$ **2**

e) Solve the simultaneous equations:
$x - y = 2$
$3x + 2y = 1$ **2**

f) Evaluate $\left(\dfrac{1}{16}\right)^{-\frac{3}{4}}$ **1**

g) Factorise $125a - ab^3$ **2**

Question 2: (12 marks) Start a new page **Marks**

a) Simplify the expression $\dfrac{6}{x^2+6x+5} - \dfrac{4}{x^2+5x}$ **3**

b) Solve the inequality $|16-4x| > 8$ and graph your solution on the number line. **3**

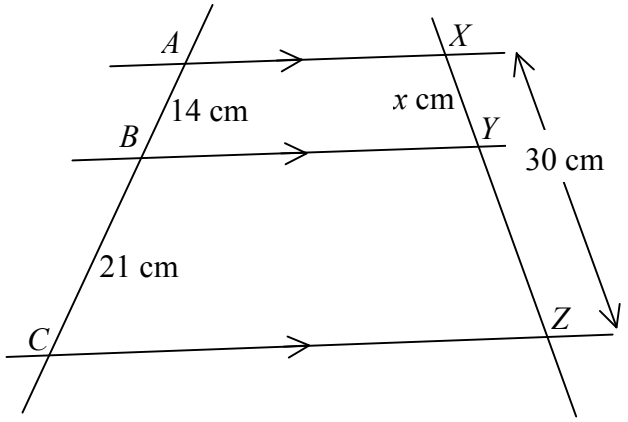
c) Solve $(2x + 3)^2 = 25$ **2**

d) Factorise completely $x^4 - x^2 - 12$ **2**

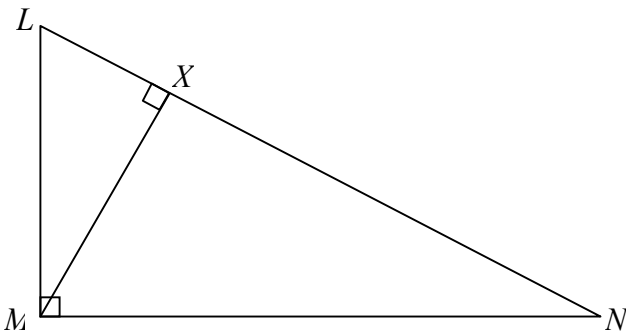
e) Find the coordinates of the centre of the circle $x^2 + y^2 - 2x + 6y + 1 = 0$. **2**

Question 3: (12 marks) Start a new page **Marks**

a) In the diagram, $AX \parallel BY \parallel CZ$.
$AB = 14$ cm; $BC = 21$ cm; $XZ = 30$ cm and $XY = x$ cm.
Find the length of XY giving reasons. **2**



b)



ΔLMN is right angled at M. X lies on LN such that $MX \perp LN$.

i) Prove that ΔLMX is similar to ΔLNM. **2**

ii) Hence prove that $LM^2 = NL \times LX$. **2**

c) Sketch each of the following, showing their important features.

i) $y = \sqrt{25 - x^2}$ **2**

ii) $y = |2x - 1|$ **2**

iii) $y = x(x + 2)(1 - x)$ **2**

Question 4: (12 marks) Start a new page **Marks**

a) A piecemeal function is defined as

$$f(x) = \begin{cases} -x^2, & x < 0 \\ 5x - 4, & x \geq 0 \end{cases}$$

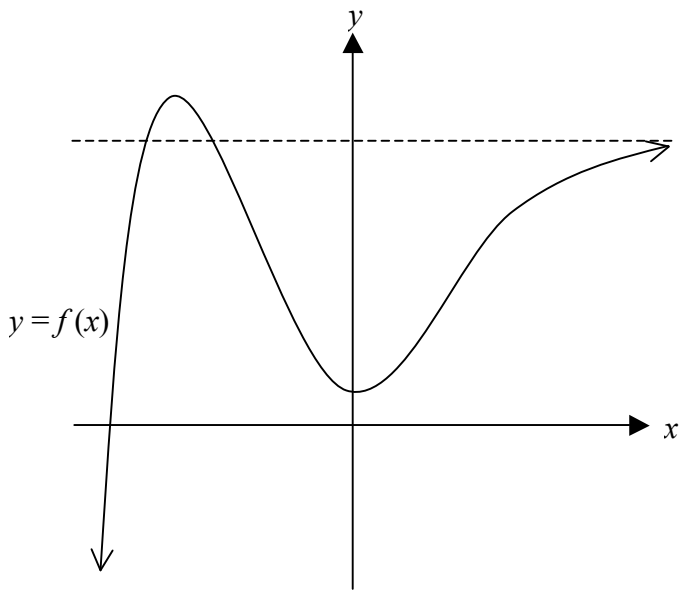
Evaluate $f(-2) + f(0) - f(1)$ **2**

b) State the domain of the function $y = \sqrt{2x+5}$. **1**

c) Sketch $y = 5^x - 3$ and hence state the range of the function. **3**

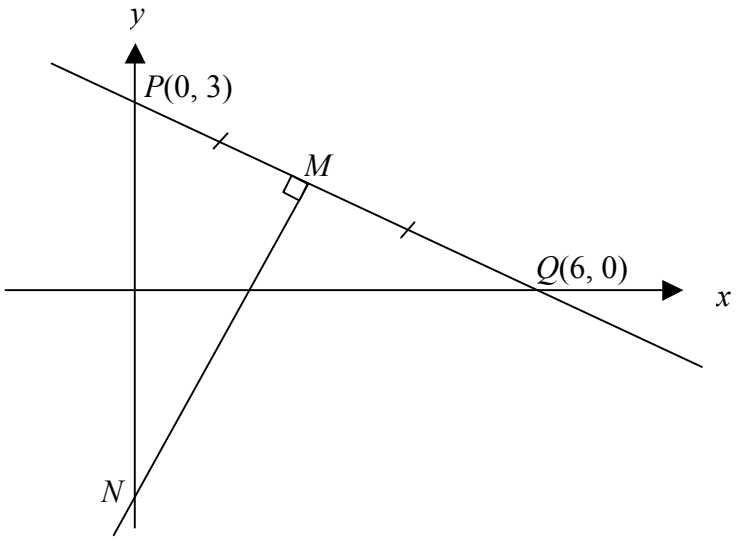
d) Sketch the region in the number plane which $y \leq 4 - x^2$ and $y > -2$ hold simultaneously. **3**

e) The diagram illustrates the graph of $y = f(x)$. Copy or trace the graph onto your examination page and, on a new set of axes directly below, sketch the graph of its gradient function, $y = f'(x)$. **3**



Question 5: (12 marks) Start a new page **Marks**

a) The diagram shows the points P (0, 3) and Q (6, 0). The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the y-axis at N.



i) Show that the gradient of PQ is $-\_$. **1**

ii) Find the coordinates of M. **2**

iii) Find the equation of MN. **2**

iv) Show that the coordinates of N are $(0, -\frac{9}{2})$. **1**

v) The quadrilateral $PNQR$ is a parallelogram. Find the coordinates of R. **1**

b) Find the perpendicular distance from the point A (3, −2) to the line $y = 2x + 5$. **2**

c) Find the equation of the line which passes through the point of intersection of the lines $2x + 3y = 5$ and $3x - 4y = 7$ and which also passes through the point (−1, 3). **3**

Question 6: (12 marks) Start a new page **Marks**

a) Find the value of θ if $0° \leq \theta \leq 90°$ and $\tan 20° = \cot(\theta + 30°)$ **1**

b) If $\sin x° = \frac{4}{5}$ and $\cos x° < 0$, find the value of $\tan x°$. **2**

c) Prove that $\cot^2\theta(1 - \cos^2\theta) = \cos^2\theta$ **3**

d) If $0° \leq \theta \leq 360°$, find all the solutions for θ for
$2\sin\theta\cos\theta - \sin\theta = 0$. **3**

e) A boat leaves port P and steams 80 nautical miles on a bearing of 110°.
It then turns to a bearing of 190° and steams a further 100 nautical miles.

i) Draw a diagram illustrating this information. **1**

ii) How far is the boat from P to the nearest nautical mile? **2**

Question 7: (12 marks) Start a new page **Marks**

a) Differentiate the following:

i) $y = 5x^2 - 3x$ **1**

ii) $f(x) = \dfrac{(x^2+3)(5x-1)}{x^2}$ **3**

b) Consider the curve $y = x + \dfrac{8}{x}$.

i) Find the gradient of the curve at the point P where $x = 2$. **2**

ii) What are the co-ordinates of P. **1**

iii) Find the equation of the normal to the curve at the point P. **2**

c) At what point on the curve $f(x) = \dfrac{x^2}{4} - 5$ is the tangent parallel to $3x - y + 2 = 0$? **3**

End of paper

Solutions

Question one

a) $2.02580\ldots$

$\doteq 2.03$

b) $\frac{21}{99}$

$= \frac{7}{33}$

c) $\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}}$

$= \frac{3+\sqrt{2} + 3-\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \frac{6}{9-2}$

$= \frac{6}{7}$

which is rational

d) $\frac{2}{3} - \frac{x-1}{4}$

$= \frac{2(4) - 3(x-1)}{12}$

$= \frac{8-3x+3}{12}$

$= \frac{11-3x}{12}$

e) $2x - 2y = 4$

$3x + 2y = 1$

$5x = 5$

$x = 1$

$x=1,\ 2 - 2y = 4$

$2y = -2$

$y = -1$

$\begin{cases} x=1 \\ y=-1 \end{cases}$

f) $(16)^{\frac{3}{4}}$

$= 8$

g) $a(125 - b^3)$

$= a(5-b)(25+5b+b^2)$

Question two

a) $\frac{6}{(x+5)(x+1)} - \frac{4}{x(x+5)}$

$= \frac{6(x) - 4(x+1)}{x(x+5)(x+1)}$

$= \frac{6x - 4x - 4}{x(x+5)(x+1)}$

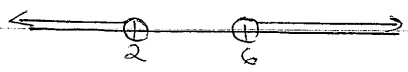
$= \frac{2x-4}{x(x+5)(x+1)}$

b) $|16 - 4x| > 8$

$16 - 4x > 8$ or $16 - 4x < -8$

$-4x > -8$ $\qquad -4x < -24$

$x < 2$ $\qquad x > 6$



c) $(2x+3)^2 = 25$

$2x + 3 = \pm 5$

$2x + 3 = 5$ $\qquad 2x + 3 = -5$

$2x = 2$ $\qquad 2x = -8$

$x = 1$ $\qquad x = -4$

$x = 1, -4$

Question two

d) let $m = x^2$

$m^2 - m - 12$

$(m-4)(m+3)$

$\therefore (x^2-4)(x^2+3)$

$= (x-2)(x+2)(x^2+3)$

e) $x^2 - 2x + 1 + y^2 + 6y + 9 = -1 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 9$

Centre $(1, -3)$

Question three

a) $\frac{AB}{BC} = \frac{XY}{YZ}$ a family of parallel lines will cut all transversals in the same ratio

$\frac{14}{21} = \frac{x}{(30-x)}$

$\frac{2}{3} = \frac{x}{(30-x)}$

$60 - 2x = 3x$

$5x = 60$

$x = 12$

$XY = 12\text{ cm}$

b) In ΔLMX, ΔLNM

$\angle MLX = \angle NLM$ Common

$\angle LXM = \angle LMN$ both 90°

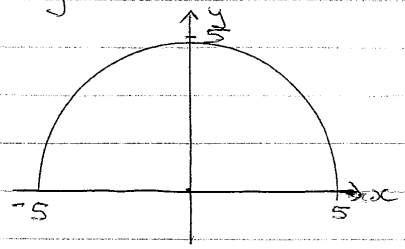
$\therefore \Delta LMX \;|||\; \Delta LMN$ equiangular

$\frac{LM}{LN} = \frac{LX}{LM}$ ratio of sides of similar triangles

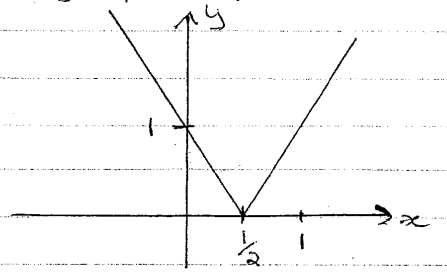
$\therefore LM^2 = NL \cdot LX$

Question three

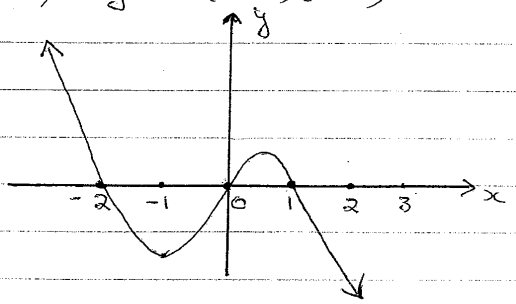
c) i) $y = \sqrt{25 - x^2}$



(ii) $y = |2x - 1|$



(iii) $y = x(x+2)(1-x)$



Question four

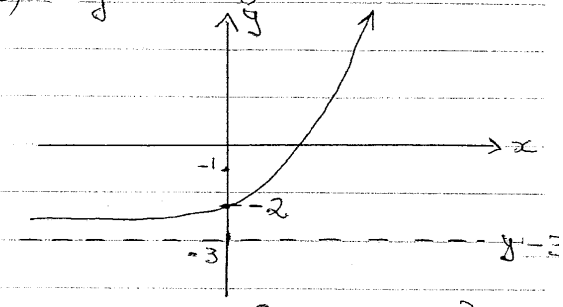
a) $f(-2) + f(0) - f(1)$

$= -4 + -4 - 1$

$= -9$

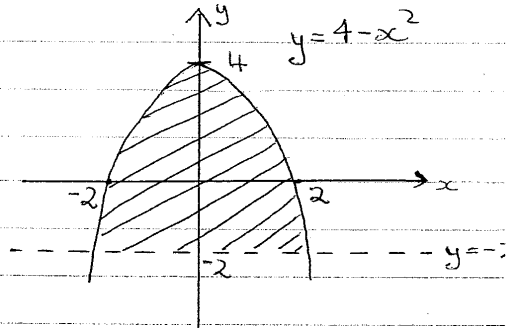
b) domain $\{x : x \geq -\frac{5}{2}\}$

c) $y = 5^x - 3$

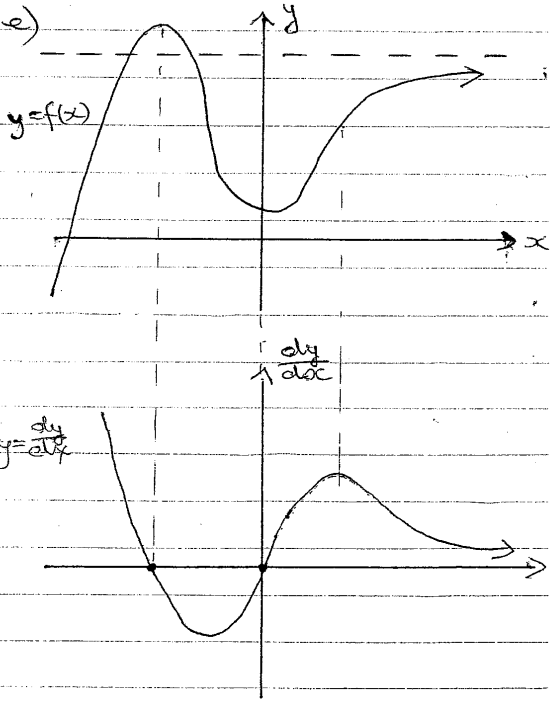


range : $\{y : y > -3\}$

d)



e)



Question five

a) i) $m = \frac{0-3}{6-0}$

$= \frac{-3}{6}$

$= -\frac{1}{2}$

(ii) $M: \left(\frac{0+6}{2}, \frac{3+0}{2}\right)$

$= \left(\frac{6}{2}, \frac{3}{2}\right)$

$= (3, 1\frac{1}{2})$

(iii) gradient MN is 2

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = 2(x - 3)$

$2y - 3 = 4(x-3)$

$2y - 3 = 4x - 12$

$0 = 4x - 2y - 9$

(iv) $x = 0, \quad 4(0) - 2y - 9 = 0$

$2y = -9$

$y = -\frac{9}{2}$

$\therefore N\left(0, -\frac{9}{2}\right)$

(v) $R\left(6, 7\frac{1}{2}\right)$

b) $d = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$

$a = 2, \ b = -1, \ c = 5$

$d = \left|\frac{2(3) - 1(-2) + (5)}{\sqrt{(2)^2 + (-1)^2}}\right|$

$= \left|\frac{6 + 2 + 5}{\sqrt{4+1}}\right|$

$= \frac{13}{\sqrt{5}}$.

c) $(2x + 3y - 5) + k(3x - 4y - 7) = 0$

$(2(-1) + 3(3) - 5) + k(3(-1) - 4(3) - 7) = 0$

$(-2 + 9 - 5) + k(-3 - 12 - 7) = 0$

$2 + k(-22) = 0$

$22k = 2$

$k = \frac{1}{11}$

$(2x + 3y - 5) + \frac{1}{11}(3x - 4y - 7) = 0$

$11(2x + 3y - 5) + (3x - 4y - 7) = 0$

$22x + 33y - 55 + 3x - 4y - 7 = 0$

$25x + 29y - 62 = 0$

Question six

a) $\tan 20^\circ = \cot 70^\circ$

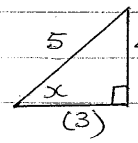
$\therefore 70 = \theta + 30^\circ$

$\theta = 40^\circ$

OR $20^\circ + 30^\circ + \theta = 90^\circ$

$\therefore \theta = 40^\circ$

b) 2nd quadrant $\therefore \tan x < 0$;



$\tan x^\circ = -\frac{4}{3}$

c) $\cot^2\theta(1 - \cos^2\theta) = \cos^2\theta$

$LHS = \frac{\cos^2\theta}{\sin^2\theta} \times (\sin^2\theta)$

$= \cos^2\theta$.

$= RHS$

Question six

d) $2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

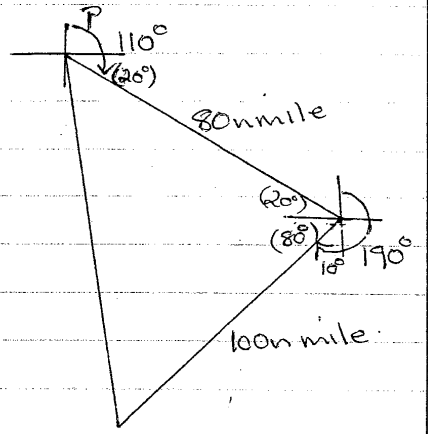
$\theta = 0°, 180°, 360°$ $\quad 2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$

$\theta = 0°, 60°, 180°, 300°, 360°$

e)



$d^2 = 80^2 + 100^2 - 2(80)(100)\cos 100°$

$d \doteq 138.4859...$

the distance is approx 138 nautical miles.

Question seven

a) $y = 5x^2 - 3x$

$\frac{dy}{dx} = 10x - 3$

(ii) $f(x) = \frac{5x^3 - x^2 + 15x - 3}{x^2}$

$= 5x - 1 + 15x^{-1} - 3x^{-2}$

$f'(x) = 5 - 15x^{-2} + 6x^{-3}$

$= 5 - \frac{15}{x^2} + \frac{6}{x^3}$

b i) $y = x + 8x^{-1}$

$\frac{dy}{dx} = 1 - 8x^{-2}$

$= 1 - \frac{8}{x^2}$

$x = 2$, $\frac{dy}{dx} = 1 - \frac{8}{4}$

$= -1$

the gradient at P is -1

(ii) $y = 2 + \frac{8}{2}$

$= 6$

P is $(2, 6)$

(iii) gradient of normal is 1

equation is

$y - 6 = 1(x - 2)$

$y - 6 = x - 2$

$y = x + 4$

c) $y = 3x + 2$ gradient 3

$f(x) = \frac{1}{4}x^2 - 5$

$f'(x) = \frac{1}{2}x$

$f'(x) = 3$

$\frac{1}{2}x = 3$

$x = 6$

$x = 6$, $y = 4$

at $(6, 4)$ the tangent is parallel to $3x - y + 2 = 0$