NORTH SYDNEY GIRLS HIGH SCHOOL

2007

YEARLY EXAMINATION

Preliminary Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

At the end of the examination, place your answer sheets in order and put this question paper on top. Submit one bundle. The bundle will be separated for marking so please ensure your name is written on EVERY PAGE.

Student Name: ________________________________ **Teacher:** ________________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84
Attempt Questions 1–7 All questions are of equal value

Begin each question on a new page.

Question 1 (12 marks) **Marks**

a) Find the value of $\frac{\sqrt[4]{389}}{3+4^2}$ correct to 3 significant figures. **2**

b) Simplify $5\sqrt{7}-\frac{7}{\sqrt{7}}$, expressing your answer with a single rational denominator. **2**

c) Factorise x^3+2x^2+x+2 **2**

d) Evaluate in exact form $\sin 120°(\sin 30° + \cos 45°)$ **2**

e) Solve $5m^2+2m-3=0$ **2**

f) Simplify $\frac{3}{x+3}-\frac{6x}{x^2-9}$ **2**

Question 2 (12 marks) **Start a new page**

a) Differentiate with respect to x. Give your answers in simplest form.

i) $y = \frac{x^3}{3} - 1$ **1**

ii) $y = x(x-1)^2$ **1**

iii) $y = \frac{3x^3 - 4x^2}{x}$ **1**

iv) $y = -\frac{3}{2}\sqrt[3]{x^2}$ **2**

b) The gradient of the tangent to the curve $y = ax^2 - 8x + 10$ is 4 when $x = 2$. Find the value of a. **2**

c) Show that the tangents at the points (1, -3) and (0, -3) on the curve $y = x^2 - x - 3$ are perpendicular to each other. **3**

d) Given that $\operatorname{cosec}\alpha = 4$ and α is acute, find the value of $\tan\alpha$. **2**

Question 3 (12 marks) **Start a new page**

a) i) Determine the gradient of a straight line with an angle of inclination of 135°. **1**

ii) Find the equation of a line with this gradient and an x intercept of $\frac{2}{5}$.
Express your answer in general form. **2**

b) Show that the perpendicular distance from a point P (1, 7) to the line $2y = x + 3$ is $2\sqrt{5}$ units. **2**

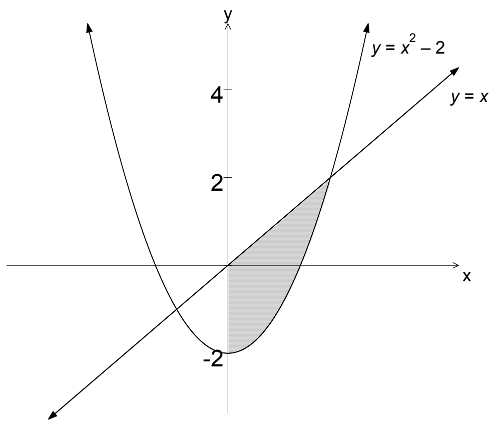
c) A function $g(x)$ is the function defined by

$$g(x) = \begin{cases} -1 & \text{for} \quad x \le -1 \\ x^2 - 2 & \text{for} \quad -1 < x < 1 \\ -\dfrac{1}{x} & \text{for} \quad x \ge 1 \end{cases}$$

i) State the range of $g(x)$. **2**

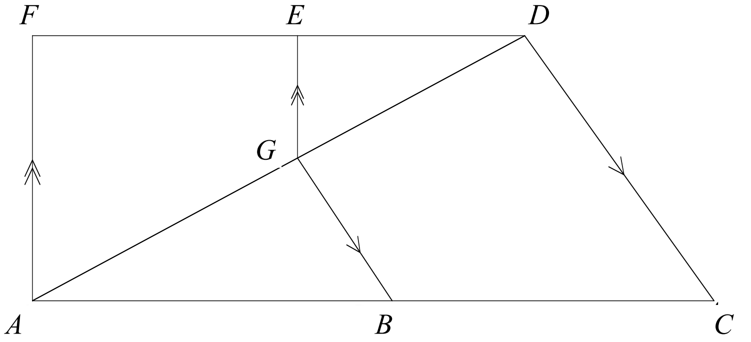
ii) Evaluate $g(2) - 2g(0) + g(-3)$ **3**

d) Write three inequations satisfied by all points in the shaded region below. **2**



Question 4 (12 marks) **Start a new page**

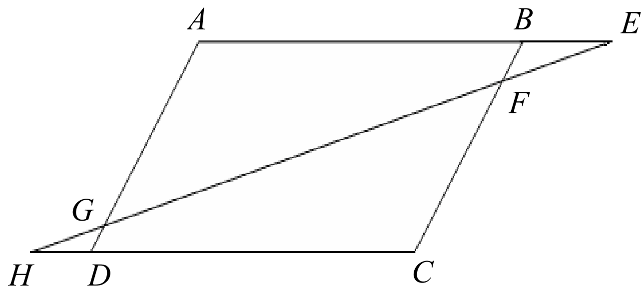
a) Using the following diagram, state three ratios equal to $BG : CD$ **2**



b) $ABCD$ is a parallelogram, shown in the diagram to the right.

AB is produced to E and CD is produced to H so that $BE = DH$.

EH meets BC and AD at F and G respectively.



i) Copy the diagram onto your answer sheet.

ii) Prove $\Delta BEF \equiv \Delta DHG$. **3**

iii) Hence prove that $AG = CF$. **2**

c) The line $ax + by - 3 = 0$ is perpendicular to the line $2x - y + 5 = 0$ and passes through the point (1, 1). Find the value of a and b. **3**

d) A straight line passes through the points $A(2k-1, 2k+1)$ and $B(k-1, k+1)$. Show that the equation of AB is $y = x + 2$. **2**

Question 5 (12 marks) Start a new page

a) Find the smallest angle of a triangle with sides 10cm, 16cm and 18cm. **2**

b) From a Qantas jet maintaining a constant altitude, the angle of depression of Sydney Tower is 20°. After flying 500m in a horizontal direction away from the tower, the angle of depression is 15°. Find the altitude of the plane, given that Sydney Tower has a height of 309m. **3**

c) Nemo swims 10km due East from his Dad and then 5km on a bearing of 050ºT.

i) Find the distance from Nemo to his Dad to the nearest km.

ii) Find the bearing of Nemo from his Dad to the nearest degree. **4**

d) Solve $\sin 2\theta = \frac{9}{20}$ for $0° \le \theta \le 360°$ **3**

Question 6 (12 marks) **Start a new page**

a) Given that $\cos x = \sin(x + 25°)$, find the value of x for $0° \le x \le 90°$. **1**

b) Prove $(\text{cosec}\, x + \cot x)^2 = \dfrac{1 + \cos x}{1 - \cos x}$ **3**

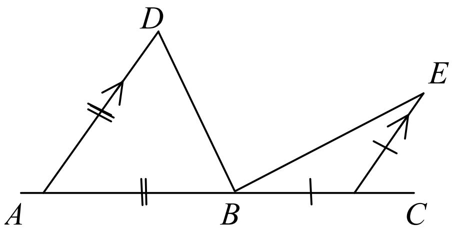
c) At what point(s) on the curve $y = x^3 - \frac{3}{2}x^2 - 12x - 1$ is the gradient of the tangent equal to – 6? **3**

d) Find $\displaystyle\lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1}$ **2**

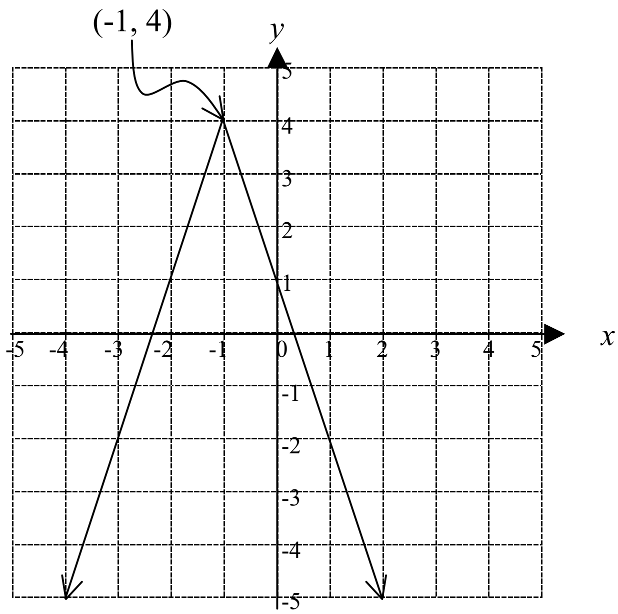
e) For the curve $y = ax^2 + bx + c$, where a, b and c are constants, it is given that $\dfrac{dy}{dx} = -y$ when $x = -2$. Show that b is equal to c. **3**

Question 7 (12 marks) **Start a new page**

a) In the diagram below, A, B and C are collinear. $AD = AB$, $BC = EC$ and AD _ EC. Prove that $\angle DBE$ is a right angle. **3**



b) Give the equation of the function whose graph appears below. **2**



c) For the function $f(x) = \left(x^2 - 5x\right)^2 - 36$:

i) Determine algebraically whether the function is odd, even or neither. **2**

ii) Factorise fully $f(x)$. **2**

iii) Evaluate $f(\frac{5}{2})$. **1**

iv) Neatly sketch $f(x)$ showing all key features and intercepts. **2**

End of paper

☺ Year 11 Preliminary Mathematics Examination 2007 – Solutions ☺

Question 1

a) $0\cdot234$ (3 sf)

b) $\frac{28\sqrt{7}}{7}$ $\left(=4\sqrt{7}\right)$

c) $= x^2(x+2)+(x+2)$

$= (x^2+1)(x+2)$

d) $= \frac{\sqrt{3}}{2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2}\left(\frac{1+\sqrt{2}}{2}\right)$

$= \frac{\sqrt{3}+\sqrt{6}}{4}$

e) $(m+1)(5m-3)=0$

$m=-1$ *or* $m=\frac{3}{5}$

f) $\frac{3}{x+3}-\frac{6x}{x^2-9}=\frac{3(x-3)-6x}{(x+3)(x-3)}$

$=\frac{-3x-9}{(x+3)(x-3)}$

$=\frac{-3}{x-3}$

Question 2

a) i) $\frac{dy}{dx}=\frac{3x^2}{3}=x^2$

ii) $y=x(x^2-2x+1)$

$=x^3-2x^2+x$

$\frac{dy}{dx}=3x^2-4x+1$

iii) $y=3x^2-4x$

$\frac{dy}{dx}=6x-4$

iv) $y=-\frac{3}{2}x^{\frac{2}{3}}$

$\frac{dy}{dx}=-\frac{3}{2}\times\frac{2}{3}x^{-\frac{1}{3}}$

$=-\frac{1}{\sqrt[3]{x}}$

b) $\frac{dy}{dx}=2ax-8$

when $x=2$,

$4=2(2)a-8$

$12=4a$

$\therefore a=3$

c) $\frac{dy}{dx}=2x-1$

When $x=1$,

$\frac{dy}{dx}=2(1)-1$

$=1$

So, gradient of tangent at $x=1$ is 1.

When $x=0$,

$\frac{dy}{dx}=2(0)-1$

$=-1$

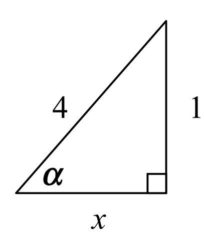
So, gradient of tangent at $x=0$ is -1.

In perpendicular lines, $m_1m_2=-1$

$1\times-1=-1$, $\therefore$ tangents at $x=1$ and $x=0$ are perpendicular.

d) $\text{cosec}\,\alpha=4$

$\sin\alpha=\frac{1}{4}$



Using Pythagoras,

$x^2+1^2=4^2$

$x=\sqrt{15}$

$\therefore\tan\alpha=\frac{1}{\sqrt{15}}$

Question 3

a) i) $m = \tan\theta$

$m = \tan 135°$

$= -1$

ii) $m = -1; \quad (\tfrac{2}{5}, 0)$

$y - y_1 = m(x - x_1)$

$y = -1(x - \tfrac{2}{5})$

$y = -x + \tfrac{2}{5}$

$5x + 5y - 2 = 0$

b) $x - 2y + 3 = 0; \quad (1, 7)$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|1(1) - 2(7) + 3|}{\sqrt{1^2 + (-2)^2}}$$

$$= \frac{|-10|}{\sqrt{5}}$$

$$= \frac{10\sqrt{5}}{5}$$

$$= 2\sqrt{5}$$

c) i) $-2 \le y < 0$

ii) $g(2) - 2g(0) + g(-3) = -\dfrac{1}{2} - 2\left(0^2 - 2\right) - 1$

$= 2\dfrac{1}{2}$

d) $x \ge 0; \quad y \ge x^2 - 2; \quad y \le x$

Question 4

a) $AG : AD \quad AB : AC \quad FE : FD$

b) ii) $\angle ADC = \angle ABC$ (opposite angles of a parallelogram are equal)

$\angle GDH = 180° - \angle ADC$ (straight angle HDC)

and $\angle FBE = 180° - \angle ABC$ (straight angle ABE)

$\therefore \angle GDH = \angle FBE$

In ΔBEF and ΔDHG:

$BE = DH$ (given)

$\angle FBE = \angle GDH$ (proven above)

$\angle BEF = \angle GHD$ (alternate angles are equal, $AB \parallel CD$)

$\therefore \Delta BEF \equiv \Delta DHG$ (AAS)

iii) $BF = GD$ (matching sides in congruent triangles, $\Delta BEF \equiv \Delta DHG$)

$BC = AD$ (opposite sides of a parallelogram are equal)

$BC \_ BF = AD \_ GD$

$\therefore CF = AG$

c) First, find the gradient of $2x - y + 5 = 0$:

$y = 2x + 5$

$\therefore m = 2$

In perpendicular lines, $m_1 m_2 = -1$, so gradient of line $ax + by - 3 = 0$ must be $-\tfrac{1}{2}$.

Using the point (1, 1):

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\tfrac{1}{2}(x - 1)$$
$$2y - 2 = -x + 1 \qquad \therefore a = 1;\ b = 2.$$
$$x + 2y - 3 = 0$$

d) Method 1: Using the two-point formula:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{y - (2k+1)}{x - (2k-1)} = \frac{k+1-(2k+1)}{k-1-(2k-1)}$$
$$\frac{y - 2k - 1}{x - 2k + 1} = \frac{-k}{-k}$$
$$= 1$$
$$y - 2k - 1 = x - 2k + 1$$
$$y = x + 2$$

Method 2: Using point and gradient:

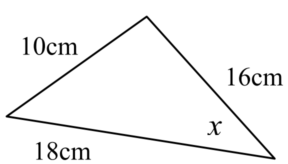
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{k+1-(2k+1)}{k-1-(2k-1)}$$
$$= \frac{-k}{-k}$$
$$= 1$$

Then, using $B(k-1,\ k+1)$:

$$y - y_1 = m(x - x_1)$$
$$y - (k+1) = x - (k-1)$$
$$y - k - 1 = x - k + 1$$
$$y = x + 2$$

Question 5

a) The smallest angle of a triangle is opposite the shortest side.

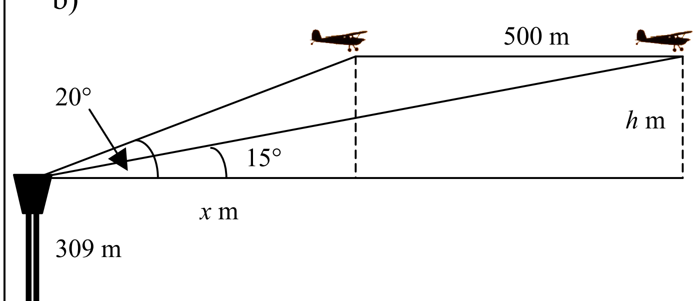


Using cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos x = \frac{18^2 + 16^2 - 10^2}{2 \times 18 \times 16}$$
$$= \frac{480}{576}$$
$$x = 33 \cdot 56°$$

b)



Using right angled triangles,

$$\tan 20° = \frac{h}{x} \quad \text{and} \quad \tan 15° = \frac{h}{500 + x}$$

$$\therefore x = \frac{h}{\tan 20°} \qquad \therefore x = \frac{h - 500\tan 15°}{\tan 15°}$$

Eliminating x,

$$\frac{h}{\tan 20°} = \frac{h - 500\tan 15°}{\tan 15°}$$

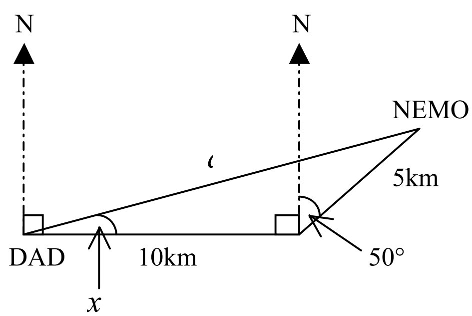
$$h\tan 15° = h\tan 20° - 500\tan 15°\tan 20°$$

$$500\tan 15°\tan 20° = h(\tan 20° - \tan 15°)$$

$$h = \frac{500\tan 15°\tan 20°}{\tan 20° - \tan 15°}$$
$$= 507 \cdot 834160.....$$

Altitude of plane = $h + 309$
= 817m (nearest m)

c)



i) Using cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\therefore d^2 = 10^2 + 5^2 - 2\times 10\times 5\times\cos 140°$$
$$= 125 + 76 \cdot 604444...$$
$$= 201 \cdot 604444...$$
$$d = 14 \cdot 198747.....$$
= 14km (nearest km)

ii) Using sine rule:

$$\frac{\sin x}{5} = \frac{\sin 140°}{14 \cdot 198747...}$$

$$\sin x = \frac{5\sin 140°}{14 \cdot 198747...}$$
$$= 0 \cdot 226353...$$
$$x = 13 \cdot 082488...$$
$$= 13° \text{ (nearest deg)}$$

Bearing of Nemo from Dad is
(90 _ 13)°T = 077°T.

d) Let $u = 2\theta$ $\qquad 0° \le 2\theta \le 720°$

$\therefore \sin u = \frac{9}{20}$

$u = 26 \cdot 74368...°$ *or* $153 \cdot 25631...°$
or $386 \cdot 74368°...$ *or* $513 \cdot 25631...°$

$\therefore 2\theta = 26 \cdot 74368...°$ *or* $153 \cdot 25631...°$
or $386 \cdot 74368°...$ *or* $513 \cdot 25631...°$

$\theta = 13 \cdot 37°,\ 76 \cdot 63°, 193 \cdot 37°,$
$256 \cdot 63° \quad (2dp)$

Question 6

a) $\cos a = \sin(90° - a)$

$$\therefore x = 90° - (x + 25°)$$
$$x = 90° - x - 25°$$
$$2x = 65°$$
$$x = 32 \cdot 5°$$

b) LHS = $(\text{cosec } x + \cot x)^2$
$= \text{cosec}^2 x + 2\,\text{cosec}\,x\cot x + \cot^2 x$

$$= \frac{1}{\sin^2 x} + 2\times\frac{1}{\sin x}\times\frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{1+2\cos x+\cos^2 x}{\sin^2 x}$$

$$= \frac{(1+\cos x)^2}{1-\cos^2 x}$$

$$= \frac{(1+\cos x)^2}{(1+\cos x)(1-\cos x)}$$

$$= \frac{1+\cos x}{1-\cos x}$$

= RHS

c) $y = x^3 - \frac{3}{2}x^2 - 12x - 1$

$$\frac{dy}{dx} = 3x^2 - 3x - 12$$

If m = _6:

$-6 = 3x^2 - 3x - 12$

$0 = 3x^2 - 3x - 6$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$\therefore x = 2 \quad or \quad x = -1$

When $x = 2$, $y = 2^3 - \frac{3}{2}(2)^2 - 12(2) - 1$

$= -23$

When $x = -1$, $y = (-1)^3 - \frac{3}{2}x(-1)^2 - 12(-1) - 1$

$= 8\cdot 5$

$\therefore (2, -23)$ and $(-1, 8\cdot 5)$ are the two points.

d) $$\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$$

$$= \lim_{x\to 1} \frac{(x-1)(x+3)}{x-1}$$

$$= \lim_{x\to 1} x+3$$

= 1+3

= 4

e) $y = ax^2 + bx + c$

when x = _2, $y = a(-2)^2 + b(-2) + c$

$= 4a - 2b + c$

$\therefore -y = -4a + 2b - c$

$$\frac{dy}{dx} = 2ax + b$$

when x = _2, $\frac{dy}{dx} = 2a(-2) + b$

$= -4a + b$

when x = _2, $\frac{dy}{dx} = -y$

$-4a + b = -4a + 2b - c$

$-b = -c$

$\therefore b = c$

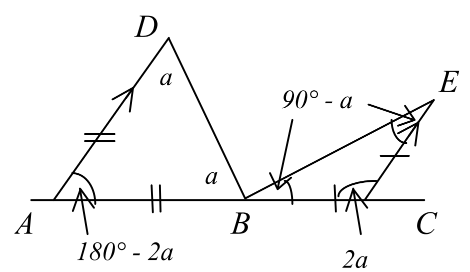
Question 7

a) Let $\angle ADB = a$

$\angle DBA = a$ (equal angles opposite equal sides)
$\angle DAB = 180° - 2a$ (angle sum ΔADB)

$\angle DAB + \angle ECB = 180°$ (cointerior $\angle s$ are supplementary, $AD \parallel EC$)
$\therefore \angle ECB = 2a$
$\angle EBC = \angle BEC$ (equal angles opposite equal sides)
$\angle EBC = \dfrac{180° - 2a}{2}$ (angle sum ΔBEC)
$= 90° - a$

$\angle DBA + \angle DBE + \angle EBC = 180°$ (straight angle ABC)
$\angle DBE = 180° - a - (90° - a)$
$= 90°$
$\therefore \angle DBE$ is a right angle.



b) $f(x) = 4 - 3|x+1|$

c) i) $f(a) = (a^2 - 5a)^2 - 36$
$f(-a) = \left[(-a)^2 - 5(-a)\right]^2 - 36$
$= (a^2 + 5a)^2 - 36$
$f(a) \neq f(-a) \text{ or } -f(-a)$

$\therefore$ function is neither odd nor even.

ii) $(x^2 - 5x)^2 - 36$
$= (x^2 - 5x)^2 - 6^2$
$= (x^2 - 5x - 6)(x^2 - 5x + 6)$
$= (x-6)(x+1)(x-3)(x-2)$

iii) $f(\frac{5}{2}) = \left[(\frac{5}{2})^2 - 5(\frac{5}{2})\right]^2 - 36$
$= (\frac{25}{4} - \frac{25}{2})^2 - 36$
$= \frac{49}{16}$

iv) Note: function is not even, but is symmetrical about $x = \frac{5}{2}$.