NORTH SYDNEY GIRLS HIGH SCHOOL

2008
YEARLY EXAMINATION

Preliminary Mathematics

General Instructions
- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Total Marks – 84
Attempt Questions 1–8

At the end of the examination, place your answer sheets in order and put this question paper on top. Submit one bundle. The bundle will be separated for marking so please ensure your name is written on EVERY PAGE.

Student Name: _____ **Teacher:**_____

	1a-g	1hi	2	3abcd	3e	4abc	4de	5	6a	6b	6cd	7a	7bc	7d	8a	8bc	Totals
P2														/3			/3
P3	/8					/4				/2							/14
P4		/4			/2		/6	/10			/6				/2		/30
P5			/10	/10					/2								/22
P6																/8	/8
P7													/6				/6
P8												/1					/1
	/12		/10		/12		/10		/10		/10		/10		/12		/84

Question 1 (12 Marks) **Marks**

(a) Evaluate $3-|4-7|$ **1**

(b) Simplify $\frac{2}{x}+\frac{5}{2x}$ **1**

(c) Express $5m^{-2}n^3$ without negative indices. **1**

(d) Rationalise the denominator of $\frac{2}{\sqrt{5}}$ **1**

(e) My electricity bill this quarter is $276.25. This represents an 7% increase on my bill for the same period of time last year. What was my electricity bill last year? **2**

(f) Given that $3+4\sqrt{5}=a+\sqrt{b}$, find integer values for a and b. **2**

(g) Solve $|x-4|>9$ **2**

(h) Simplify $\frac{a^3-b^3}{a-b}$ **2**

Question 2 (10 Marks) **Start a new page** **Marks**

(a) Given that $f(x) = 2x + 4$,

(i) Find a if $f(a) = -2$ **2**

(ii) Find $\dfrac{f(2x)}{4}$ **2**

(b) Sketch the following graphs on separate number planes, showing all key features.

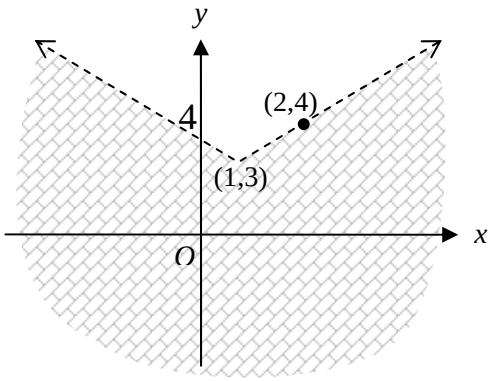
(i) $y = |x| + 3$ **2**

(ii) $y = (x-4)^2$ **2**

(iii) $y = -2^x$ **2**

Question 3 (12 Marks) **Start a new page**

(a) Write an inequality to describe the shaded region. **3**



(b) It is given that $f(x) = ax^3 + bx$ and that $f(-1) = -11$. **3**
State the numerical coordinates of three points that the graph of $y = f(x)$ passes through.

(c) What is the domain of $f(x) = \dfrac{2}{x}$? **1**

(d) What is the domain and range of $g(x) = \sqrt{4-x}$? **3**

(e) Solve $x^2 - 3 \le 2x$ **2**

Question 4 (10 Marks) **Start a new page** **Marks**

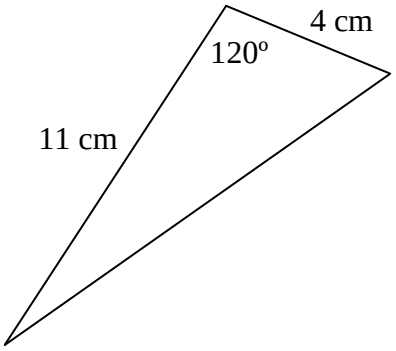
(a) Write the value of $\dfrac{16\sin 11^\circ}{\sin 165^\circ}$ correct to 3 decimal places. **1**

(b) State the exact value of $\operatorname{cosec} 225^\circ$. **1**

(c) Simplify $\dfrac{\tan x}{\sin x}$ **2**

(d) Solve $\sin 2\theta = \dfrac{1}{2}$ if $0^\circ \le \theta \le 360^\circ$ **3**

(e) Find the area of this triangle, giving your answer as an exact value. **3**



Question 5 (10 Marks) **Start a new page**

(a) What is the x-intercept of the line $y = 4x + 8$? **1**

(b) P is the point $(4, 5)$ and Q is the point $(14, -1)$

(i) Find the midpoint of PQ **2**

(ii) Find the gradient of PQ **2**

(iii) Find the equation of a line that is perpendicular to PQ and passes through Q. **2**

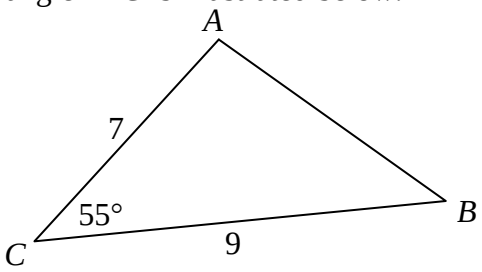
(c) Find the angle of inclination of the line $3x - 4y = -5$ to the positive direction of the x-axis. **3**

Question 6 (10 marks) **Start a new page** **Marks**

(a) Show that $f(x) = 6x^3 + 2x$ is an odd function **2**

(b) Show that $\dfrac{3\sin^2 x - 5}{3\cos^2 x + 2} = -1$ **2**

(c) The triangle *ABC* is illustrated below.



(i) Find *AB* correct to 2 decimal places. **2**

(ii) Find $\angle ABC$ to the nearest minute. **2**

(d) Find the perpendicular distance from the point $(1,3)$ to the line $3x - 4y + 5 = 0$ **2**

Question 7 (10 marks) **Start a new page**

(a) Evaluate: $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ **1**

(b) Differentiate:

(i) $4x^3$ **1**

(ii) $(2x + 6)^7$ **1**

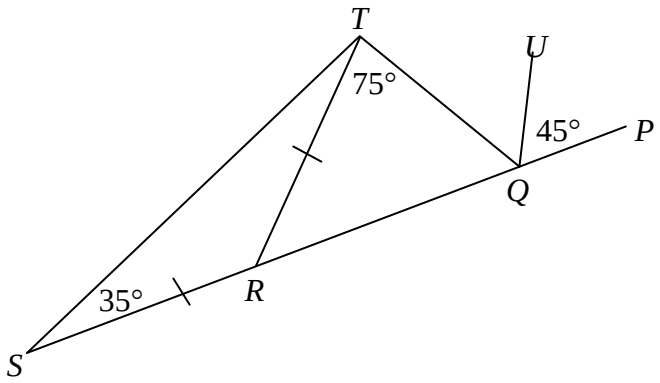
(iii) $\dfrac{1}{\sqrt{x-1}}$ **2**

(c) (i) Express $\dfrac{x^2 + 3x^3}{x^5}$ as a sum of two fractions **1**

(ii) Hence differentiate $\dfrac{x^2 + 3x^3}{x^5}$ **1**

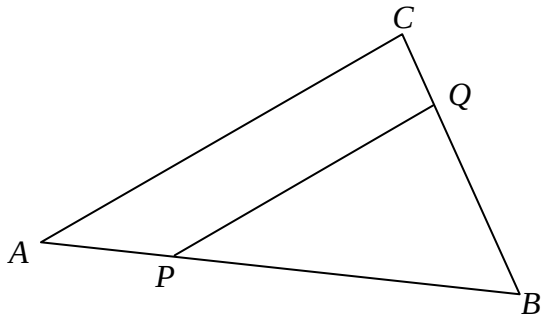
Question 7 continues on page 6

(d) In the diagram below $TR = RS$, $\angle TSP = 35°$, $\angle UQP = 45°$ and $\angle RTQ = 75°$. **3**
Find the size of $\angle UQT$, giving reasons.



Question 8 (10 marks) **Start a new page**

(a) In the diagram below, you are given that $\Delta ABC \,|||\, \Delta PBQ$ and $CQ : QB = 1 : 3$.



(i) Find the value of $PQ : AC$. **1**

(ii) Given that $AB = 12$, find PB. **1**

(b) (i) Show that the equation of the normal to $y = \dfrac{x}{3+x}$ at the point $(2, 0{\cdot}4)$ is $125x + 15y - 256 = 0$. **3**

(ii) This normal meets the x-axis at T. Find the co-ordinates of T. **1**

(c) (i) Find the equation of the tangent to $y = x^4$ at the point where $x = 2$. **3**

(ii) Explain how your answer to part (i) shows that $x^4 \geq 32x - 48$ for all real x. **1**

End of Paper

Solutions 2008 Year 11 2U T3
Question 1 (12 Marks)

(a) 0

(b)

$$\frac{2}{x}+\frac{5}{2x}$$
$$=\frac{4}{2x}+\frac{5}{2x}$$
$$=\frac{9}{2x}$$

(c) $\dfrac{5n^3}{m^2}$

(d) $\dfrac{2\sqrt{5}}{5}$

(e) Let x be the cost last year
$1.07x = 276.25$
$x = \dfrac{276.25}{1.07}$
The cost was \$258.20 to the nearest 5c.

(f)
$3+4\sqrt{5} = a+\sqrt{b}$
$3+\sqrt{5\times 16} = a+\sqrt{b}$
Equating rational and irrational parts
$a=3, b=80$

(g) $x<-5, x>13$

(h)
$$\frac{a^3-b^3}{a-b}$$
$$=\frac{(a-b)(a^2+ab+b^2)}{a-b}$$
$$=a^2+ab+b^2$$

Question 2 (10 Marks)

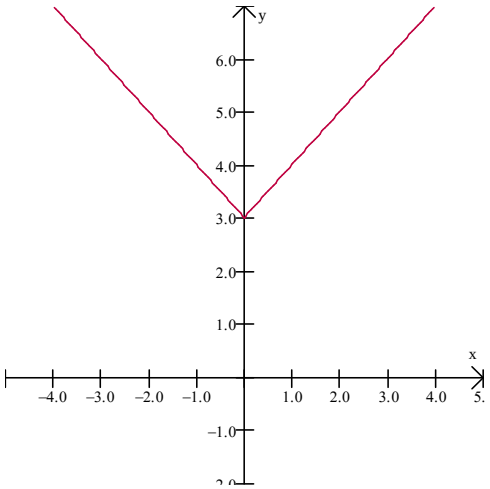
(a) Given that $f(x)=2x+4$,

(i)
$2a+4=-2$
$a=-3$

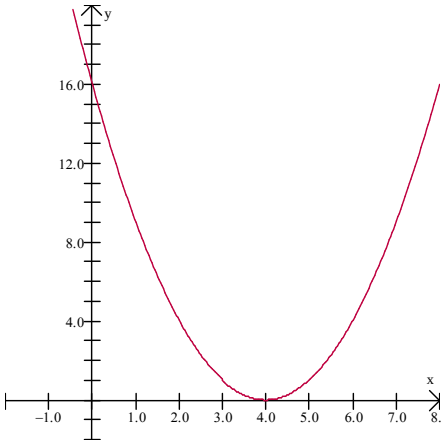
(ii)
$$\frac{f(2x)}{4}$$
$$=\frac{2(2x)+4}{4}$$
$$=\frac{4(x+1)}{4}$$
$$=x+1$$

(b)

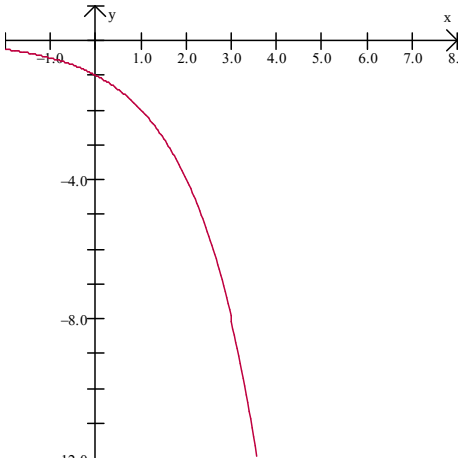
(i) $y=|x|+3$



(ii) $y=(x-4)^2$



(iii) $y=-2^x$



Question 3 (12 Marks)

(a) The boundary is $y = |x-1|+3$

The region is $y < |x-1|+3$

Or $y < x+2$ or $y < 4-x$

(b) (0,0) (-1, -11) and (1,11)

(c) All real x except $x = 0$

(d)

Domain: $4 - x \geq 0$

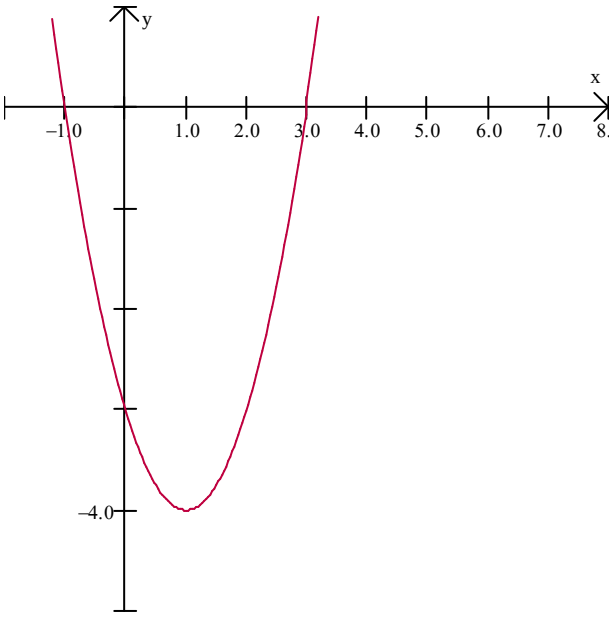
$x \leq 4$

Range: $y \geq 0$

(e)

$x^2 - 2x - 3 \leq 0$

Let $y = x^2 - 2x - 3$



From the graph $y \leq 0$ when $-1 \leq x \leq 3$

Question 4 (10 Marks)

(a) 11.796

(b) $\text{cosec}(225°) = -\dfrac{1}{\sin(45°)}$

$= -\sqrt{2}$

(c) $\dfrac{\tan x}{\sin x}$

$= \dfrac{\sin x}{\cos x} \times \dfrac{1}{\sin x}$

$= \dfrac{1}{\cos x}$

$= \sec x$

(d)

$0 \leq \theta \leq 360$

$0 \leq 2\theta \leq 720$

Sine is positive in quadrants 1 and 2

$2\theta = 30, 150, 390, 510$

$\theta = 15, 75, 195, 255$

(e) Find the area of this triangle, giving your answer as an exact value.

$A = \dfrac{1}{2}ab\sin C$

$= \dfrac{1}{2} \times 4 \times 11 \times \sin(120°)$

$\dfrac{1}{2} \times 4 \times 11 \times \dfrac{\sqrt{3}}{2}$

$= 11\sqrt{3}$

The area is $11\sqrt{3}$ square units

Question 5 (10 Marks)

(a)

$4x+8=0$

$x=-2$

(b)

(i) The midpoint is $\left(\frac{4+14}{2},\frac{5+-1}{2}\right)$ ie.

$(9,2)$

(ii)

$m=\frac{-1-5}{14-4}$

$=-\frac{3}{5}$

(iii)

$m_2=\frac{5}{3}$

$y--1=\frac{5}{3}(x-14)$

$5x-3y-73=0$

(c)

$y=\frac{3x+5}{4}$

$m=\frac{3}{4}$

$\tan\theta=\frac{3}{4}$

$\theta=36°52'$

Question 6 (10 marks)

(a)

$f(-x)=6(-x)^3+2(-x)$

$=-6x^3-2x$

$=-\left(6x^3-2x\right)$

$=-f(x)$

So $f(x)=6x^3+2x$ is an odd function

(b)

$LHS=\frac{3\sin^2 x-5}{3\cos^2 x+2}$

$=\frac{3\left(1-\cos^2 x\right)-5}{3\cos^2 x+2}$

$=\frac{3-3\cos^2 x-5}{3\cos^2 x+2}$

$=\frac{-\left(3\cos^2 x+2\right)}{3\cos^2 x+2}$

$=-1$

$=RHS$

So $\frac{3\sin^2 x-5}{3\cos^2 x+2}=-1$

(c)

(i)

$AB^2=7^2+9^2-2\times7\times9\times\cos55°$

$AB=7.60\,(correct\ to\ 2dp)$

(ii)

$\cos\angle ABC=\frac{9^2+(AB)^2-7^2}{2\times9\times AB}$

$\angle ABC=49°0'$ (correct to nearest minute)

(d)

$d=\frac{|3(1)-4(3)+5|}{\sqrt{3^2+4^2}}$

$=\frac{4}{5}$

The distance is $\frac{4}{5}$ units.

Question 7 (10 marks)

(a)

$$\frac{x^2-9}{x-3}$$
$$=\frac{(x-3)(x+3)}{x-3}$$
$$=x+3 \text{ provided } x\neq 3$$

So

$$\lim_{x\to 3}\frac{x^2-9}{x-3}$$
$$=\lim_{x\to 3} x+3$$
$$=3+3$$
$$=6$$

The limit is 6

(b)

(i)

$$\frac{d\left(4x^3\right)}{dx}$$
$$=12x^2$$

(ii)

$$\frac{d\left((2x+6)^7\right)}{dx}$$
$$=7(2x+6)^6\times 2$$
$$=14(2x+6)^6$$

(iii)

$$\frac{d\left((x-1)^{-\frac{1}{2}}\right)}{dx}$$
$$=-\frac{1}{2}(x-1)^{-\frac{3}{2}}$$
$$=-\frac{1}{2\sqrt{(x-1)^3}}$$

(c)

(i)

$$\frac{x^2+3x^3}{x^5}=\frac{1}{x^3}+\frac{3}{x^2}$$

(ii)

$$\frac{d\left(x^{-3}+3x^{-2}\right)}{dx}$$
$$=-3x^{-4}-6x^{-3}$$
$$=-\frac{3}{x^4}-\frac{6}{x^3}$$

(d)

$$\angle STR=\angle TSR \text{ (opposite equal sides)}$$
$$=35$$
$$\angle TQP=35+(35+75) \text{ (exterior angle of } \Delta TQS)$$
$$=145$$
$$\angle TQU=145-45 \text{ (adjacent angles)}$$
$$=100$$

Question 8 (10 marks)

(a)

$$PQ:AC=BQ:BC \text{ (corr.sides , } \Delta ABC|||\Delta PBQ)$$
$$=BQ:BQ+QC$$
$$=3:3+1$$
$$=3:4$$

(ii)

$$\frac{PB}{12}=\frac{3}{4} \text{ (corresponding sides, } \Delta ABC|||\Delta PBQ)$$
$$PB=\frac{3}{4}\times 12$$
$$=9$$

(b)(i) $$y'=\frac{(3+x)(1)-x(1)}{(3+x)^2}$$
$$=\frac{3}{(3+x)^2}$$
$$=\frac{3}{(3+2)^2} \text{ when } x=2$$
$$=\frac{3}{25}$$

So the gradient of the normal is $-\frac{25}{3}$

$$y-0\cdot 4=-\frac{25}{3}(x-2)$$
$$3y-1\cdot 2=-25x+50$$
$$250x+30y-512=0$$
$$125x+15y-256=0$$

(ii) This line cuts the x-axis when $y = 0$

$$125x + 15(0) - 256 = 0$$

$$125x = 256$$

$$x = \frac{256}{125}$$

$$\therefore \quad T = \left(\frac{256}{125}, 0\right)$$

(c)

$$y' = 4x^3$$

$$= 4(2)^3$$

$$= 32$$

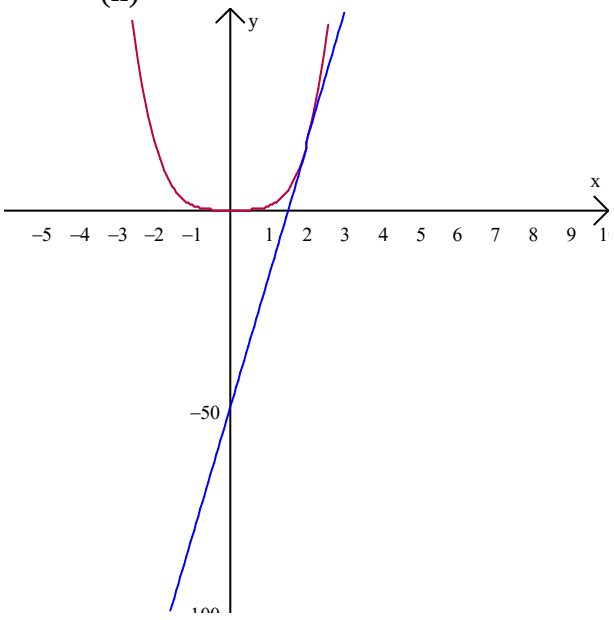
When $x = 2$, $y = 2^4$

So the equation is

$$y - 16 = 32(x - 2)$$

$$y = 32x - 48$$

(ii)



The diagram shows the graph of $y = x^4$ with the tangent $y = 32x - 48$ drawn at $x = 2$. The domain of both functions is all real x.

Note that $y = x^4$ is never below the tangent $y = 32x - 48$. This means that for any given x-coordinate, the corresponding y-coordinate of $y = x^4$ is greater than or equal to the corresponding y-coordinate of $y = 32x - 48$.

i.e. $x^4 \geq 32x - 48$ for all real x.