NORTH SYDNEY GIRLS HIGH SCHOOL

2009

YEAR 11 YEARLY EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7
All questions are of equal value.

At the end of the examination, place your solution booklets in order and put this question paper on top.
Submit one bundle.

Student Number: ______________________ **Teacher:** ______________________

Student Name: ______________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Begin each question in a SEPARATE writing booklet.

Question 1 (12 marks) **Marks**

a) Express $\dfrac{\sqrt{3}}{7-4\sqrt{3}}$ in the form $a+b\sqrt{3}$. 2

b) Simplify $\dfrac{4x^2+8xy+4y^2}{x^2-y^2}$ 2

c) Find the value of $5\left[0\cdot 32+(1\cdot 12)^{\frac{4}{3}}\right]$ correct to 3 significant figures. 2

d) Solve $2x^2+5x-3>0$. 2

e) Express $9^n \times 81 \times 3^n$ in the form 3^x. 2

f) Solve $|2x-3| \leq 4$ 2

Question 2 (12 Marks) **Start a new booklet.** **Marks**

a) Given that the equation of a line l is $3x-2y+7=0$:

i) Find the obtuse angle that the line makes with the x axis, correct to the nearest minute. 2

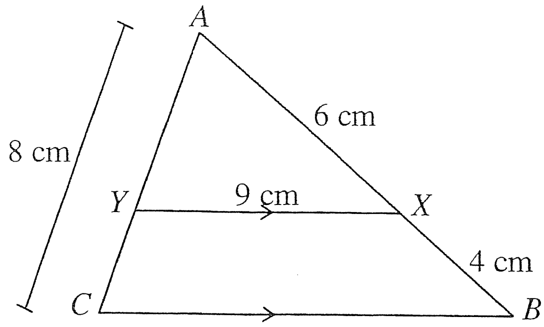
ii) Show that the perpendicular distance from the line to the point $(4,-2)$ is $\dfrac{23\sqrt{13}}{13}$ units. 2

b) i) Sketch the function $y=(x-2)(x+1)$ showing x and y intercepts. 2

ii) Hence sketch the graph of $y=|(x-2)(x+1)|$. 1

Question 2 continued over the page....

c) In the diagram below, $XY \parallel BC$.



i) Prove that triangles AXY and ABC are similar. 3

ii) Hence, or otherwise, find the length of YC. 2

Question 3 (12 Marks) **Start a new booklet.** **Marks**

a) Differentiate with respect to x, giving your answers in simplest form.

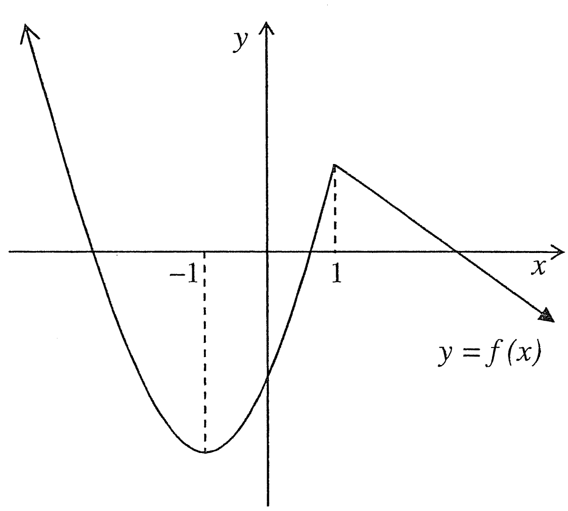
i) $y = 3x^2 - \dfrac{2}{x}$ 2

ii) $y = (6x+1)^{\frac{3}{2}}$ 2

iii) $y = x^2\sqrt{x+1}$ 2

b) Find the equation of the normal to the curve $y = x^3 - 4x - 8$ at the point $(2, -8)$. Give your answer in general form. 3

c) The diagram below shows the graph of $y = f(x)$. Sketch a possible graph of $y = f'(x)$. 3



Question 4 (12 Marks) **Start a new booklet.** **Marks**

a) Evaluate in exact form $\dfrac{\tan 120°}{\cos^2 120°}$ 2

b) Solve $2\sin 2\alpha = \sqrt{3}$ in the domain $0° \le \alpha° \le 360°$. 3

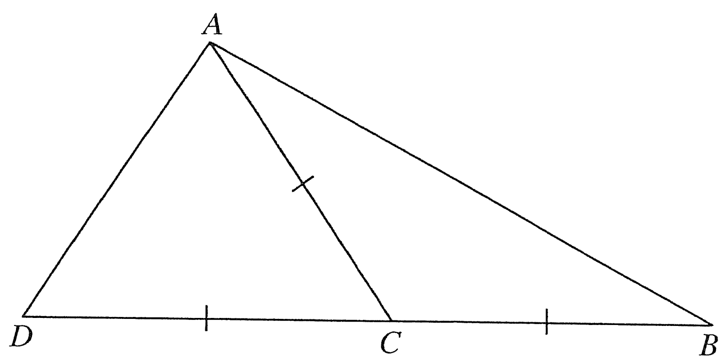
c) If $\sin\beta = \dfrac{3}{4}$ and β is obtuse, find the exact value of $\tan\beta$. 2

d) State the domain and range of the function $f(x) = \dfrac{1}{\sqrt{x-2}}$. 2

e) Sketch the graphs of $y \le x+1$ and $xy \le 4$ on the same set of coordinate axes, shading the region(s) where both inequalities hold true. (You do not need to find the coordinates of the points of intersection.) 3

Question 5 (12 Marks) **Start a new booklet.** **Marks**

a) In the diagram below, $AC = DC = CB$. Show that $\angle DAB = 90°$, giving reasons. 3



b) Show that the points $D(6a, -2b)$, $E(2a, 0)$ and $F(0, b)$ are collinear. 2

c) Find the length of the interval joining $A(4a, -2a)$ and $B(a, a)$ in simplest surd form. 2

d) Given that a circle has the equation $x^2 - 4x + y^2 + 6y - 36 = 0$, find the centre and radius of the circle. 2

e) Differentiate $f(x) = 3x^2 - 5$ from first principles. 3

Question 6 (12 Marks) **Start a new booklet.** **Marks**

a) Determine whether the function $f(x) = x^2 - 4x + 5$ is odd, even or neither. 1

b) For what values of k is the parabola $y = x^2 + (k-3)x + k$ positive definite? 2

c) If one root of $3x^2 - 8x + d = 0$ is three times the other root, find d. 2

d) Form a quadratic equation whose roots are $(3+\sqrt{2})$ and $(3-\sqrt{2})$. 2

e) A boat sails 10 km from a Marina on a bearing of S30°E. It then sails 15 km on a bearing of N20°E before dropping anchor.

i) Draw a diagram showing all key information about the boat's journey. 1

ii) How far is the boat from the Marina? Give your answer correct to the nearest metre. 2

iii) What is the bearing of the Marina from the boat where it is anchored? 2

Question 7 (12 Marks) **Start a new booklet.** **Marks**

a) Sketch the following piecemeal function in the domain $-2 \le x \le 5$ 4

$$f(x) = \begin{cases} -x^2+2 & \text{for } x < 1 \\ 3 & \text{for } 1 \le x \le 3 \\ 6-x & \text{for } x > 3 \end{cases}$$

b) Find a value of x if $\operatorname{cosec}(x-25)^\circ = \sec 65^\circ$. 1

c) Simplify $\dfrac{\sec\theta}{\tan\theta + \cot\theta}$ 2

d) Evaluate $\lim\limits_{x\to\infty} \dfrac{4x^2 - 2x^3 + 1}{7x^3 - 6x}$ 1

e) i) Show that the derivative of $y = \dfrac{x^3}{(x^2+1)^2}$ is $y' = \dfrac{x^2(3-x^2)}{(x^2+1)^3}$ 3

ii) For which value(s) of x is the slope of this curve positive? 2

End of paper

SOLUTIONS

YEAR 11 MATHEMATICS YEARLY EXAM 2009

Question 1

a) $\dfrac{\sqrt{3}}{7-4\sqrt{3}} \times \dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}$

$= \dfrac{7\sqrt{3}+12}{49-48}$

$= 7\sqrt{3}+12$

b) $\dfrac{4(x^2+2xy+y^2)}{x^2-y^2}$

$= \dfrac{4(x+y)^2}{(x+y)(x-y)}$

$= \dfrac{4(x+y)}{x-y}$

c) $= 7.41559\ldots$

$= 7.42 \ (3sf)$

d) $2x^2+5x-3$ $\quad x-6$

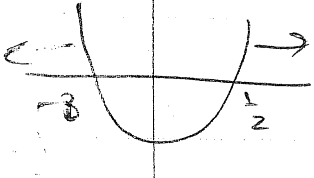
$= 2x^2+6x-x-3$ $\quad +5$

$= 2x(x+3)-(x+3)$ $\quad (6,-1)$

$= (2x-1)(x+3)$

$\therefore (2x-1)(x+3)>0$

$x=\frac{1}{2}, -3$



$\therefore x<-3,$

$x>\frac{1}{2}$

e) $9^n \times 81 \times 3^n$

$= (3^2)^n \times 3^4 \times 3^n$

$= 3^{2n} \times 3^4 \times 3^n$

$= 3^{3n+4}$

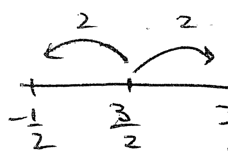
f) $|2x-3| \leq 4$

$-4 \leq 2x-3 \leq 4$

$-1 \leq 2x \leq 7$

$-\frac{1}{2} \leq x \leq \frac{7}{2}$

OR $|x-\frac{3}{2}| \leq 2$

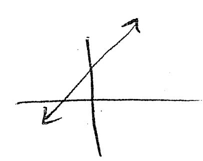


$-\frac{1}{2} \leq x \leq \frac{7}{2}$

Question 2

a) $3x-2y+7=0$

$y=\frac{3}{2}x+7/2$



i) $m=\tan\theta$

$\frac{3}{2}=\tan\theta$

$\theta = 56°19'$

$\therefore$ obtuse $\angle = 180° - 56°19'$

$= 123°41'$

ii) $a=3 \quad b=-2 \quad c=7$

$x_1=4 \quad y_1=-2$

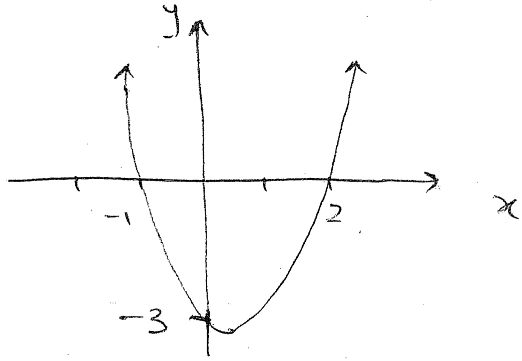
$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$=\frac{|3(4)+(-2)(-2)+7|}{\sqrt{3^2+(-2)^2}}$$

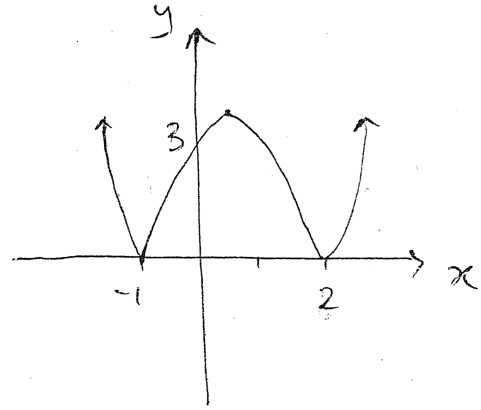
$$=\frac{|23|}{\sqrt{13}}$$

$$=\frac{23\sqrt{13}}{13} \text{ units.}$$

b) i)



ii)



c) i) In ΔAXY & ΔABC:

$\angle YAX$ is common

$\angle AYX = \angle ACB$ (corresponding $\angle$'s, $YX \parallel CB$)

$\therefore \Delta AXY \parallel\!\mid \Delta ABC$ (equiangular)

ii) $\frac{AY}{AC}=\frac{AX}{AB}$ (corresponding sides in similar Δs)

$\frac{AY}{8}=\frac{6}{10}$

$AY=4.8$

$YC = AC-AY$

$= 8-4.8$

$= 3.2\text{cm}$.

Question 3

a) i) $y=3x^2-2x^{-1}$

$\frac{dy}{dx}=6x+2x^{-2}$

$=6x+\frac{2}{x^2}$

ii) $y=(6x+1)^{3/2}$

$\frac{dy}{dx}=\frac{3}{2}(6x+1)^{1/2}\times 6$

$=\frac{18}{2}(6x+1)^{1/2}$

iii) $y=x^2(x+1)^{1/2}$

$u=x^2 \qquad v=(x+1)^{\frac{1}{2}}$

$u'=2x \qquad v'=\frac{1}{2}(x+1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = uv' + vu'$$
$$= x^2\left(\tfrac{1}{2}\right)(x+1)^{-\frac{1}{2}} + 2x(x+1)^{\frac{1}{2}}$$
$$= \tfrac{1}{2}x(x+1)^{-\frac{1}{2}}\left[x + 4(x+1)\right]$$
$$= \tfrac{1}{2}x(x+1)^{-\frac{1}{2}}(5x+1)$$

b) $y = x^3 - 4x - 8$

$$\frac{dy}{dx} = 3x^2 - 4$$

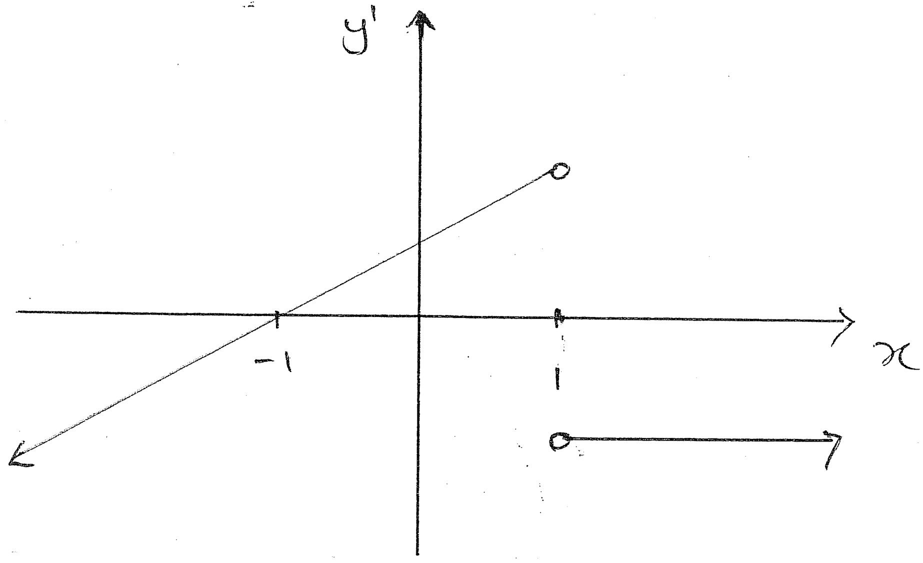
m of tangent at $x=2$ is $3(2)^2 - 4$
$= 8$

$\therefore$ m of normal is $-\frac{1}{8}$.

$\therefore$ $y - y_1 = m(x - x_1)$
$y + 8 = -\frac{1}{8}(x-2)$

$8y + 64 = -x + 2$
$\underline{x + 8y + 62 = 0}$.

c)



Question 4

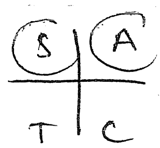
a) $\dfrac{\tan 120^\circ}{\cos^2 120^\circ} = \dfrac{-\sqrt{3}}{\left(-\frac{1}{2}\right)^2}$

$= \dfrac{-\sqrt{3}}{\frac{1}{4}}$

$= -4\sqrt{3}$

b) $2\sin 2\alpha = \sqrt{3}$

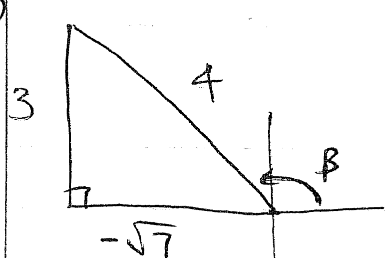
$\sin 2\alpha = \dfrac{\sqrt{3}}{2}$



$2\alpha = 60^\circ, 120^\circ, 420^\circ, 480^\circ$
(note $0^\circ \leq 2\alpha^\circ \leq 720^\circ$)

$\therefore \alpha = 30^\circ, 60^\circ, 210^\circ, 240^\circ$.

c)



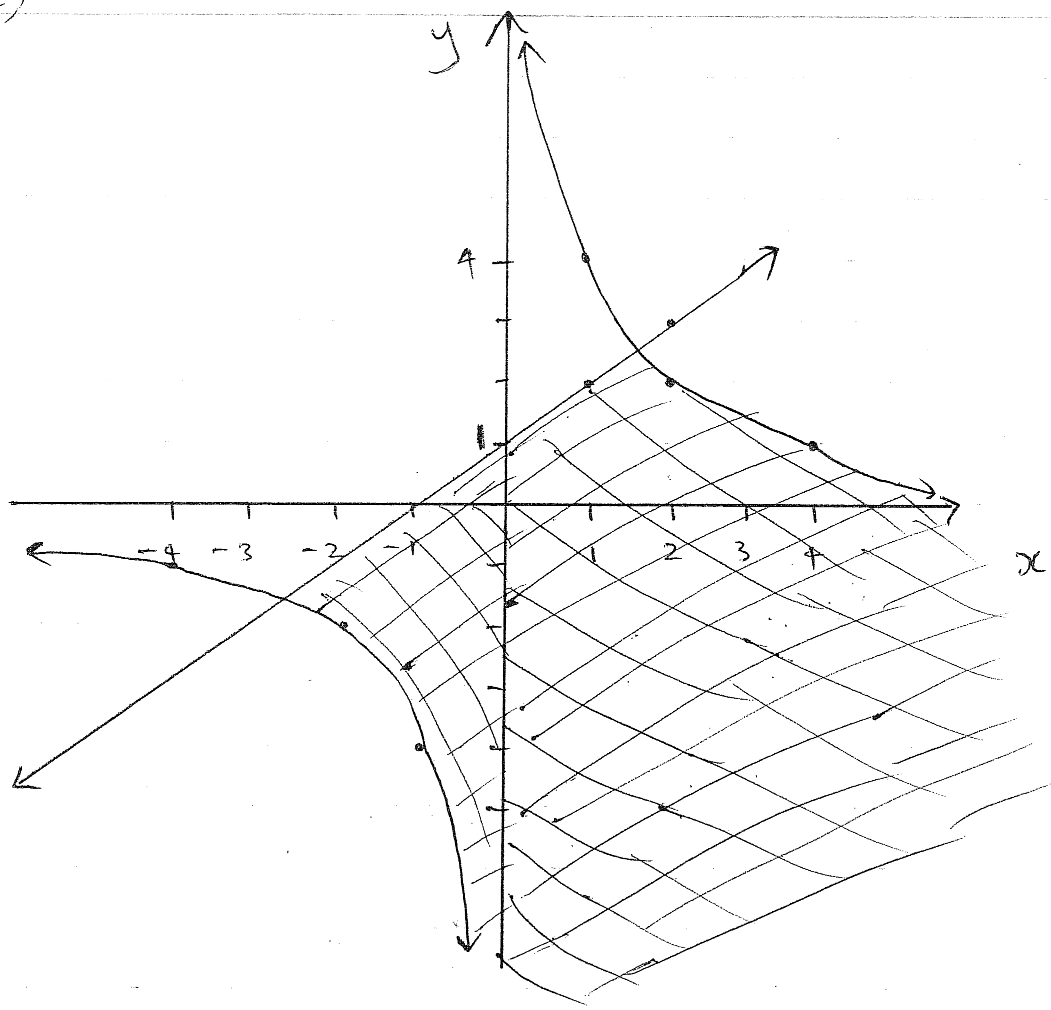
$\therefore \tan\beta = \dfrac{-3}{\sqrt{7}}$

d) $x - 2 > 0$
$\therefore x > 2$ is the domain

since $\sqrt{x-2}$ is always +ve,
So is $f(x) = \dfrac{1}{\sqrt{x-2}}$

$\therefore y > 0$ is the range.

e)



Question 5

a) let $\angle CDA = x$

$\therefore \angle DAC = x$ (equal $\angle$s opp equal sides)

$\therefore \angle BCA = 2x$ (exterior $\angle$ of ΔADC)

$\angle CAB = \angle CBA$ (equal $\angle$s opp equal sides)

$\therefore 2\times\angle CAB + 2x = 180^\circ$ ($\angle$ sum ΔCAB)

$$\therefore \angle CAB = \frac{180^\circ - 2x}{2}$$

$$= 90^\circ - x$$

$\angle DAB = \angle DAC + \angle CAB$ (adjacent $\angle$s)

$$= x + 90^\circ - x$$

$$= 90^\circ.$$

b) Collinear points lie on the same line, so should have the same gradient.

$$m_{DE} = \frac{-2b-0}{6a-2a} \qquad m_{EF} = \frac{b-0}{0-2a}$$

$$= \frac{-2b}{4a} \qquad = \frac{b}{-2a}$$

$$= \frac{-b}{2a} \qquad = \frac{-b}{2a} = m_{DE}.$$

$\therefore$ D, E, F are collinear.

c) $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$= (4a-a)^2 + (-2a-a)^2$$

$$= (3a)^2 + (-3a)^2$$

$$= 9a^2 + 9a^2$$

$$= 18a^2.$$

$\therefore d = \sqrt{18}\,a$ (d must be +ve)

$= 3\sqrt{2}\,a$ units

d) $x^2 - 4x + y^2 + 6y - 36 = 0$

$$x^2 - 4x \quad + y^2 + 6y \quad = 36$$
$$x^2 - 4x + 4 + y^2 + 6y + 9 = 36 + 4 + 9$$
$$(x-2)^2 \quad + (y+3)^2 \quad = 49$$

$\therefore$ centre is $(2,-3)$ and radius is 7.

e) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \to 0} 6x + 3h$$

$$= 6x + 3(0)$$

$$= 6x$$

Question 6

a) $f(a) = a^2 - 4a + 5 \qquad f(-a) = (-a)^2 - 4(-a) + 5$

$$= a^2 + 4a + 5$$

no relationship between $f(a)$ and $f(-a)$ $\therefore$ neither odd nor even.

b) $\Delta < 0$ for true definite parabola $(a > 0$ as $a = 1)$.

$\therefore \quad b^2 - 4ac < 0$

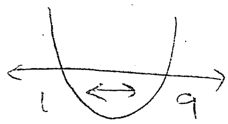
$(k-3)^2 - 4k < 0$

$k^2 - 6k + 9 - 4k \leq 0$

$k^2 - 10k + 9 \leq 0$

$(k-1)(k-9) < 0$

$k = 1 \text{ or } 9$



$1 < k < 9$

c) The roots are α and 3α

$\alpha + 3\alpha = \frac{8}{3}$

$4\alpha = \frac{8}{3}$

$\underline{\alpha = \frac{2}{3}}$ ①

$\alpha(3\alpha) = \frac{d}{3}$

$3\alpha^2 = \frac{d}{3}$ ②

① → ② $\quad 3\left(\frac{2}{3}\right)^2 = \frac{d}{3}$

$\frac{3 \times 4}{9} = \frac{d}{3}$

$\underline{d = 4}$

d) $(3+\sqrt{2}) + (3-\sqrt{2}) = \frac{-b}{a}$

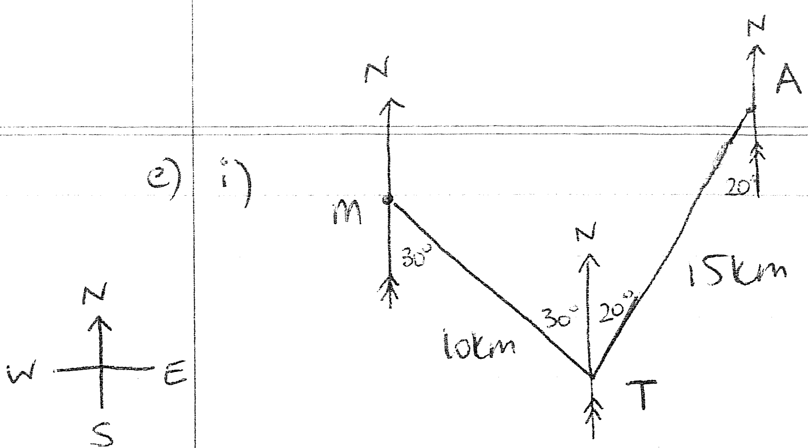
$6 = \frac{-b}{a}$

$(3+\sqrt{2})(3-\sqrt{2}) = \frac{c}{a}$

$7 = \frac{c}{a}$

$\therefore$ eqn is $x^2 - 6x + 7 = 0$

e) i)



M = Marina
T = turning pt
A = anchored pt.

ii) $AM^2 = 10^2 + 15^2 - 2(10)(15)\cos 50°$
$= 132.1637\ldots$
$AM = 11.496$ km (nearest m)

iii) bearing °T $= 180° + 20° + \angle MAT$

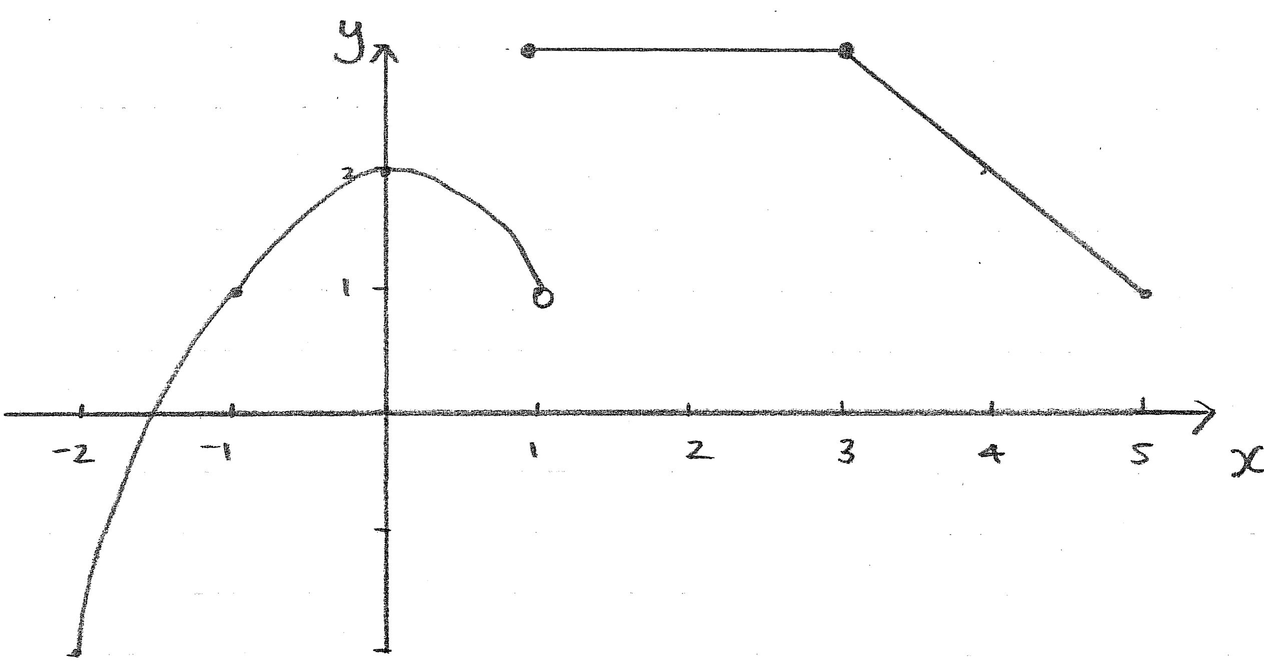
$$\frac{\sin\angle MAT}{10} = \frac{\sin 50°}{AM}$$

$\angle MAT = 41.79°$ (2dp)

∴ bearing of Marina from boat is 242°T (nearest deg)

Question 7

a)



b) $\operatorname{cosec} a° = \sec(90-a)°$
∴ $\operatorname{cosec}(x-25)° = \operatorname{cosec}(90-65)°$
$x = 90° - 65° + 25° = \underline{50°}$

c) $\dfrac{\sec\theta}{\tan\theta + \cot\theta} = \dfrac{\sec\theta}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}$

$$= \dfrac{\sec\theta}{\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}$$

$$= \sec\theta \div \dfrac{1}{\cos\theta\sin\theta}$$

$$= \dfrac{1}{\cos\theta} \times \dfrac{\cos\theta\sin\theta}{1}$$

$$= \sin\theta.$$

d) $\lim\limits_{x\to\infty} \dfrac{4x^2/x^3 - 2x^3/x^3 + 1/x^3}{7x^3/x^3 - 6x/x^3}$

$$= \dfrac{0-2+0}{7-0}$$

$$= \dfrac{-2}{7}$$

e) i) $y = \dfrac{x^3}{(x^2+1)^2}$ $\quad u = x^3 \quad u' = 3x^2 \quad v = (x^2+1)^2$

$v' = 2(x^2+1) \times 2x$

$= 4x(x^2+1)$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{3x^2(x^2+1)^{\cancel{2}} - 4x^4(x^2+1)}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{3x^4 + 3x^2 - 4x^4}{(x^2+1)^3}$$

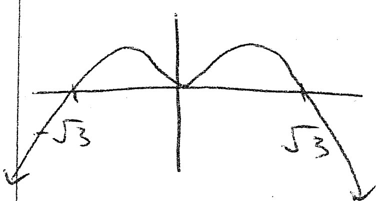
$$= \frac{3x^2 - x^4}{(x^2+1)^3} = \frac{x^2(3-x^2)}{(x^2+1)^3}.$$

ii) Slope +ve = $y' > 0$

$$\frac{x^2(3-x^2)}{(x^2+1)^3} > 0$$

$\therefore$ $x^2(3-x^2) > 0$

$x^2(\sqrt{3}-x)(\sqrt{3}+x) > 0$



$\therefore$ $-\sqrt{3} < x < \sqrt{3}$ but $x \neq 0$.

SOLUTIONS

YEAR 11 MATHEMATICS YEARLY EXAM 2009

Question 1

a) $\dfrac{\sqrt{3}}{7-4\sqrt{3}} \times \dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}$

$= \dfrac{7\sqrt{3}+12}{49-48}$

$= 7\sqrt{3}+12$

b) $\dfrac{4(x^2+2xy+y^2)}{x^2-y^2}$

$= \dfrac{4(x+y)^2}{(x+y)(x-y)}$

$= \dfrac{4(x+y)}{x-y}$

c) $= 7.41559\ldots$

$= 7.42$ (3sf)

d) $2x^2+5x-3$ $\quad \times -6$

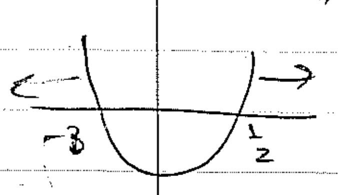
$= 2x^2+6x-x-3$ $\quad +5$

$= 2x(x+3)-(x+3)$ $\quad (6,-1)$

$= (2x-1)(x+3)$

$\therefore (2x-1)(x+3)>0$

$x=\frac{1}{2}, -3$



$\therefore x<-3,$

$x>\frac{1}{2}$

e) $9^n \times 81 \times 3^n$

$= (3^2)^n \times 3^4 \times 3^n$

$= 3^{2n} \times 3^4 \times 3^n$

$= 3^{3n+4}$.

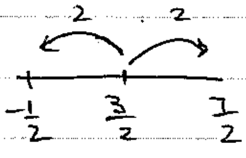
f) $|2x-3| \leq 4$

$-4 \leq 2x-3 \leq 4$

$-1 \leq 2x \leq 7$

$-\frac{1}{2} \leq x \leq \frac{7}{2}$

OR $|x-\frac{3}{2}| \leq 2$

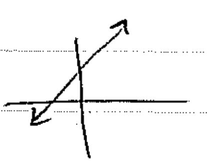


$-\frac{1}{2} \leq x \leq \frac{7}{2}$.

Question 2

a) $3x-2y+7=0$

$y=\frac{3}{2}x+7/2$



i) $m=\tan\theta$

$\frac{3}{2}=\tan\theta$

$\theta = 56°19'$

$\therefore$ obtuse $\angle = 180° - 56°19'$

$= 123°41'$

ii) $a=3 \quad b=-2 \quad c=7$

$x_1=4 \quad y_1=-2$

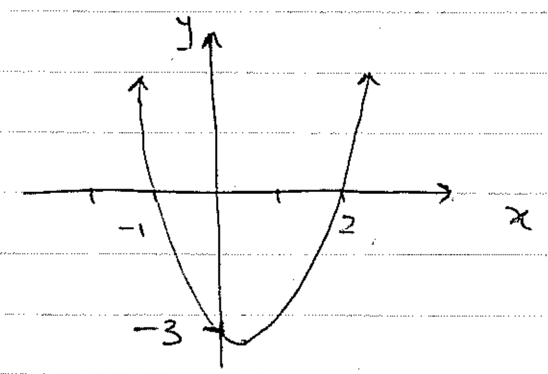
$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$=\frac{|3(4)+(-2)(-2)+7|}{\sqrt{3^2+(-2)^2}}$$

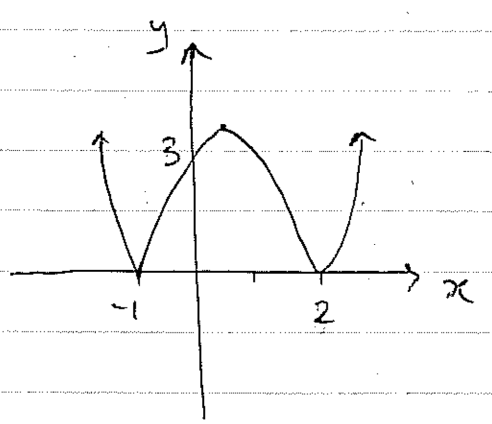
$$=\frac{|23|}{\sqrt{13}}$$

$$=\frac{23\sqrt{13}}{13} \text{ units.}$$

b) i)



ii)



c) i) In ΔAXY & ΔABC:

$\angle YAX$ is common

$\angle AYX = \angle ACB$ (corresponding $\angle$'s, $YX \parallel CB$)

$\therefore \Delta AXY \parallel\!\mid \Delta ABC$ (equiangular)

ii) $\frac{AY}{AC}=\frac{AX}{AB}$ (corresponding sides in similar Δs)

$$\frac{AY}{8}=\frac{6}{10}$$

$$AY=4.8$$

$$YC=AC-AY$$

$$=8-4.8$$

$$=\underline{3.2\text{cm}}.$$

Question 3

a) i) $y=3x^2-2x^{-1}$

$$\frac{dy}{dx}=6x+2x^{-2}$$

$$=6x+\frac{2}{x^2}$$

ii) $y=(6x+1)^{3/2}$

$$\frac{dy}{dx}=\frac{3}{2}(6x+1)^{1/2}\times 6$$

$$=\frac{18}{2}(6x+1)^{1/2}$$

iii) $y=x^2(x+1)^{1/2}$

$u=x^2 \qquad v=(x+1)^{\frac{1}{2}}$

$u'=2x \qquad v'=\frac{1}{2}(x+1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = uv' + vu'$

$= x^2\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}} + 2x(x+1)^{\frac{1}{2}}$

$= \frac{1}{2}x(x+1)^{-\frac{1}{2}}\left[x + 4(x+1)\right]$

$= \frac{1}{2}x(x+1)^{-\frac{1}{2}}(5x+1)$

b) $y = x^3 - 4x - 8$

$\frac{dy}{dx} = 3x^2 - 4$

m of tangent at $x=2$ is $3(2)^2 - 4$

$= 8$

$\therefore$ m of normal is $-\frac{1}{8}$.

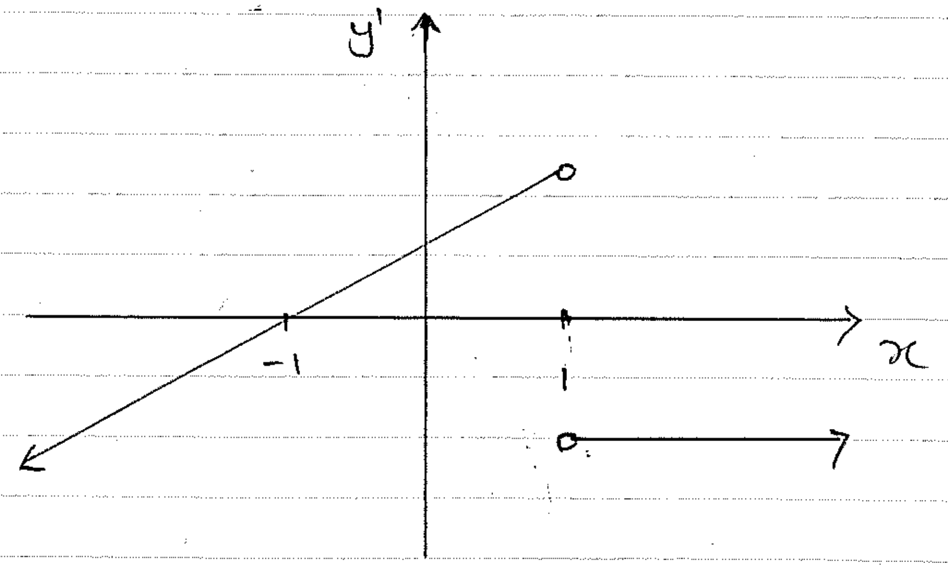
$\therefore\ y - y_1 = m(x - x_1)$

$y + 8 = -\frac{1}{8}(x-2)$

$8y + 64 = -x + 2$

$\underline{x + 8y + 62 = 0}$.

c)



Question 4

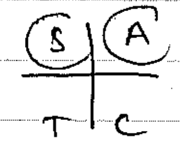
a) $\dfrac{\tan 120^\circ}{\cos^2 120^\circ} = \dfrac{-\sqrt{3}}{\left(-\frac{1}{2}\right)^2}$

$= \dfrac{-\sqrt{3}}{\frac{1}{4}}$

$= -4\sqrt{3}$

b) $2\sin 2\alpha = \sqrt{3}$

$\sin 2\alpha = \dfrac{\sqrt{3}}{2}$

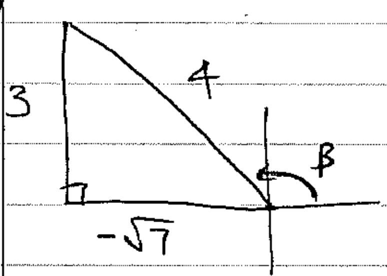


$2\alpha = 60^\circ, 120^\circ, 420^\circ, 480^\circ$

(note $0^\circ \leq 2\alpha^\circ \leq 720^\circ$)

$\therefore \alpha = 30^\circ, 60^\circ, 210^\circ, 240^\circ$

c)



$\therefore \tan\beta = \dfrac{-3}{\sqrt{7}}$

d) $x - 2 > 0$

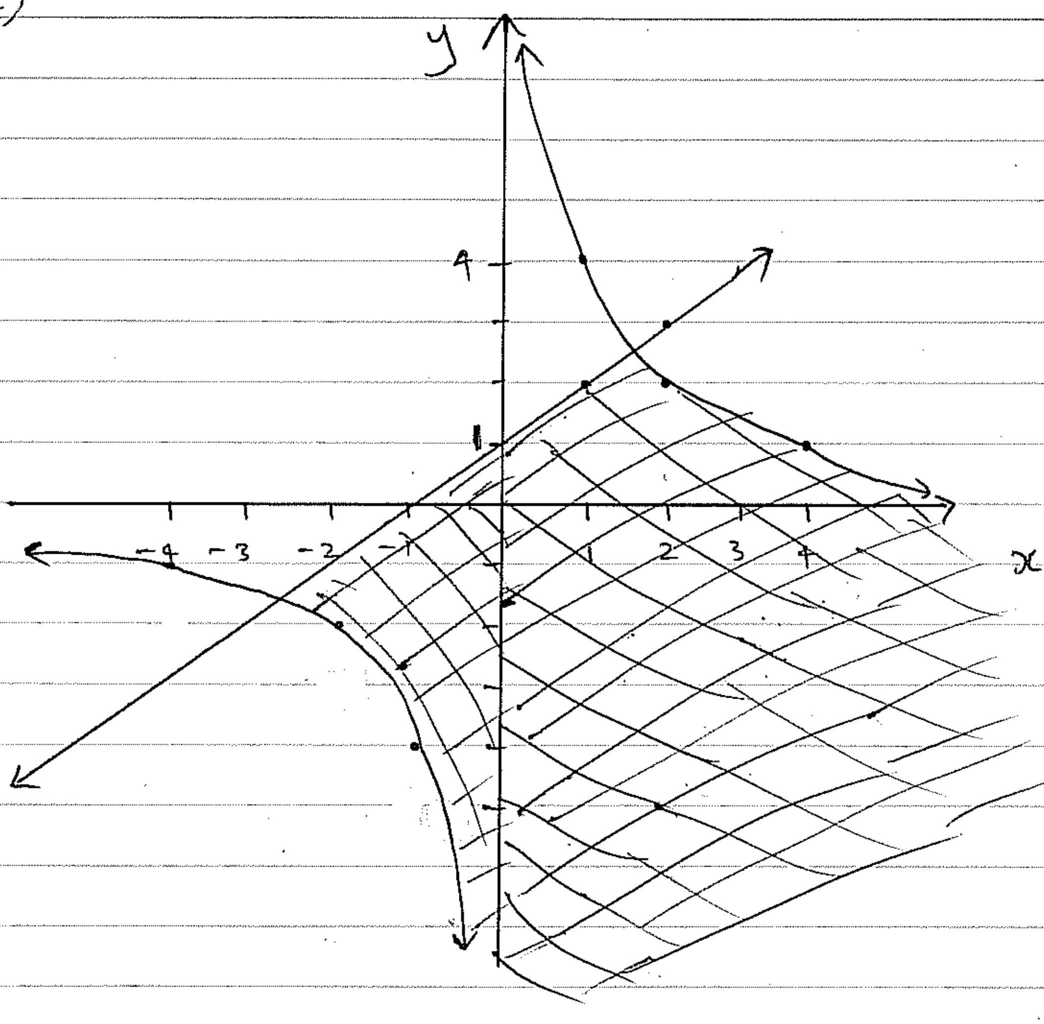
$\therefore x > 2$ is the domain

since $\sqrt{x-2}$ is always +ve,

So is $f(x) = \dfrac{1}{\sqrt{x-2}}$

$\therefore y > 0$ is the range.

e)



Question 5

a) let $\angle CDA = x$

$\therefore \angle DAC = x$ (equal $\angle$s opp equal sides)

$\therefore \angle BCA = 2x$ (exterior $\angle$ of $\triangle ADC$)

$\angle CAB = \angle CBA$ (equal $\angle$s opp equal sides)

$\therefore 2\times\angle CAB + 2x = 180^\circ$ ($\angle$ sum $\triangle CAB$)

$\therefore \angle CAB = \dfrac{180^\circ - 2x}{2}$

$= 90^\circ - x$

$\angle DAB = \angle DAC + \angle CAB$ (adjacent $\angle$s)

$= x + 90^\circ - x$

$= 90^\circ$.

b) Collinear points lie on the same line, so should have the same gradient.

$m_{DE} = \dfrac{-2b-0}{6a-2a}$ $\qquad$ $m_{EF} = \dfrac{b-0}{0-2a}$

$= \dfrac{-2b}{4a}$ $\qquad$ $= \dfrac{b}{-2a}$

$= \dfrac{-b}{2a}$ $\qquad$ $= \dfrac{-b}{2a} = m_{DE}$.

$\therefore$ D, E, F are collinear.

c) $d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (4a-a)^2 + (-2a-a)^2$

$= (3a)^2 + (-3a)^2$

$= 9a^2 + 9a^2$

$= 18a^2$.

$\therefore d = \sqrt{18}\,a$ (d must be +ve)

$= 3\sqrt{2}\,a$ units

d) $x^2 - 4x + y^2 + 6y - 36 = 0$

$$x^2 - 4x \quad + y^2 + 6y \quad = 36$$
$$x^2 - 4x + 4 + y^2 + 6y + 9 = 36 + 4 + 9$$
$$(x-2)^2 \quad + (y+3)^2 \quad = 49$$

$\therefore$ centre is $(2, -3)$ and radius is 7.

e) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h}$$

$$= \lim_{h \to 0} \frac{6x\cancel{h} + 3h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} 6x + 3h$$

$$= 6x + 3(0)$$

$$= 6x.$$

Question 6

a) $f(a) = a^2 - 4a + 5 \qquad f(-a) = (-a)^2 - 4(-a) + 5$
$$= a^2 + 4a + 5$$

no relationship between $f(a)$ and $f(-a)$ $\therefore$ neither odd nor even.

b) $\Delta < 0$ for the definite parabola $(a > 0$ as $a = 1)$.

$\therefore \quad b^2 - 4ac < 0$

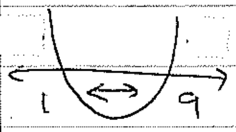
$(k-3)^2 - 4k \leq 0$

$k^2 - 6k + 9 - 4k \leq 0$

$k^2 - 10k + 9 \leq 0$

$(k-1)(k-9) \leq 0$

$k = 1 \text{ or } 9$



$1 < k < 9$

c) The roots are α and 3α

$\alpha + 3\alpha = \frac{8}{3}$ $\qquad \alpha(3\alpha) = \frac{d}{3}$

$4\alpha = \frac{8}{3}$

$3\alpha^2 = \frac{d}{3}$ ②

$\underline{\alpha = \frac{2}{3}}$ ①

① → ② $\quad 3\left(\frac{2}{3}\right)^2 = \frac{d}{3}$

$\frac{3 \times 4}{9} = \frac{d}{3}$

$\underline{d = 4}$.

d) $(3+\sqrt{2}) + (3-\sqrt{2}) = \frac{-b}{a}$

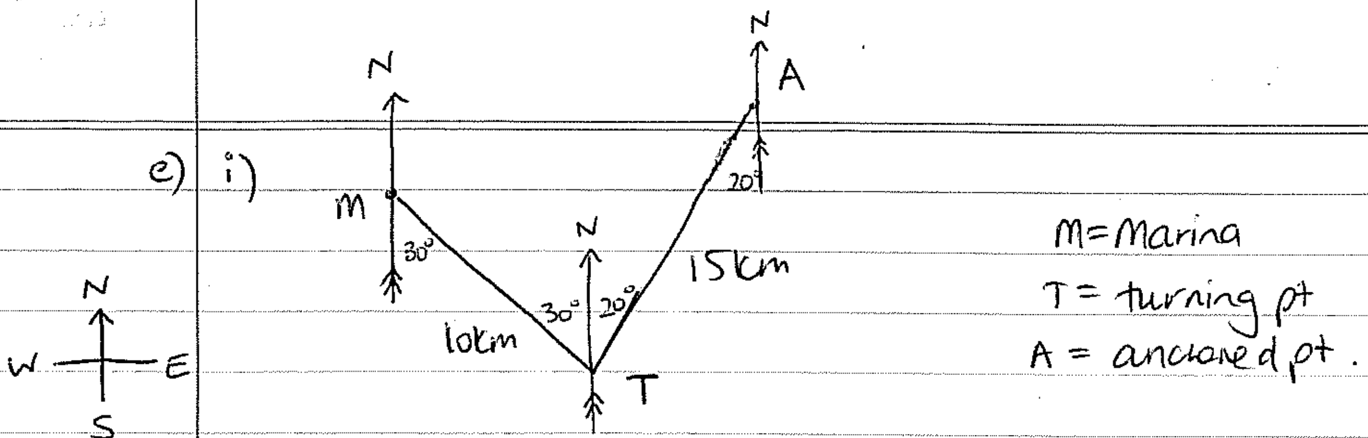
$6 = \frac{-b}{a}$

$(3+\sqrt{2})(3-\sqrt{2}) = \frac{c}{a}$

$7 = \frac{c}{a}$

$\therefore$ eqn is $x^2 - 6x + 7 = 0$

e) i)



ii) $AM^2 = 10^2 + 15^2 - 2(10)(15)\cos 50°$

$= 132.1637\ldots$

$AM = 11.496$ km (nearest m)

iii) bearing °T $= 180° + 20° + \angle MAT$

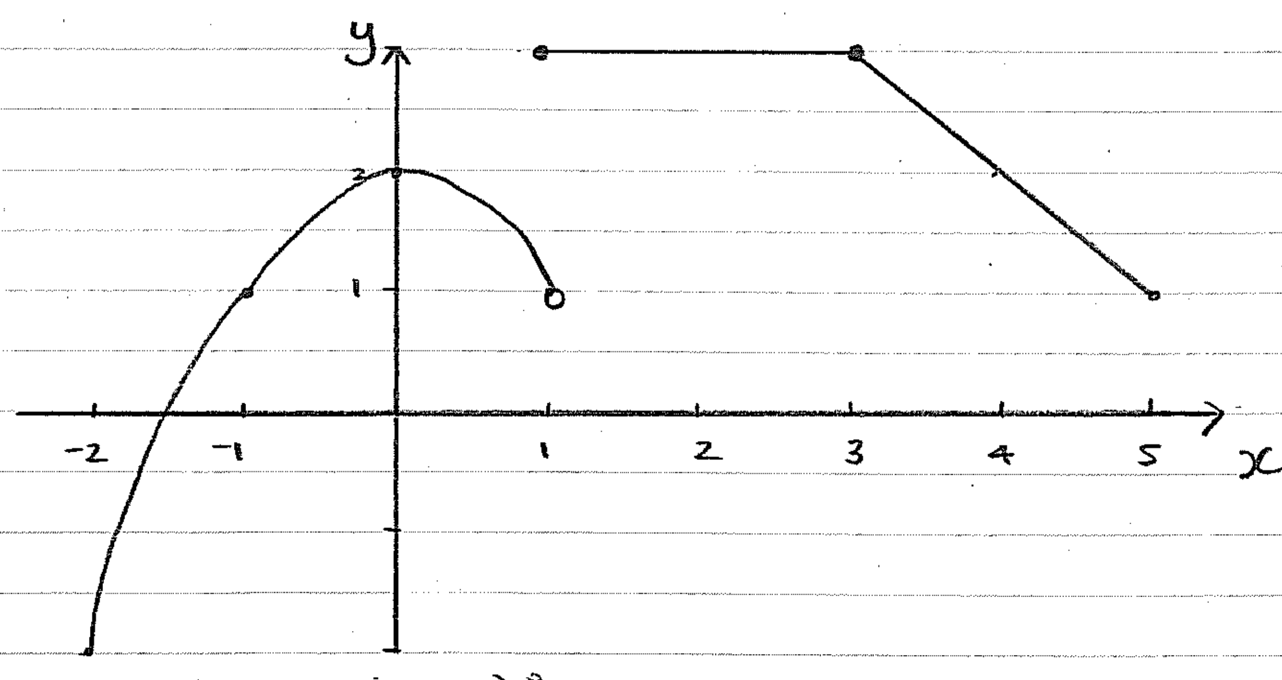
$\frac{\sin \angle MAT}{10} = \frac{\sin 50°}{AM}$

$\angle MAT = 41.79°$ (2dp)

$\therefore$ bearing of Marina from boat is 242°T (nearest deg)

Question 7

a)



b) $\text{cosec}\, a° = \sec(90-a)°$

$\therefore \text{cosec}(x-25)° = \text{cosec}(90-65)°$

$x = 90° - 65° + 25° = \underline{50°}$

c) $\dfrac{\sec\theta}{\tan\theta+\cot\theta} = \dfrac{\sec\theta}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{\sec\theta}{\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}$

$= \sec\theta \div \dfrac{1}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta} \times \dfrac{\cos\theta\sin\theta}{1}$

$= \sin\theta$.

d) $\displaystyle\lim_{x\to\infty} \dfrac{4x^2/x^3 - 2x^3/x^3 + 1/x^3}{7x^3/x^3 - 6x/x^3}$

$= \dfrac{0-2+0}{7-0}$

$= \dfrac{-2}{7}$

e) i) $y = \dfrac{x^3}{(x^2+1)^2}$ $\quad u = x^3$ $\quad v = (x^2+1)^2$

$u' = 3x^2$ $\quad v' = 2(x^2+1) \times 2x$

$= 4x(x^2+1)$

$\dfrac{dy}{dx} = \dfrac{vu'-uv'}{v^2}$

$= \dfrac{3x^2(x^2+1)^{\cancel{2}} - 4x^4\cancel{(x^2+1)}}{(x^2+1)^{\cancel{4}\,3}}$

$= \dfrac{3x^4+3x^2-4x^4}{(x^2+1)^3}$

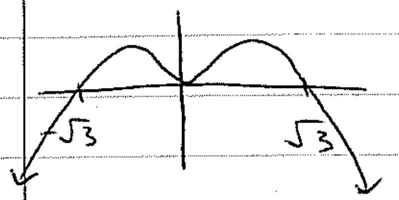
$= \dfrac{3x^2-x^4}{(x^2+1)^3} = \dfrac{x^2(3-x^2)}{(x^2+1)^3}$.

ii) Slope +ve = $y' > 0$

$$\frac{x^2(3-x^2)}{(x^2+1)^3} > 0$$

$\therefore\ x^2(3-x^2) > 0$

$x^2(\sqrt{3}-x)(\sqrt{3}+x) > 0$



$\therefore\ -\sqrt{3} < x < \sqrt{3}$ but $x \neq 0$.