NORTH SYDNEY GIRLS HIGH SCHOOL

2010
YEARLY EXAMINATION

Preliminary Mathematics

General Instructions

Reading Time – 5 minutes

Working Time – 2 hours

Write using black or blue pen

Board-approved calculators may be used

All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7

All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top. Submit ONE bundle. The bundle will be separated for marking so please ensure your name is written on each solution booklet.

Student Name: ______________________________

Teacher: ______________________________

	1	2a	2b	3ab	3c	4	5	6a	6bc	6d	7abc	7d	Totals
P2					/6					/3			/9
P3	/12												/12
P4						/12	/12				/7		/31
P5				/6					/7				/13
P6			/5									/5	/10
P7		/7						/2					/9
	/12		/12		/12	/12	/12			/12		/12	/84

Total Marks – 84
Attempt Questions 1 – 7
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 Marks)

(a) Evaluate $\dfrac{\sin^2 63°}{\tan 14°}$ to two significant figures. **2**

(b) Express $\dfrac{x}{y}$ without negative indices if $x = a^{-2}b^3$ and $y = a^5b^{-2}$. **2**

(c) Solve $(3-x)(4+x) > 0$. **2**

(d) Simplify $\dfrac{6x^2+42x}{49-x^2}$. **2**

(e) Rationalise the denominator of $\dfrac{\sqrt{3}-1}{2\sqrt{3}+1}$. **2**

(f) Solve $|x-7| = 3$. **2**

Question 2 (12 Marks) **Start a new booklet.**

(a) Differentiate with respect to x:

(i) $5x^3 + \frac{1}{4}x + 3$ **1**

(ii) $\dfrac{x^3-4x^2}{\sqrt{x}}$ **2**

(iii) $\dfrac{1}{(2x+3)^4}$ **2**

(iv) $\left(x^2+\dfrac{1}{x}\right)^2$. **2**

(b) (i) Find the equation of the tangent to $y = x^3 - 9x^2 + 20x - 8$ at the point (1, 4). **3**

(ii) For what values of x are the tangents parallel to the line $y = -4x + 3$? **2**

Question 3 (12 Marks) **Start a new booklet.**

(a) Sketch on a number plane the region defined by $0 \le x + y \le 1$. **2**

(b) A function is defined as $f(x) = 4 - 2^{-x}$.

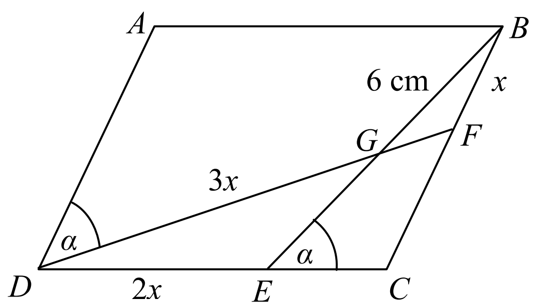
(i) Find, in simplest form:

(α) $f(x^2)$ **1**

(β) $[f(x)]^2$. **2**

(ii) Determine if $f(x)$ is even, odd or neither. **1**

(c) *ABCD* is a parallelogram.
F and *E* are two points on *BC* and *DC* respectively such that $BF = x$, $DE = 2x$ and $\angle ADF = \angle BEC = \alpha$.
FD meets *BE* at *G* such that $DG = 3x$ and $BG = 6$ cm.



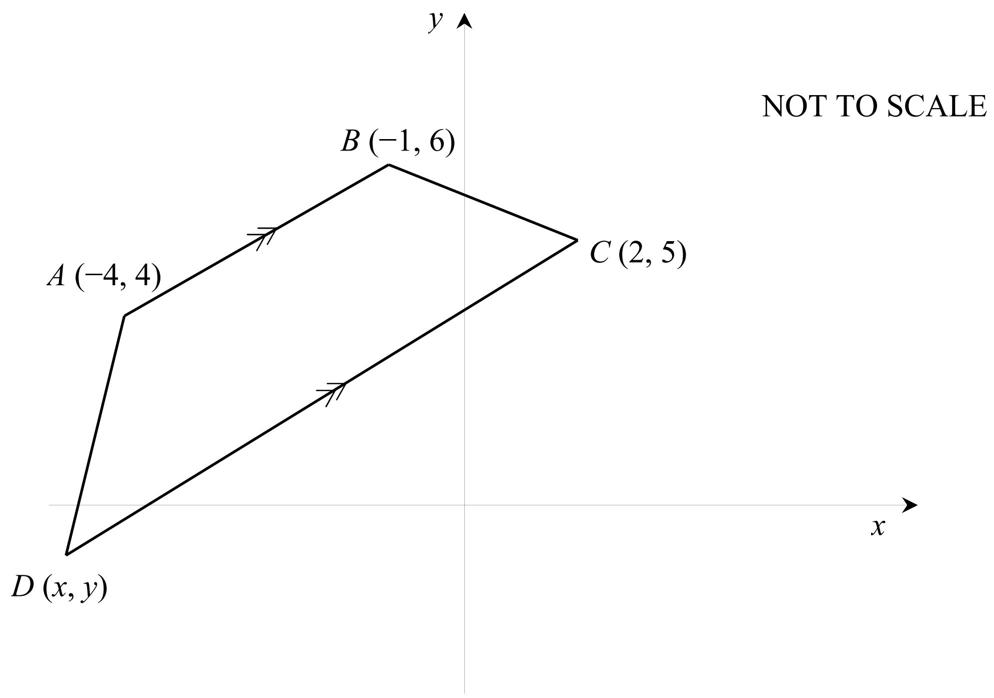
(i) Prove that $\angle GDE = \angle GBF$. **2**

(ii) Prove that ΔGDE is similar to ΔGBF. **2**

(iii) Hence find the value of x. **2**

Question 4 (12 Marks) **Start a new booklet.**

Below is a diagram showing a quadrilateral *ABCD*, where $AB \parallel CD$.



(a) Find the gradient of the line *AB*. **1**

(b) Find the exact value of the distance *AB*. **2**

(c) Show that the equation of the line *CD* is $2x-3y+11=0$. **2**

(d) Find the perpendicular distance from the line *CD* to the point *A*. **2**

(e) The distance between *C* and *D* is to be $\sqrt{117}$ units.

(i) Show that $\left(x-2\right)^2+\left(\dfrac{2x+11}{3}-5\right)^2=117$. **1**

(ii) Find the coordinates of point *D*, assuming $x<0$. **3**

(f) Find the area of the quadrilateral *ABCD*. **1**

Question 5 (12 Marks) **Start a new booklet.**

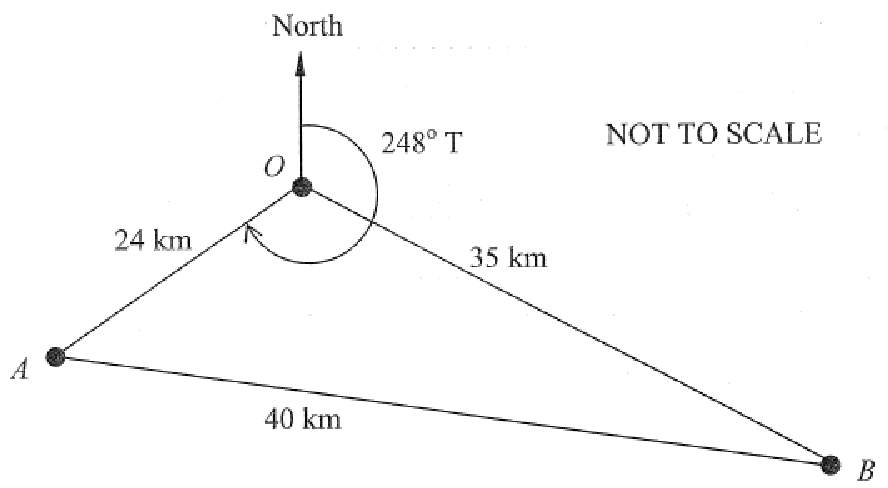
(a) Simplify $\operatorname{cosec}(90° - A)\cos(90° - A)$. **2**

(b) Find β in the domain $90° < \beta < 270°$ if $2\sin^2 \beta = \dfrac{1}{2}$. **2**

(c) (i) Find the angle, in degrees and minutes, that a line with gradient −2.5 makes with the positive x-axis. **2**

(ii) Find the acute angle that the same line makes with the y-axis. **1**

(d) A section of rainforest is to be scoured in the search for new species. The shape is shown below.
The bearing of landmark A from landmark O is 248ºT and is 24 km in distance.
The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



(i) Find the size of $\angle AOB$. **2**

(ii) Hence or otherwise calculate the area of this section of the rainforest. **1**

(iii) What is the bearing of landmark O from landmark B? **2**

Question 6 (12 Marks) **Start a new booklet.**

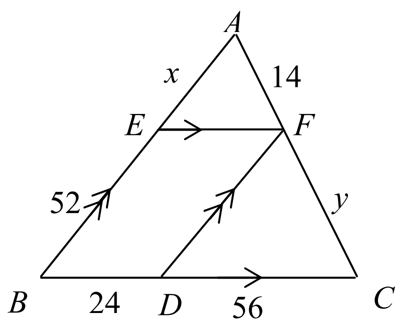
(a) For the curve $y = ax^2 + bx + c$ where a, b and c are constants, it is given that $y = 1$ and $\dfrac{dy}{dx} = 1$ when $x = 1$. Find the relationship between a and c. **2**

(b) (i) Sketch on the same number plane the functions $y = |x| - 2$ and $y = 2x + 1$. **3**

(ii) Use your answer to (i) to solve $|x| > 2x + 3$. **2**

(c) State the domain and range of $f(x) = \sqrt{1 - x^2}$. **2**

(d) In the diagram below, $BA \parallel DF$ and $EF \parallel BC$.
It is also given that $BE = 52$, $AF = 14$, $DC = 56$, $BD = 24$, $EA = x$ and $FC = y$.



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Find x and y, giving reasons. **3**

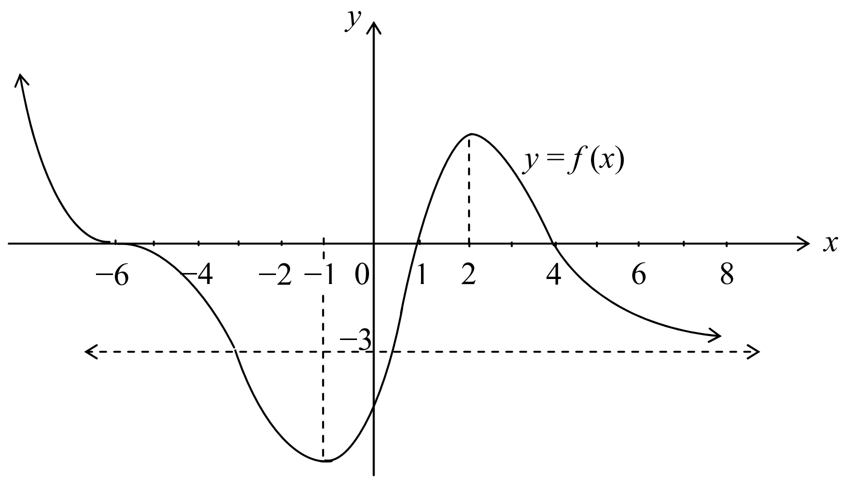
Question 7 (12 Marks) **Start a new booklet.**

(a) The line $2x + 3y - 13 = 0$ is translated 5 units up and parallel to the line. Find the equation of this transformation. **2**

(b) Prove the identity $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$. **2**

(c) Evaluate $\lim\limits_{x\to 3}\dfrac{x^3 - 27}{3 - x}$. **3**

(d) The diagram below shows the graph of $y = f(x)$.



(i) For which values of x is $f'(x) > 0$? **1**

(ii) Explain what happens to $f'(x)$ for large positive values of x. **1**

(iii) Sketch the graph $y = f'(x)$. **3**

End of Paper

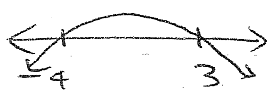
YEAR 11 MATHEMATICS YEARLY - SOLUTIONS

Q1

a) $3.1841\ldots$
$= 3.2 \ (2 \text{ sf})$

b) $\dfrac{x}{y} = \dfrac{a^{-2}b^{3}}{a^{5}b^{-2}}$
$= \dfrac{b^{5}}{a^{7}}$

c) $(3-x)(4+x) > 0$

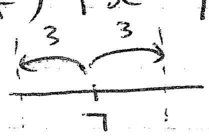


$\therefore \underline{-4 < x < 3}$.

d) $\dfrac{6x^2+42x}{49-x^2}$
$= \dfrac{6x\cancel{(x+7)}}{\cancel{(7+x)}(7-x)}$
$= \dfrac{6x}{7-x}$

e) $\dfrac{\sqrt{3}-1}{2\sqrt{3}+1} \times \dfrac{2\sqrt{3}-1}{2\sqrt{3}-1}$
$= \dfrac{6-\sqrt{3}-2\sqrt{3}+1}{12-1}$
$= \dfrac{7-3\sqrt{3}}{11}$

f) $|x-7| = 3$



$\therefore x = 4, 10$.

Q2

a) i) $y = 5x^3 + \frac{1}{4}x + 3$
$\dfrac{dy}{dx} = 15x^2 + \frac{1}{4}$

ii) $y = \dfrac{x^3 - 4x^2}{\sqrt{x}} = x^{5/2} - 4x^{3/2}$
$\dfrac{dy}{dx} = \frac{5}{2}x^{3/2} - 6x^{\frac{1}{2}}$
$= \dfrac{5\sqrt{x^3}}{2} - 6\sqrt{x}$

iii) $y = (2x+3)^{-4}$
$\dfrac{dy}{dx} = -4(2x+3)^{-5} \times 2$
$= \dfrac{-8}{(2x+3)^5}$

iv) $y = \left(x^2 + \frac{1}{x}\right)^2 = x^4 + 2x + x^{-2}$
$\dfrac{dy}{dx} = 4x^3 - 2x^{-3} + 2$
$= 4x^3 - \dfrac{2}{x^3} + 2$

b) $y = x^3 - 9x^2 + 20x - 8 \quad (1,4)$
i) $\dfrac{dy}{dx} = 3x^2 - 18x + 20$

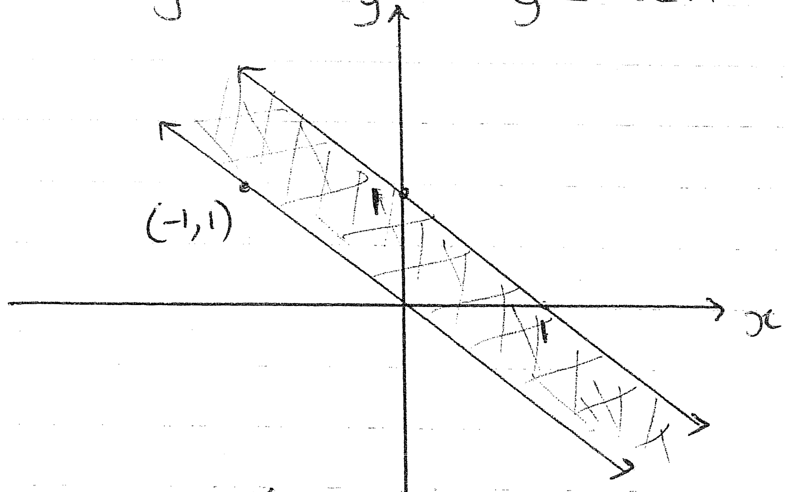
when $x=1$, $m = 3(1)^2 - 18(1) + 20$
$= 5$

$\therefore$ eqn tangent $y - 4 = 5(x-1)$
$\underline{y = 5x - 1}$.

ii) tangents parallel $\rightarrow$ $m = -4$
$\therefore -4 = 3x^2 - 18x + 20$
$0 = 3x^2 - 18x + 24$
$= x^2 - 6x + 8$
$= (x-4)(x-2)$
$\therefore x = 2$ and $x = 4$.

Q3 a) sketch $x+y \geq 0$ and $x+y \leq 1$

$y \geq -x$ $\quad$ $y \leq -x+1$



b) $f(x) = 4 - 2^{-x}$

i) α) $f(x^2) = 4 - 2^{-x^2}$

β) $[f(x)]^2 = (4-2^{-x})^2$

$= 16 - 2 \times 4 \times 2^{-x} + (2^{-x})^2$

$= 16 - 2^1 \times 2^2 \times 2^{-x} + 2^{-2x}$

$= 16 - 2^{3-x} + 2^{-2x}$

ii) $f(a) = 4 - 2^{-a}$ $\quad$ $f(-a) = 4 - 2^{-(-a)}$

$= 4 - 2^a$

$f(a) \neq f(-a)$, nor does $f(a) = -f(-a)$

$\therefore$ neither odd nor even.

c) i) $\angle ABE = \angle BEC = \alpha$ (alternate $\angle$s, $AB \parallel DC$)

$\angle ABF = \angle ADE$ (opp $\angle$s of parallelogram)

$\therefore \angle ABE + \angle GBF = \angle ADF + \angle GDE$

$\alpha + \angle GBF = \alpha + \angle GDE$

$\therefore \angle GBF = \angle GDE$.

ii) In $\triangle GDE$ & $\triangle GBF$:

$\angle GDE = \angle GBF$ (proven above)

$\angle DGE = \angle BGF$ (vert opp $\angle$s)

$\therefore \triangle GDE \;|||\; \triangle GBF$ (equiangular).

iii) $\frac{DG}{GB} = \frac{DE}{BF}$ (corresponding sides in congruent $\triangle$s in ratio)

$\therefore \frac{3x}{6} = \frac{2x}{x}$

$\frac{x}{2} = 2$

$\therefore x = 4$

Q4 a) $m = \frac{rise}{run} = \frac{6-4}{-1-(-4)}$

$= \frac{2}{3}$

b) $d_{AB}^2 = (-1+4)^2 + (6-4)^2$

$= 3^2 + 2^2$

$\therefore d_{AB} = \sqrt{13}$ units.

c) $CD \parallel AB \therefore m_{CD} = \frac{2}{3}$ $\quad C(2,5)$

$\therefore y - 5 = \frac{2}{3}(x-2)$

$3y - 15 = 2x - 4$

$0 = 2x - 3y + 11$

d) $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$ $\quad a = 2,\ b = -3,\ c = 11$

$x_1 = -4 \quad y_1 = 4$.

$= \left|\frac{2(-4) - 3(4) + 11}{\sqrt{2^2 + (-3)^2}}\right|$

$$= \left|\frac{-8-12+11}{\sqrt{13}}\right|$$

$$= \frac{9}{\sqrt{13}} \text{ units}$$

e) line CD: $2x-3y+11=0$ $\qquad C(2,5)$

i)

$$3y = 2x+11$$

$$y = \frac{2x+11}{3}$$

$\therefore D(x,y)$ is actually $D\left(x, \frac{2x+11}{3}\right)$

$$\therefore d_{CD}^2 = (x-2)^2 + \left(\frac{2x+11}{3} - 5\right)^2$$

$$\therefore 117 = (x-2)^2 + \left(\frac{2x+11}{3} - 5\right)^2$$

ii)

$$\therefore 117 = (x-2)^2 + \frac{1}{9}(2x+11-15)^2$$

$$1053 = 9(x-2)^2 + (2x-4)^2 \quad * \text{ see below:}$$

$$1053 = 9x^2 - 36x + 36 + 4x^2 - 16x + 16$$

$$0 = 13x^2 - 52x - 1001$$

$$= x^2 - 4x - 77$$

$$= (x-11)(x+7)$$

$\therefore \cancel{x=11}$ or $x=-7$

$(x<0)$

when $x=-7$, $y = \frac{2(-7)+11}{3} = -1$

$\therefore D(-7,-1)$.

*

$$1053 = 9(x-2)^2 + 4(x-2)^2$$

$$= 13(x-2)^2$$

$$81 = (x-2)^2$$

$$\pm 9 = x-2$$

$x = 11$ or -7

$(x<0)$

$\therefore x = -7$

$\therefore y = -1$

f) $A = \frac{1}{2}h(a+b)$

$$= \frac{1}{2} \times \frac{9}{\sqrt{13}}\left(\sqrt{13} + \sqrt{117}\right)$$

$$= 18 \text{ units}^2$$

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Q5 a) $\operatorname{cosec}(90° - A)\cos(90° - A)$

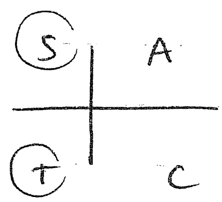
$= \sec A \times \sin A$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$.

b) $2\sin^2 B = \frac{1}{2}$ $90° < B < 270°$.

$\sin^2 B = \frac{1}{4}$

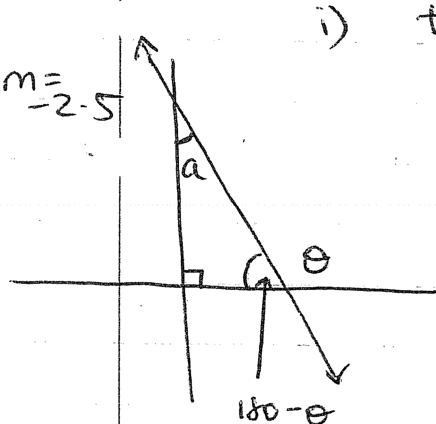


$\therefore \sin B = \pm \frac{1}{2}$

related $\angle = 30°$

$\therefore B = 150°, 210°$.

c)



i) $\tan\theta = -2.5$

$\theta = 180 - 68°12'$

$= 111°48'$.

ii) let acute $\angle = a$

$\therefore a + 90° + (180 - \theta)° = 180°$ (angle sum Δ)

$a = \theta - 90°$

$= 111°48' - 90°$

$\therefore$ acute $\angle = 21°48'$.

d) i) use cosine rule: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\therefore \cos \angle AOB = \dfrac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}$

$= \dfrac{201}{1680}$

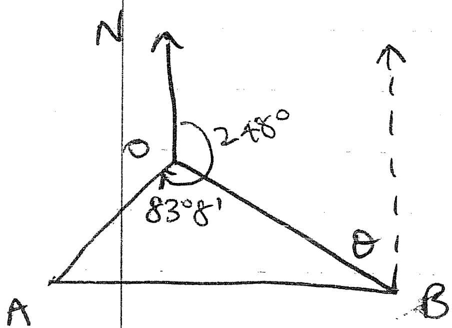
$\therefore \angle AOB = 83°7'$ or $83.1°$

ii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 24 \times 35 \times \sin 83°8'$

$= 417\,m^2$ (nearest m^2)

iii)



$\angle NOB = 248° - 83°8'$

$= 164°52'$

$\therefore \theta = 180 - 164°52'$ (co-int <s)

$= 15°8'$

$\therefore$ Bearing of O from B $= 360° - 15°8'$

$= \underline{\underline{344°52'T}}$.

Q6 a) $y = ax^2 + bx + c$ $\quad \therefore \frac{dy}{dx} = 2ax + b$.

when $x=1$, $y=1$

$\therefore 1 = a(1)^2 + b(1) + c$

$1 = a + b + c$

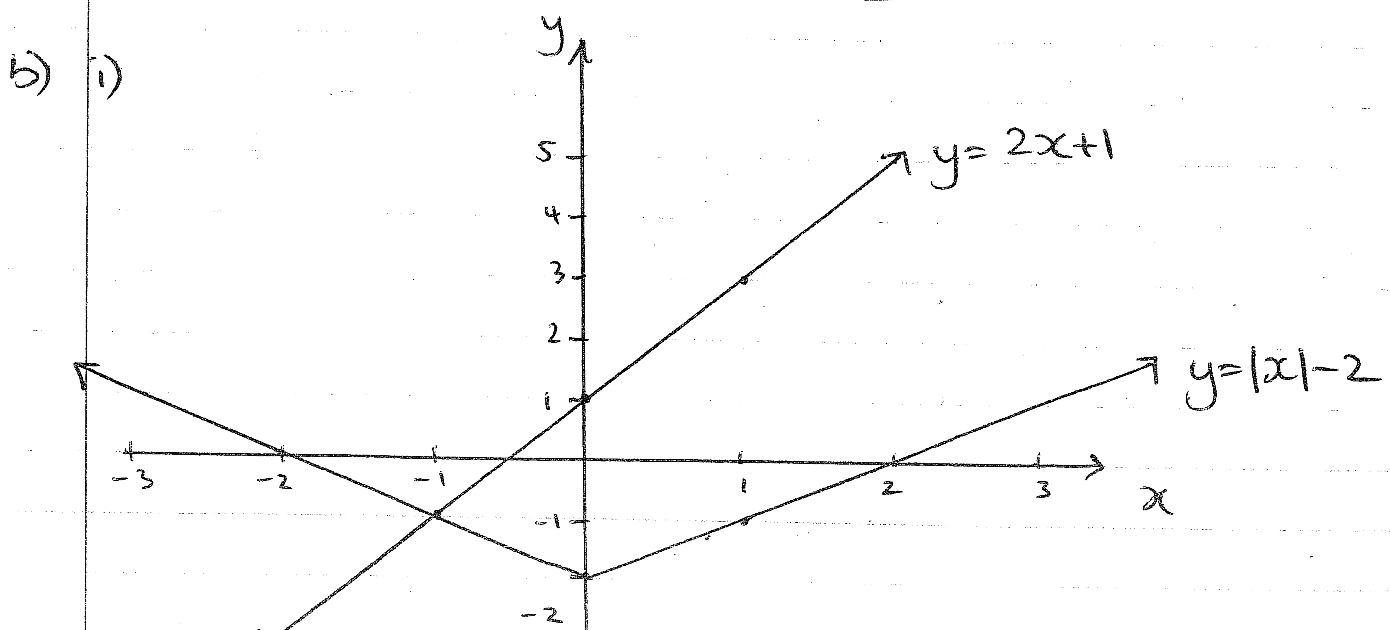
when $x = 1$, $\frac{dy}{dx} = 1$

$\therefore 1 = 2a(1) + b$

$1 = 2a + b$.

So, $a + b + c = 2a + b \Rightarrow \underline{\underline{a = c}}$.

b) i)



ii) $|x| - 2 > 2x + 1$ is identical to $|x| > 2x + 3$

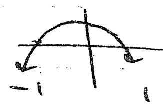
graphical solution : $x < -1$.

(Think: "when is the absolute value graph above the straight line?")

c) domain : $1 - x^2 \geq 0$

$(1-x)(1+x) \geq 0$

$\therefore -1 \leq x \leq 1$.



range : $0 \leq y \leq 1$.

d) $\frac{y}{14} = \frac{56}{24}$ (parallel lines preserve ratios, $BA \parallel DF$)

$\therefore y = 32\,^{2}/_{3}$.

$\frac{x}{52} = \frac{14}{y}$ (parallel lines preserve ratios, $BC \parallel EF$)

$\therefore x = 22\frac{2}{7}$

Q7 a) 5 units up $\Rightarrow$ y intercept moves up 5.

$2x + 3y - 13 = 0$

$3y = -2x + 13$

$y = -\frac{2}{3}x + \frac{13}{3}$

$\therefore$ new line : $y = -\frac{2}{3}x + \frac{13}{3} + 5$

$= -\frac{2}{3}x + \frac{28}{3}$

ie, $2x + 3y - 28 = 0$.

b) $LHS = \dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta}$

$= \dfrac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \dfrac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$

$= \dfrac{\cos^2\theta}{\cos\theta-\sin\theta} + \dfrac{\sin^2\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\cancel{(\cos\theta-\sin\theta)}(\cos\theta+\sin\theta)}{\cancel{\cos\theta-\sin\theta}}$

$= \cos\theta + \sin\theta$

$= RHS.$

c) $\lim\limits_{x\to 3} \dfrac{x^3-27}{3-x}$

$= \lim\limits_{x\to 3} \dfrac{(x-3)(x^2+3x+9)}{3-x}$

$= \lim\limits_{x\to 3} -(x^2+3x+9)$

$= -(3^2+3(3)+9)$

$= -27.$

d) i) $f'(x) > 0$ for $-1<x<2$ only.

ii) $f'(x)$ is approaching zero (The gradient is approaching 0) ie $f'(x) \to 0$.

iii)