NORTH SYDNEY GIRLS HIGH SCHOOL

2012

YEARLY EXAMINATION

Preliminary Mathematics

General Instructions

Reading Time – 5 minutes
Working Time – 2 hours
Write using black or blue pen
Diagrams may be done in pencil
Board approved calculators may be used

All necessary working should be shown in every question.

Total marks –

Attempt Questions 1-7

All questions are of equal value.
At the end of the examination, place your solution booklets in order and place them inside this question paper.
Submit one bundle. The bundle will be separated for marking so please ensure your number is written on each solution booklet.

Student Name:____

Teacher: ____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Question 1 (12 marks)

(a)	Find the value of $\dfrac{3+2^{2.5}}{\sqrt{5^2\times 1.5}}$ correct to three decimal places.	**2**
(b)	Solve $5m^2+3m-2=0$.	**2**
(c)	Express $\dfrac{2}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$ by rationalising the denominator.	**3**
(d)	Simplify fully $\dfrac{x^3+8}{x^2-x-6}$.	**2**
(e)	Simplify $(2x)^5 . \left(\dfrac{3}{x}\right)^{-2}$, expressing your answer without negative indices.	**2**
(f)	Evaluate $\sec^2 30^\circ - \cos 180^\circ$	**1**

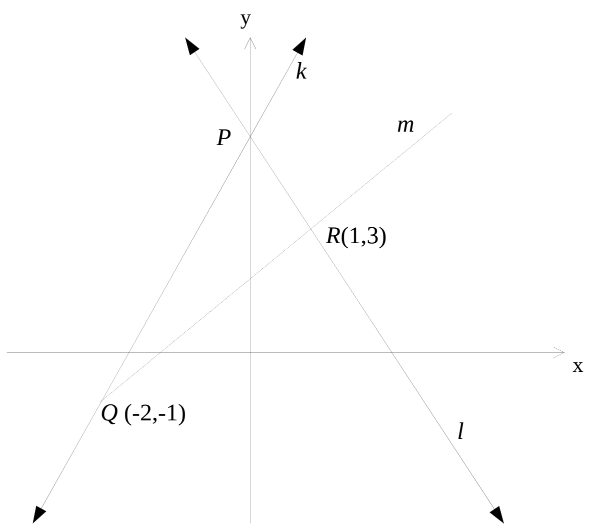
Question 2 (12 marks). Start a new booklet.

(a) Given that $\cos\alpha = \frac{1}{4}$ and α is acute, find the exact value of $\sin^2\alpha + \tan\alpha$. **2**

(b) Simplify $\frac{a}{a+b} - \frac{b}{a^2-b^2}$. **2**

(c) The point $Q(-2,-1)$ lies on the line k whose equation is $7x - 2y + 12 = 0$.

The point $R(1,\ 3)$ lies on the line l whose equation is $3x + y - 6 = 0$.



(i) Find the coordinates of the point P where the lines k and l intersect on the y axis. **2**

(ii) Find the equation of the line m which joins Q and R. **2**

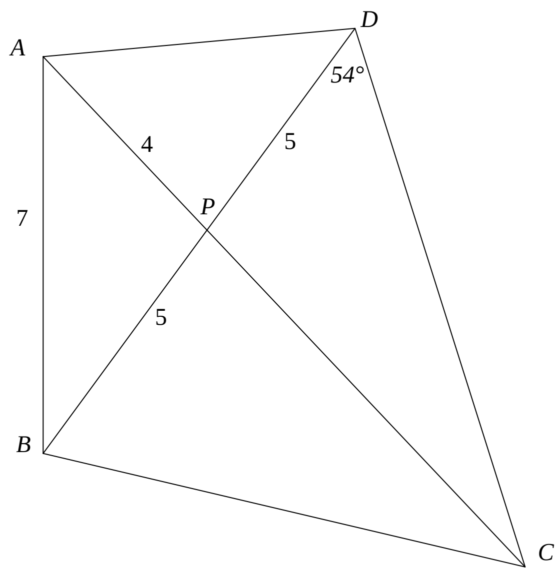
(iii) Find the perpendicular distance from P to the line m. **2**

(iv) Hence, or otherwise, find the exact value of the area of the triangle PQR. **2**

Question 3 (12 marks) Start a new booklet.

(a) Solve $\tan\theta = -1.2$ for $0° \le \theta \le 360°$, correct to the nearest minute. **2**

(b) In the diagram, $\angle PDC = 54°$, $AB = 7$, $AP = 4$, and $BP = PD = 5$.



(i) Show that $\angle APB = 102°$ to the nearest degree. **2**

(ii) Find the length of DC to the nearest whole number. **2**

(iii) Calculate the area of ΔBAP correct to 1 decimal place. **2**

(c) (i) Show that the point $\left(-2, -\frac{7}{4}\right)$ lies on the curve $y = 2^x - 2$. **1**

(ii) Sketch its graph showing all the main features. **2**

(iii) If this graph is shifted 1 unit to the right what will its new equation be? **1**

Question 4 (12 marks) Start a new booklet.

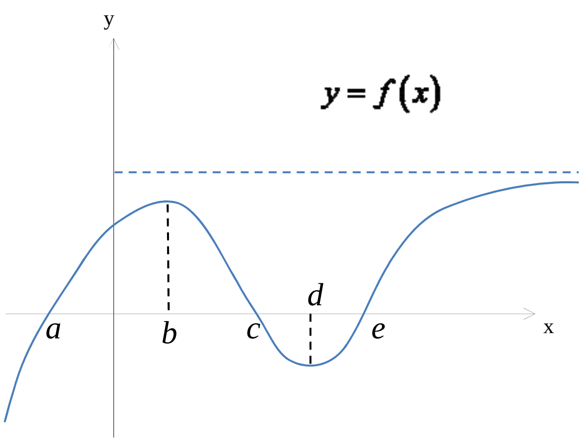
(a) Differentiate each of the following with respect to x. Give your answers in simplest form.

(i) $y = 2x^3 + 5x - 1$ **1**

(ii) $y = \dfrac{7x^2 - 3}{x}$ **2**

(iii) $y = \dfrac{2x+3}{1-x}$ **2**

(b) The following questions refer to the function $y = f(x)$ which has been sketched below.



(i) For which values of x is $f'(x) < 0$? **1**

(ii) Copy or trace the graph onto your writing booklet. On a separate number plane, sketch the gradient function $y = f'(x)$. **3**

(c) Sketch the region on the number plane that satisfies $y \geq 4 - 3x - x^2$ and $x \geq 0$ simultaneously. **3**

Question 5 (12 marks) Start a new booklet.

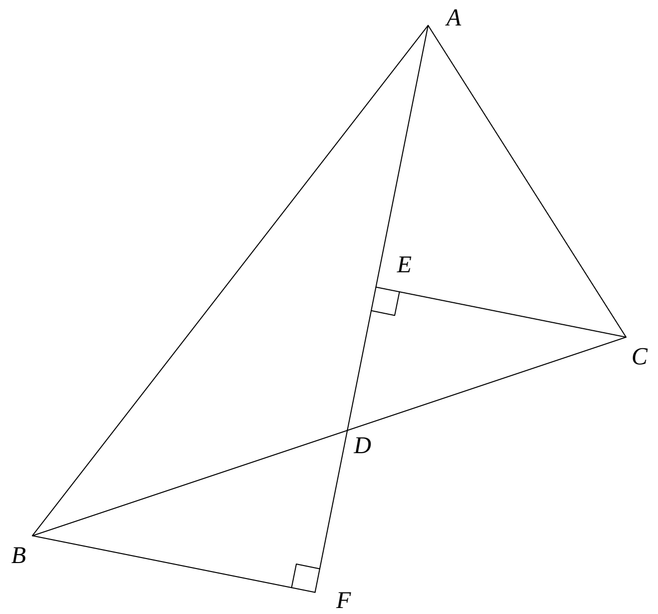
(a) (i) Find the equation of the tangent to the curve $y = x^2 + x - 3$ at the point where $x = -2$. **3**

(ii) Find the co-ordinates of another point on this curve where the tangent is perpendicular to the tangent found in (i). **3**

(b) Find the centre and radius of the circle $x^2 - 6x + y^2 + 10y = 0$ **2**

(c) In the diagram, BC is bisected by AF at D. $CE \perp AD$ and $BF \perp DF$.

Copy or trace the diagram onto your page.



(i) Prove $\Delta BDF \equiv \Delta CDE$. **3**

(ii) Prove $CE = BF$. **1**

Question 6 (12 marks) Start a new booklet.

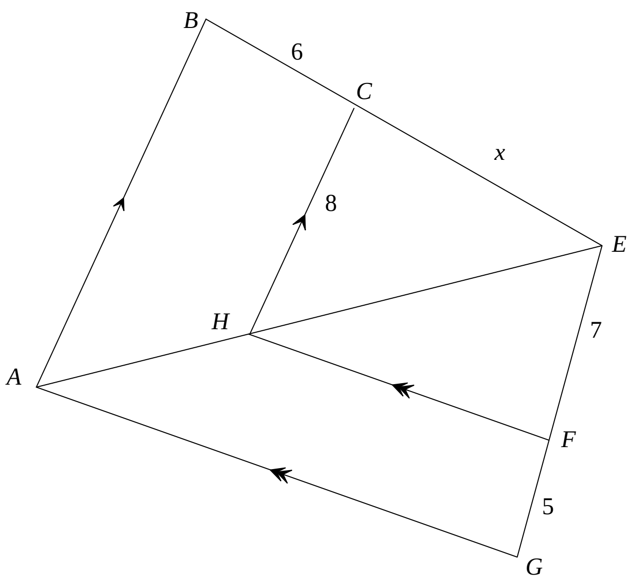
(a) (ii) Show that the equation of the normal to the curve $y = 1 + \dfrac{3}{x-1}$ at the point $(2,4)$ is $x - 3y + 10 = 0$. **3**

(ii) Find the angle that the normal makes with the positive x axis **1**

(b) (i) Prove the identity $\dfrac{1}{\cot\theta - \cos\theta} = \dfrac{\tan\theta}{1-\sin\theta}$ **2**

(ii) Hence prove the identity $\dfrac{1}{\cot\theta - \cos\theta} = \dfrac{\sin\theta + \sin^2\theta}{\cos^3\theta}$. **2**

(c) In the diagram $FH \parallel GA$ and $CH \parallel BA$, $BC = 6$, $CE = x$, $EF = 7$ and $FG = 5$.



(i) Find the value of x, giving reasons. **2**

(ii) Find the length of BA, without giving reasons. **1**

(iii) Find the ratio of the area of triangle EHF to the area of trapezium $FHAG$. No reasons are needed. **1**

Question 7 (12 marks) Start a new booklet

(a) Consider the function $f(x)=\sqrt{x^2-5}$.

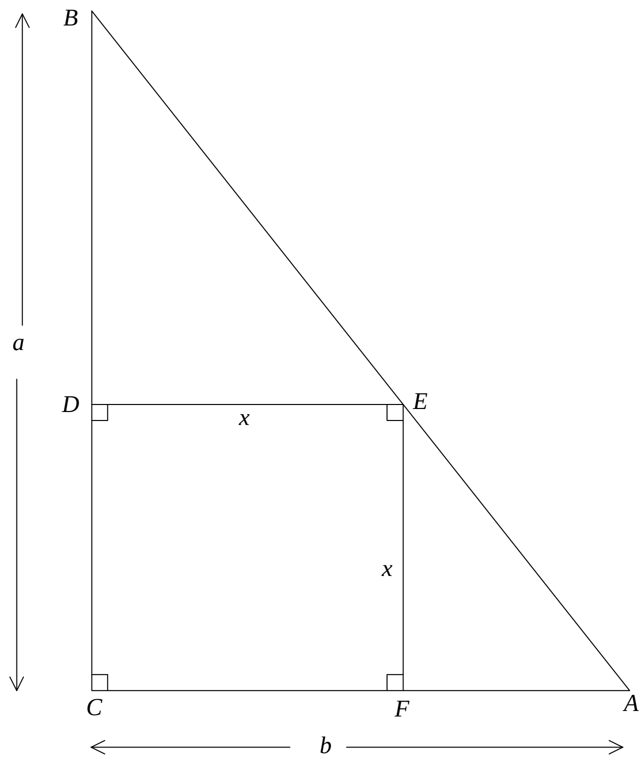
(i) State the domain of the function. **2**

(ii) Find $f'(x)$. **2**

(iii) Explain why $f'(x)$ is never zero. **1**

(b) Sketch the curve $y=|5-2x|$ showing all important features. **2**

(c) Triangle *ABC* is right-angled at *C*. A square of side length *x* is inscribed inside the triangle, as shown.



(i) State the similarity test used to show that $\Delta BDE \parallel\!| \Delta EFA$. You do not have to complete a proof. **1**

(ii) Prove that $x=\dfrac{ab}{a+b}$. **2**

(ii) Prove that if the area of the square is to be half of the area of the triangle *ABC*, then *a* must equal *b*. **2**

End of paper

Question 1 SOLUTIONS YR 11 PRELIMINARY MATHEMATICS EXAM 2012

a) $1.41365 = 1.414$

b) $(5m-2)(m+1) = 0$

$m = \frac{2}{5}, -1$

c) $\frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$\frac{2(3+\sqrt{5})}{9-5}$

$= \frac{3+\sqrt{5}}{2}$

$= \frac{3}{2} + \frac{1}{2}\sqrt{5}$ in the form $a+b\sqrt{5}$

d) $\frac{(x+2)(x^2-2x+4)}{(x+2)(x-3)}$

$= \frac{x^2-2x+4}{x-3}$

e) $32x^5 \times \frac{x^{+2}}{9}$

$= \frac{32x^7}{9}$

f) $\sec^2 30° - \cos 180°$

$= \left(\frac{1}{\frac{\sqrt{3}}{2}}\right)^2 - (-1)$

$= \frac{4}{3} + 1$

$= \frac{7}{3}$

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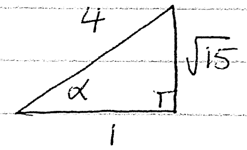
Question 2

$\cos \alpha = \frac{1}{4}$



$\sin^2 \alpha + \tan \alpha$

$= \left[1 - \left(\frac{1}{4}\right)^2\right] + \sqrt{15}$

$= \frac{15}{16} + \sqrt{15}$

b) $\frac{a}{a+b} - \frac{b}{(a+b)(a-b)}$

$= \frac{a^2 - ab - b}{(a+b)(a-b)}$

c) (i) If they intersect on y axis, then sub $x=0$ in into line k:

$7 \times 0 - 2y + 12 = 0$

$\therefore y = 6$

Check $x=0$, $y=6$ satisfies line l:

$3x + y - 6$

$= 0 + 6 - 6$

$= 0$

$\therefore$ Lines k and l meet at $(0,6)$

(ii) $m_{QR} = \frac{3+1}{1+2}$

$= \frac{4}{3}$

Equation of QR: $y - 3 = \frac{4}{3}(x-1)$

$3y - 9 = 4x - 4$

$\therefore 4x - 3y + 5 = 0$

(iii) $d = \frac{|4 \times 0 - 3 \times 6 + 5|}{\sqrt{4^2 + 3^2}}$

$= \frac{13}{5}$

(iv) $\therefore$ Distance $QR = \sqrt{(1+2)^2 + (3+1)^2}$

$= 5$

Area of $\Delta PQR = \frac{1}{2} \times \frac{13}{5} \times 5 = 6.5$ sq units

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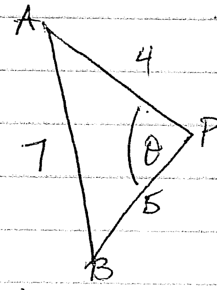
Question 3

a) $\tan\theta = -1.2 \qquad 0 \leq \theta \leq 360^\circ$

$\theta = 180^\circ - 50^\circ 12' \ , \ 360^\circ - 50^\circ 12'$

$= 129^\circ 48', \ 309^\circ 48'$

b) By the cosine rule



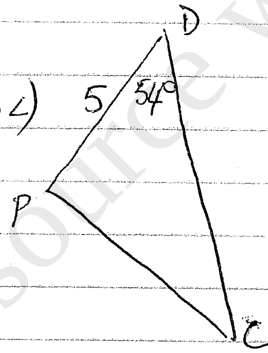
$$\cos\theta = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5}$$

$$= -\frac{1}{5}$$

$\theta = 101.53$

$= 102^\circ$ (nearest degree)

$\therefore \angle APB = 102^\circ$



(ii) $\angle DPC = 102^\circ$ (vertically opp $\angle$)

$\angle DCP = 180 - (102 + 54)$

$= 24^\circ$

Using the sine rule

$$\frac{DC}{\sin 102^\circ} = \frac{5}{\sin 24^\circ}$$

$DC = 12$ (nearest whole number)

(iii) Area $BAP = \frac{1}{2} 4 \times 5 \times \sin 102^\circ$

$= 9.8$ sq units

c) Sub $x = -2$ into $y = 2^x - 2$

$= 2^{-2} - 2$

$= \frac{1}{4} - 2$

$= -\frac{7}{4}$

$\therefore \left(-2, -\frac{7}{4}\right)$ lies on the curve.

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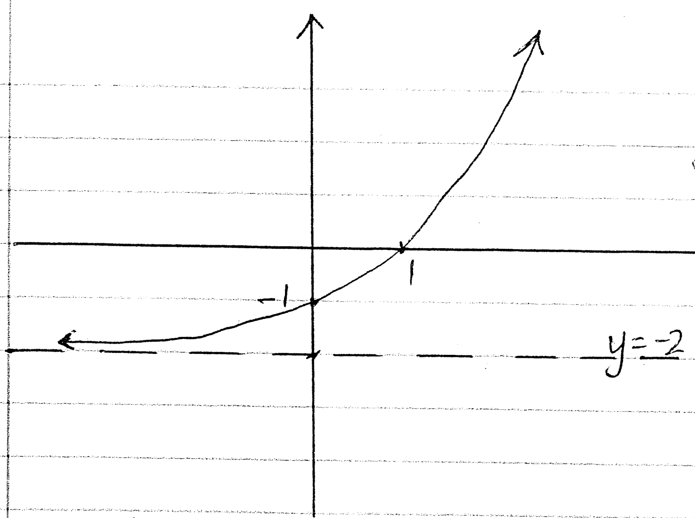
(ii)



$y = 2^x - 2$

$x = 0, \quad y = 2^0 - 2$
$= -1$

$x = 1, \quad y = 0$

(iii) Shifted one unit to the right: replace x with $x-1$

$\therefore \; y = 2^{x-1} - 2$

<u>Question 4</u>

(i) $y = 2x^3 + 5x - 1$

$\frac{dy}{dx} = 6x^2 + 5$

(ii) $y = \frac{7x^2}{x} - \frac{3}{x}$

$= 7x - 3x^{-1}$

$\frac{dy}{dx} = 7 + 3x^{-2}$

$= 7 + \frac{3}{x^2}$

(iii) $y = \frac{2x+3}{1-x}$ $\qquad u = 2x+3 \qquad v = 1-x$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = -1$

$\frac{dy}{dx} = \frac{(1-x)(2) - (2x+3)(-1)}{(1-x)^2}$

$= \frac{2 - 2x + 2x + 3}{(1-x)^2}$

$= \frac{5}{(1-x)^2}$

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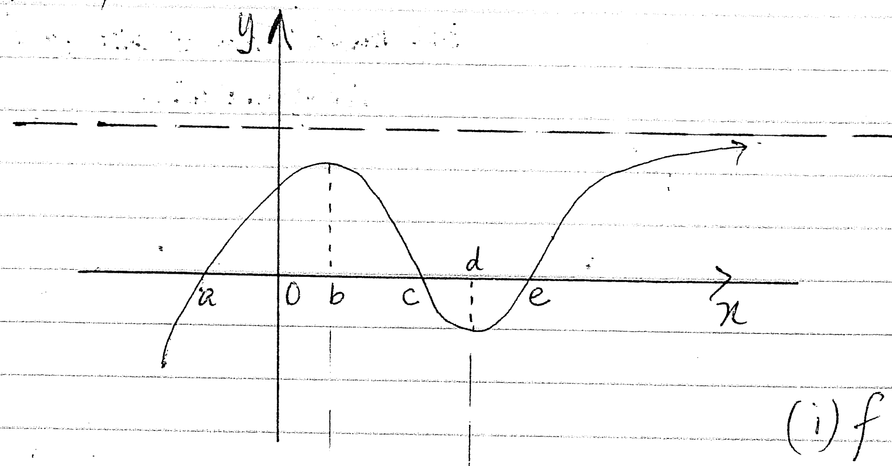
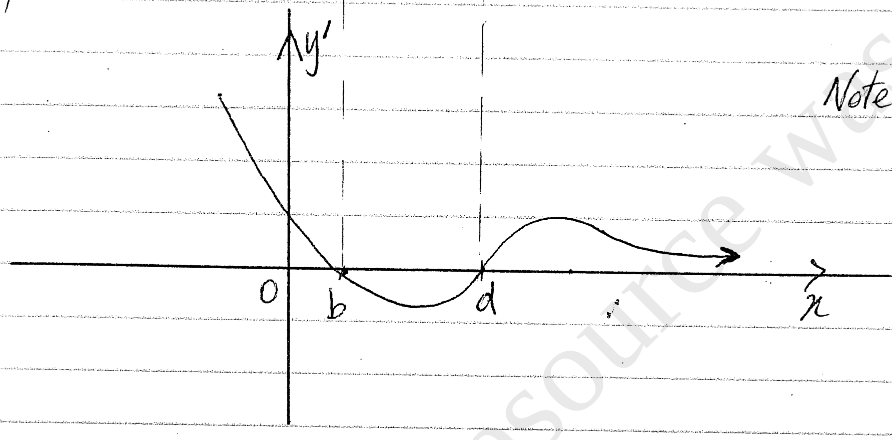
Question 4



(i) $f'(x) < 0$ when $b < x < d$

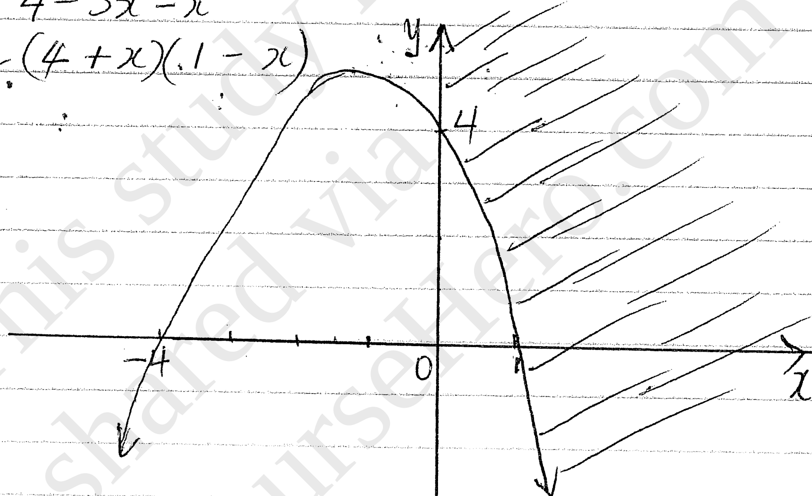
(ii) :



Note: as $x \to \infty$, $f'(x) \to 0$

c) i $y \geq 4 - 3x - x^2$

$y \geq (4+x)(1-x)$



$\therefore$ $x \geq 0$ (right half plane)

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Question 5

a) (i) $y = x^2 + x - 3$

$\frac{dy}{dx} = 2x + 1$

When $x = -2$, $\frac{dy}{dx} = 2(-2) + 1$
$= -3$

Equation tangent, passing through $(-2, -1)$

$$y + 1 = -3(x+2)$$

$$y = -3x - 7 \quad \text{or} \quad 3x + y + 7 = 0.$$

(ii) If other tangent is perpendicular then $m = \frac{1}{3}$

Solve $\frac{1}{3} = \frac{dy}{dx}$

$$2x + 1 = \frac{1}{3}$$

$$x = -\frac{1}{3}$$

$$\therefore y = \left(-\frac{1}{3}\right)^2 - \frac{1}{3} - 3$$

$$= -3\tfrac{2}{9} \quad \text{OR} \quad -\frac{29}{9}$$

The other point has co-ordinates $\left(-\frac{1}{3}, -3\tfrac{2}{9}\right)$

b) $x^2 - 6x + y^2 + 10y = 0$

Completing the square

$$x^2 - 6x + 9 + y^2 + 10y + 25 = 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 34$$

$\therefore$ Centre $(3, -5)$ and radius $= \sqrt{34}$

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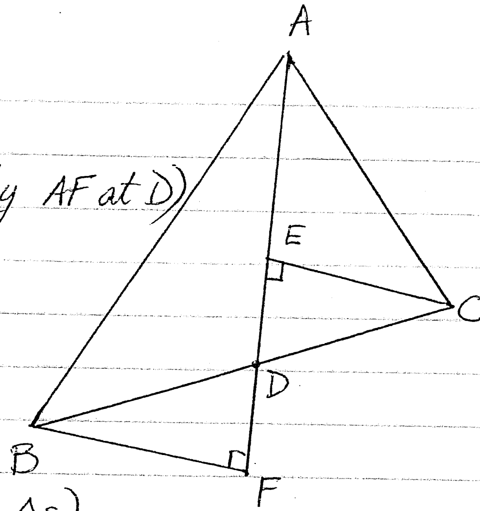
c)

(i) In $\triangle BDF$ $\triangle CDE$



$BD = DC$ (given BC is bisected by AF at D)

$\angle BDF = \angle CDE$ (vertically opposite)

$\angle DFB = \angle CED$ (both $90°$)

$\therefore \triangle BDF \equiv \triangle CDE$ (AAS)

(ii) $CE = BF$ (matching sides of congruent $\triangle$s)

Question 6

a) $y = 1 + 3(x-1)^{-1}$

$\frac{dy}{dx} = -3(x-1)^{-2} \times 1$

$= -\frac{3}{(x-1)^2}$

At $(2,4)$, $\frac{dy}{dx} = -3$

Equation of normal: $y - 4 = \frac{1}{3}(x-2)$ $\boxed{m_1 m_2 = -1}$

$3y - 12 = x - 2$

$0 = x - 3y + 10$ is required equation

(ii) $\tan\theta = \frac{1}{3}$

$\theta = 18°26'$ (nearest minute)

b) (i)

$LHS = \frac{1}{\cot\theta - \cos\theta} \times \frac{\tan\theta}{\tan\theta}$

$= \frac{\tan\theta}{1 - \cos\theta \frac{\sin\theta}{\cos\theta}}$

$= \frac{\tan\theta}{1 - \sin\theta}$

(ii) $\frac{1}{\cot\theta - \cos\theta} = \frac{\tan\theta}{1 - \sin\theta}$ by part (i)

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1-\sin^2\theta}$

$= \frac{\sin\theta + \sin^2\theta}{\cos^3\theta}$

Since $1 - \sin^2\theta = \cos^2\theta$

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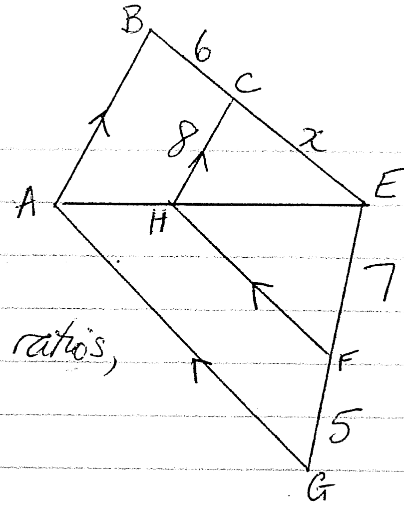
Question 6 (cont)

6 c)



(i) $\frac{EF}{FG} = \frac{EH}{HA}$ (parallel lines preserve ratios, $HF \parallel AG$)

$\frac{EH}{HA} = \frac{EC}{CB}$ (parallel lines preserve ratios, $HC \parallel AB$)

$\therefore \frac{EF}{FG} = \frac{EC}{CB}$

$\therefore \frac{7}{5} = \frac{x}{6}$

$x = 8 \cdot 4$

(ii) $\frac{CH}{BA} = \frac{EC}{EB}$ (matching sides of similar Δs) ← reason not required

$\therefore \frac{8}{BA} = \frac{x}{6+x}$

$8(14 \cdot 4) = 8 \cdot 4\,(BA)$

$BA = 13 \cdot 7$ (1 d.p)

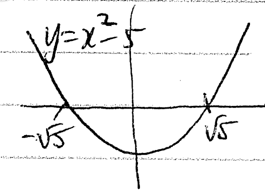
(iii) $\frac{\text{Area } \Delta EHF}{\text{Area Trapezium } FHAG} = \frac{7^2}{12^2 - 7^2}$

$= \frac{49}{9}$

7 a) $f(x) = (x^2 - 5)^{1/2}$

(i) Domain: $x^2 - 5 \geq 0$

$x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$



(ii) $f'(x) = \frac{1}{2}(x^2 - 5)^{-1/2} \cdot (2x)$

$= \frac{x}{\sqrt{x^2 - 5}}$

(iii) $f'(x) = 0$ when $x = 0$ but $x = 0$ is not in the domain

Since $x \geq \sqrt{5}$ or $x \leq -\sqrt{5}$ then $f'(x)$ is never zero.

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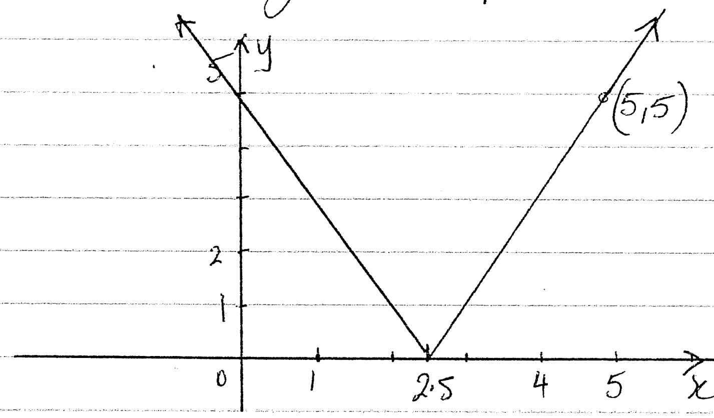
Q 7 (cont)

b) $y = |5-2x|$ is the same as $y = |2x-5|$

$= 2|2\tfrac{1}{2} - x|$

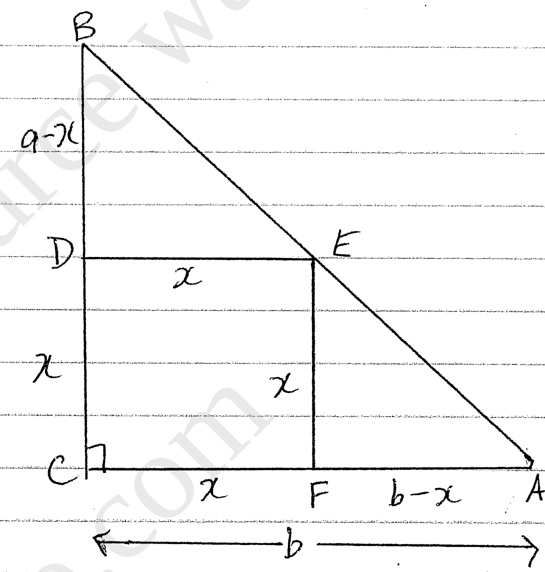


$x=0,\ y=5$

$y=0,\ x=2\tfrac{1}{2}$

gradient 2 and -2

c) (i) $\Delta BDE \ ||| \ \Delta EFA$ (equiangular)



(ii) $\dfrac{a-x}{x} = \dfrac{x}{b-x}$

$(a-x)(b-x) = x^2$

$ab - x(a+b) + x^2 = x^2$

$ab - x(a+b) = 0$

$\therefore ab = x(a+b)$

$x = \dfrac{ab}{a+b}$

(iii) Area of square $= \frac{1}{2}$ area of triangle

$x^2 = \frac{1}{2}\left[\frac{1}{2} \cdot ab\right]$

$\left(\dfrac{ab}{a+b}\right)^2 = \dfrac{1}{4}ab$

$4a^2b^2 = ab(a+b)^2$

$0 = ab(a+b)^2 - 4a^2b^2$

$= ab[a^2+2ab+b^2-4ab]$

$= ab[a^2-2ab+b^2]$

$= ab(a-b)^2$

Since $ab \neq 0$ then $(a-b)^2 = 0 \Rightarrow a = b$

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