NORTH SYDNEY GIRLS HIGH SCHOOL

2014

YEARLY EXAMINATION

Preliminary Mathematics

General Instructions
Reading Time – 5 minutes
Working Time – 2 hours
Write using black or blue pen
Diagrams may be done in pencil
All necessary working should be shown in every question.

Total Marks – 100
Section 1 – Q 1-7 worth 7 marks
Section 2 – Q 8-13 each worth 13 marks

At the end of the examination, place your solution booklets in order and place them inside this question paper.
Submit one bundle. The bundle will be separated for marking so please ensure your number is written on each solution booklet.

Student Name:________________________________

Student Number: ________________________________

Teacher: ________________________________

QUESTION	MARK
1-7	/7
8	/12
9	/12
10	/12
11	/12
12	/12
13	/12
TOTAL	/85

Section 1

7 marks
Attempt Questions 1–7
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–7.

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1. What is the correct factorisation of $8x^3 - 27y^3$?

(A) $(2x-3y)(4x^2+6xy+9y^2)$

(B) $(2x-3y)(4x^2-6xy+9y^2)$

(C) $(2x-3y)(4x^2+12xy+9y^2)$

(D) $(2x-3y)(4x^2-12xy+9y^2)$

2. What is the correct solution to the inequation $x^2 \geq 4x$?

(A) $x \geq 4$

(B) $x \leq -2,\ x \geq 2$

(C) $x \leq 0,\ x \geq 4$

(D) $0 \leq x \leq 4$

3. What is the value of $\lim_{x \to 10} \dfrac{x^2 - 100}{x - 10}$?

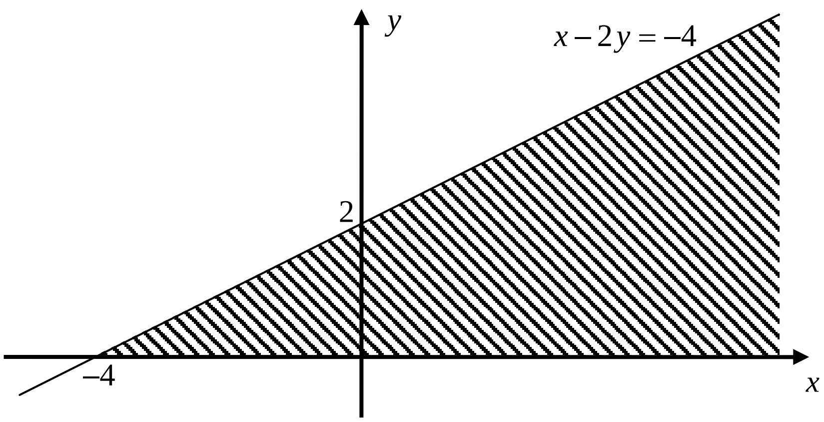
(A) undefined

(B) 0

(C) 10

(D) 20

4. Consider the following diagram showing the line $x - 2y = -4$:



Which pair of inequations describes the shaded region?

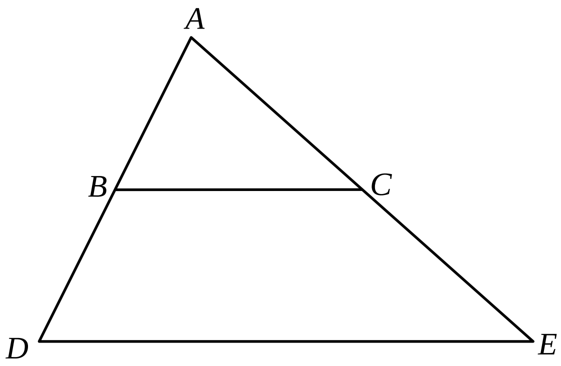
(A) $x - 2y \leq -4$ and $y \geq 0$

(B) $x - 2y \geq -4$ and $y \geq 0$

(C) $x - 2y \leq -4$ and $x \geq 0$

(D) $x - 2y \geq -4$ and $x \geq 0$

5. Consider the following diagram:



Which statement is not **necessarily** true?

(A) If $BC \parallel DE$, then $AB : BD = AC : CE$.

(B) If $AB : BD = AC : CE$, then $BC \parallel DE$.

(C) If B and C are the midpoints of AD and AE respectively, then $BC : DE = 1 : 2$.

(D) If $BC : DE = 1 : 2$, then B and C are the midpoints of AD and AE respectively.

6. Which value(s) of x are the solutions of the equation $\log_7\left(36 - x^2\right) - \log_7 x = \log_7 5$?

(A) $x = -9$ only

(B) $x = 4$ only

(C) $x = -9$ and $x = 4$

(D) There are no real solutions

7. Given that $f(x) = x^3$ and $g(x) = \sin x$ are both ODD functions, which one of the following functions is EVEN?

(A) $h(x) = x^3 + \sin x$

(B) $h(x) = x^3 - \sin x$

(C) $h(x) = x^3 \times \sin x$

(D) $h(x) = \sin\left(x^3\right)$

Section II

78 marks
Attempt Questions 8–13
Allow about 1 hour and 50 minutes for this section

Answer each question a separate writing booklet. Extra writing paper is available.
In Questions 8–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (13 marks)

(a) Solve the equation $\frac{x}{3}-\frac{2x+1}{4}=5$. **2**

(b) Write $0.\dot{2}\dot{4}$ as a fraction in simplest terms. Show all working. **2**

(c) Rationalise the denominator of $\frac{\sqrt{2}+4}{2\sqrt{2}-1}$. **2**

Write your answer in the form $a+b\sqrt{2}$, where a and b are rational.

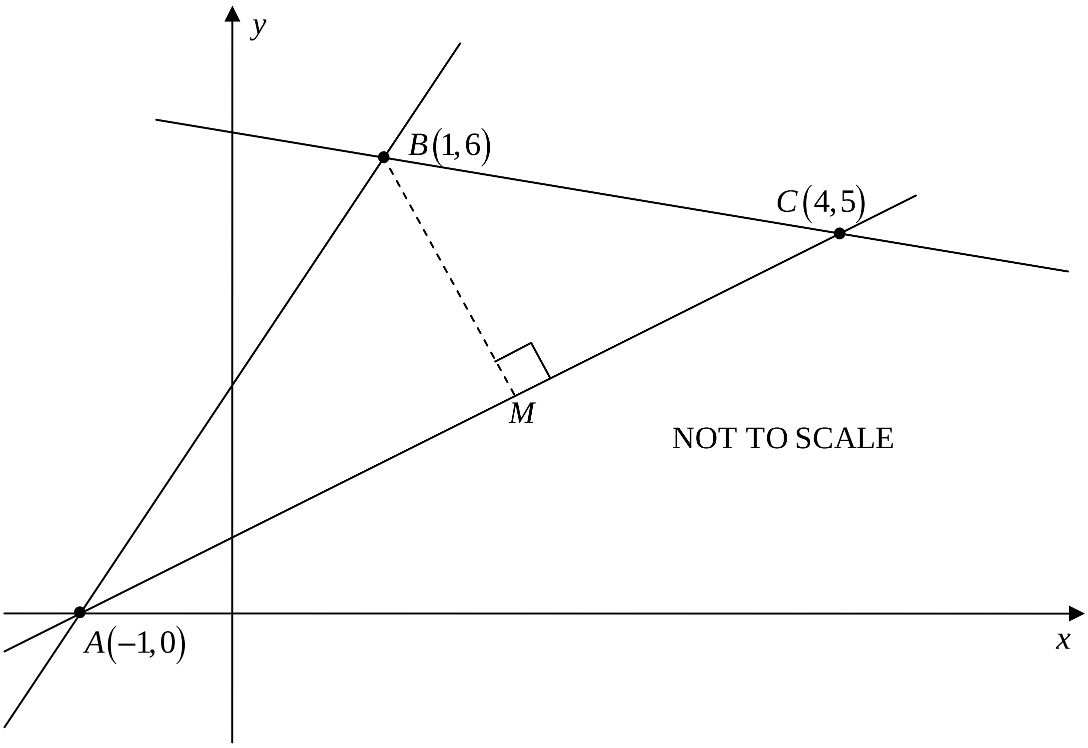
(d) Solve the inequality $|2x-1|\le 5$. **2**

(e) Find the point(s) of intersection of the line $y=1-2x$ and the parabola $y=5-2x-x^2$. **2**

(f) Fully simplify $\frac{pq^{-1}-p^{-1}q}{p^2q^{-2}-p^{-2}q^2}$, writing your answer as a single fraction without the use of negative indices. **3**

Question 9 (13 marks) (Use a SEPARATE writing booklet)

(a) The diagram shows the points $A(-1,\ 0)$, $B(1,\ 6)$ and $C(4,\ 5)$, and $BM \perp AC$.



(i) Show that the gradient of AB is 3. **1**

(ii) Show that $AB \perp BC$. **2**

(iii) Find the length of AB as a simplified surd. **2**

(iv) Show that the equation of the line AC is $x - y + 1 = 0$. **2**

(v) Find the length of BM. **2**

(vi) A line l is drawn through B and parallel to AC. The line l crosses the y-axis at D. Find the area of ΔADC **2**

(b) (i) Find the domain of the function $y = \dfrac{1}{\sqrt{5-x}}$. **1**

(ii) Find the domain of the function $y = \dfrac{1}{\sqrt{5-x}} - \dfrac{1}{\sqrt{x-2}}$. **1**

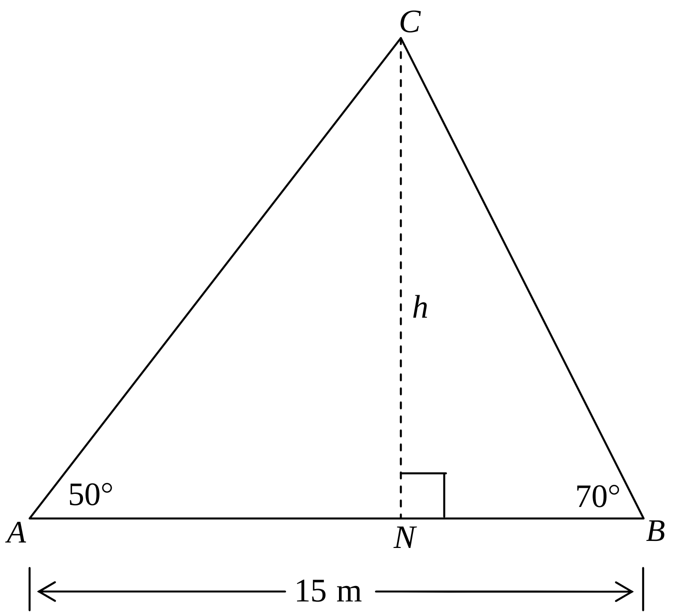
Question 10 (13 marks) (Use a SEPARATE writing booklet)

(a) (i) Solve $8^x = 3$, giving your answer correct to three significant figures. **1**

(ii) Write $2 + \frac{1}{2}\log_3 x$ as a single logarithm. **1**

(b) Solve the equation $\cos 2x = -\frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$. **3**

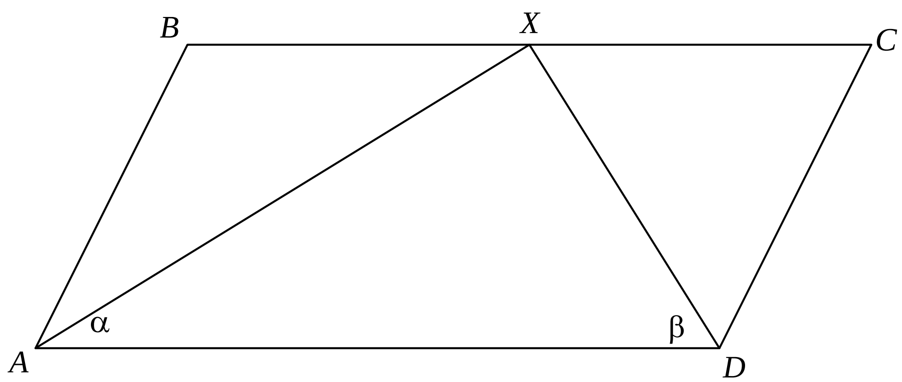
(c) In the following diagram, NC is a vertical pole of height h standing on level ground. N lies on AB, a horizontal line of length 15 metres.



(i) Show that $AN = h\tan 40°$. **1**

(ii) By finding a similar expression for BN or otherwise, find the height of the pole. **2**

(d) $ABCD$ is a parallelogram. AX bisects $\angle BAD$ and DX bisects $\angle CDA$.
Let $\angle DAX = \alpha$ and $\angle ADX = \beta$.



(i) Prove that $AX \perp DX$. **2**

(ii) Prove also that $BC = 2AB$. **3**

Question 11 (13 marks) (Use a SEPARATE writing booklet)

(a) Differentiate:

(i) $9x^2 + x + 2$ **1**

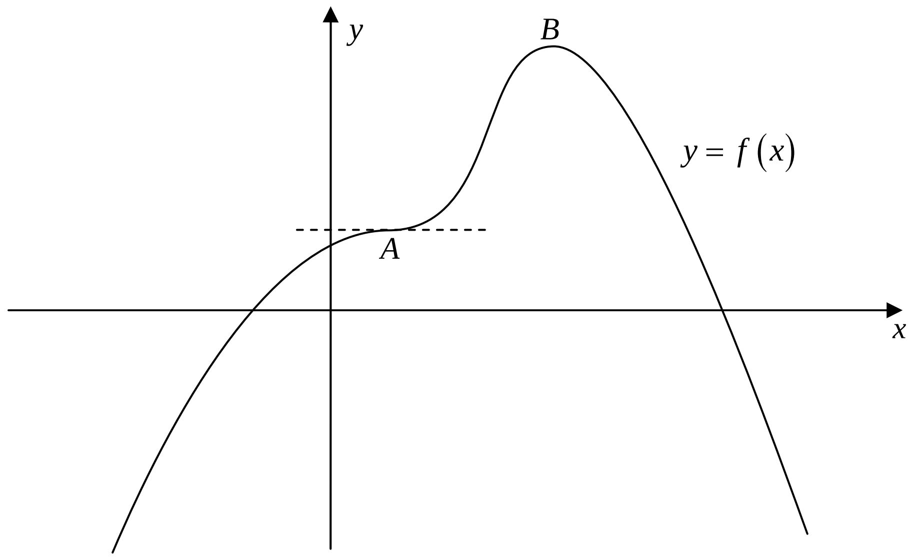
(ii) $\dfrac{3}{\sqrt{1-2x}}$ **2**

(iii) $\dfrac{2x}{1+x^2}$ **2**

(b) Copy the following curve $y = f(x)$ into your answer booklet. **2**

Below this sketch, draw a sketch of the gradient function $y = f'(x)$.

Be sure to line up the features of the two graphs.



(c) Find the equation of the normal to the curve $y = \left(\dfrac{x}{3} - 1\right)^3$ at the point where $x = 9$. **3**

(d) (i) If $f(x) = x^2 - 3x$, write down an expression for $f(x+h) - f(x)$. **1**

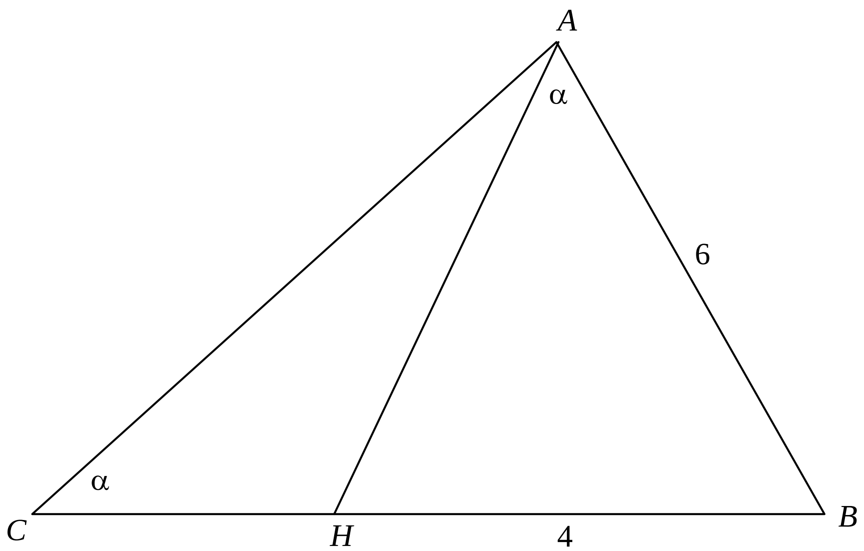
(ii) Hence differentiate $f(x) = x^2 - 3x$ by first principles. **2**

Question 12 (13 marks) (Use a SEPARATE writing booklet)

(a) For what value(s) of x is the tangent to the curve $y = 2x + \frac{1}{x}$ perpendicular to the line $4x + 7y = 0$? **3**

(b) Find the equation of the line that passes through the point of intersection of the lines $7x + 2y - 3 = 0$ and $5x + 9y - 1 = 0$, and which also passes through the point $(-3, 4)$. **3**

(c) In the diagram, $\angle BCA = \angle BAH = \alpha$, $AB = 6$ and $BH = 4$.



(i) Show that $\Delta ABC \,|||\, \Delta HBA$. **2**

(ii) Hence or otherwise find the length of HC. **2**

(d) If $f(x) = (x+1)^2$, solve the equation $f\left[f(x)\right] = 100$. **3**

Question 13 (13 marks) (Use a SEPARATE writing booklet)

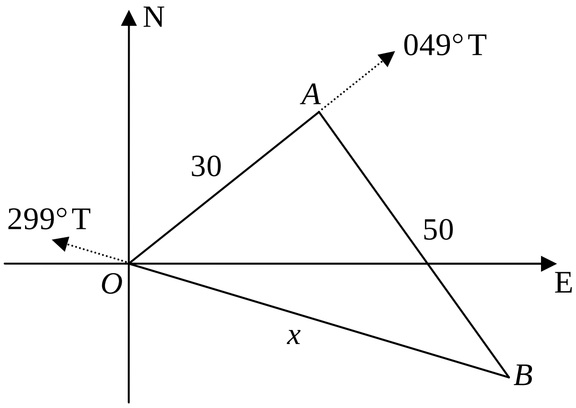
(a) The function $f(x)$ is defined by $f(x) = x^2 - k^2$ on the domain $-k \le x \le 2k$. **2**

Find the range of $f(x)$ in terms of k over this domain.

(b) (i) Prove that $\dfrac{\cos\theta}{\sin\theta + 1} - \dfrac{\cos\theta}{\sin\theta - 1} = 2\sec\theta$. **2**

(ii) Hence or otherwise solve the equation $\dfrac{\cos\theta}{\sin\theta + 1} = 4 + \dfrac{\cos\theta}{\sin\theta - 1}$ for $0 \le \theta \le 360°$. **2**

(c) A ship sails for 30 nautical miles from O to A on a bearing of $049°\,\mathrm{T}$.
It then turns and sails to a point B, 50 nautical miles away.
From B, the starting point O is observed on a bearing of $299°\,\mathrm{T}$.



(i) Show that $\angle AOB = 70°$. **1**

(ii) Show that x satisfies the quadratic equation $x^2 - (60\cos 70°)x - 1600 = 0$. **2**

(iii) Hence find the distance of B from O, giving your answer in nautical miles correct to one decimal place. **2**

(iv) By how many degrees did the ship turn at A? **2**

End of Paper

Year 11 2 Unit Yearly 2014 Solutions

Section I

1. A
2. C
3. D
4. B
5. D
6. B
7. C

Worked Solutions

2.

$$x^2 \ge 4x$$
$$x^2 - 4x \ge 0$$
$$x(x-4) \ge 0$$

$$x \le 0,\ \ x \le 4$$

3.

$$\lim_{x\to 10} \frac{x^2-100}{x-10} = \lim_{x\to 10} \frac{(x-10)(x+10)}{x-10}$$
$$= \lim_{x\to 10}(x+20)$$
$$= 20$$

6. Best done by substitution. But if working was required:

$$\log_7\left(36-x^2\right) - \log_7 x = \log_7 5$$
$$\log_7 \frac{36-x^2}{x} = \log_7 5$$
$$\frac{36-x^2}{x} = 5$$
$$36 - x^2 = 5x$$
$$x^2 + 5x - 36 = 0$$
$$(x+9)(x-4) = 0$$
$$x = -9,\ 4$$

Checking answers: $x = -9$ leads to the log of a negative number
$\therefore$ $x = 4$ is the only solution

7. (A)

$$h(-x) = (-x)^3 + (-\sin x)$$
$$= -x^3 - \sin x$$
$$= -\left(x^3 + \sin x\right)$$
$$= -h(x)$$

(B)

$$h(-x) = (-x)^3 - (-\sin x)$$
$$= -x^3 + \sin x$$
$$= -\left(x^3 - \sin x\right)$$
$$= -h(x)$$

(C)

$$h(-x) = (-x)^3 \times (-\sin x)$$
$$= \left(-x^3\right) \times (-\sin x)$$
$$= x^3 \sin x$$
$$= h(x)$$

(D)

$$h(-x) = \sin\left[(-x)^3\right]$$
$$= \sin\left(-x^3\right)$$
$$= -\sin\left(x^3\right)$$
$$= -h(x)$$

$\therefore$ (C) in the only even function

Section II

Question 8

(a) Solve the equation $\frac{x}{3}-\frac{2x+1}{4}=5$. **2**

$$\frac{x}{3}-\frac{2x+1}{4}=5$$

$$(\times 12)\quad 4x-3(2x+1)=60$$

$$4x-6x-3=60$$

$$2x=-63$$

$$\boxed{x=-\frac{63}{2}}$$

(b) Write $0.\dot{2}\dot{4}$ as a fraction in simplest terms. Show all working. **2**

$$\text{Let } x=0.\dot{2}\dot{4}$$

$$100x=24.\dot{2}\dot{4}$$

$$(\text{subtracting})\quad 99x=24$$

$$x=\frac{24}{99}$$

$$\boxed{x=\frac{8}{33}}$$

(c) Rationalise the denominator of $\frac{\sqrt{2}+4}{2\sqrt{2}-1}$. **2**

Write your answer in the form $a+b\sqrt{2}$, where a and b are rational.

$$\frac{\sqrt{2}+4}{2\sqrt{2}-1}\times\frac{2\sqrt{2}+1}{2\sqrt{2}+1}=\frac{4+\sqrt{2}+8\sqrt{2}+4}{8-1}$$

$$=\boxed{\frac{8}{7}+\frac{9}{7}\sqrt{2}}$$

(d) Solve the inequality $|2x-1|\le 5$ **2**

$$|2x-1|\le 5$$

$$-5\le 2x-1\le 5$$

$$-4\le 2x\le 6$$

$$\boxed{-2\le x\le 3}$$

(e) Find the point(s) of intersection of the line $y = 1 - 2x$ and the parabola $y = 5 - 2x - x^2$ **2**

$$1 - 2x = 5 - 2x - x^2$$
$$x^2 = 4$$
$$x = \pm 2$$

$(x = 2) \quad y = -3$

$(x = -2) \quad y = 5$

$\therefore$ Points of intersection are **$(2, -3)$ and $(-2, 5)$**

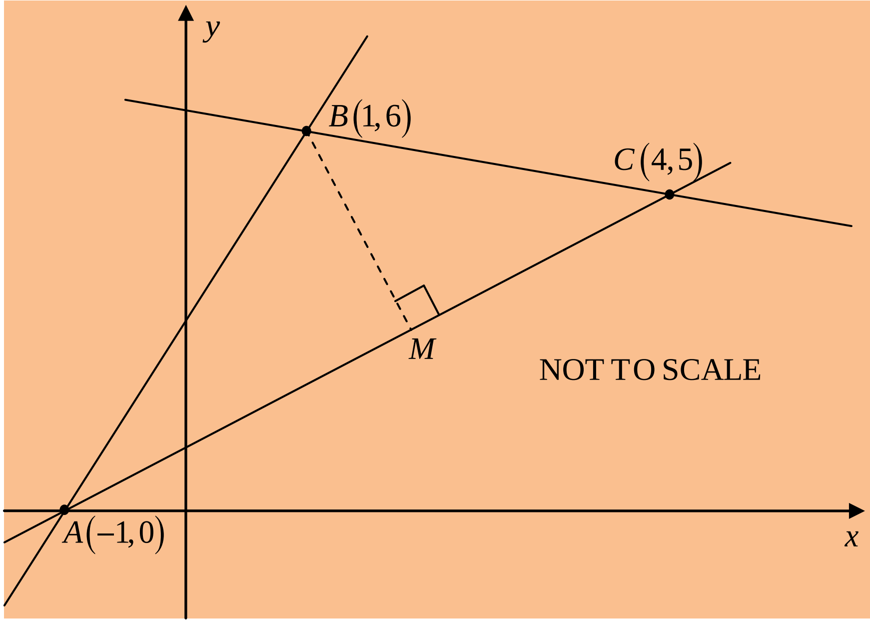
(f) Fully simplify $\dfrac{pq^{-1} - p^{-1}q}{p^2q^{-2} - p^{-2}q^2}$, writing your answer as a single fraction without the use of negative indices. **3**

$$\frac{pq^{-1} - p^{-1}q}{p^2q^{-2} - p^{-2}q^2} = \frac{\dfrac{p}{q} - \dfrac{q}{p}}{\dfrac{p^2}{q^2} - \dfrac{q^2}{p^2}} \times \frac{p^2q^2}{p^2q^2}$$
$$= \frac{p^3q - pq^3}{p^4 - q^4}$$
$$= \frac{pq\left(p^2 - q^2\right)}{\left(p^2 - q^2\right)\left(p^2 + q^2\right)}$$

$$= \boldsymbol{\frac{pq}{p^2 + q^2}}$$

Question 9

(a) The diagram shows the points $A(-1,0)$, $B(1,6)$ and $C(4,5)$, and $BM \perp AC$.



(i) Show that the gradient of AB is 3. **1**

$$m_{AB} = \frac{6-0}{1-(-1)}$$
$$= \frac{6}{2}$$

$$m_{AB} = 3$$

(ii) Show that $AB \perp BC$. **2**

$$m_{BC} = \frac{5-6}{4-1}$$
$$= -\frac{1}{3}$$

$$m_{AB} \cdot m_{BC} = 3 \cdot \left(-\tfrac{1}{3}\right)$$
$$= -1$$

$$\therefore \quad AB \perp BC$$

(iii) Find the length of AB as a simplified surd. **2**

$$AB^2 = \left[1-(-1)\right]^2 + (6-0)^2$$
$$= 40$$
$$AB = \sqrt{40}$$

$$\mathbf{AB = 2\sqrt{10}\ \text{units}}$$

(iv) Show that the equation of the line AC is $x - y + 1 = 0$. **2**

$$m_{AC} = \frac{5-0}{4-(-1)}$$
$$= 1$$

$$AC: \quad y - 5 = 1(x-4)$$
$$x - y + 1 = 0$$

OR: Show by substituting the coordinates of A AND C into the given equation.

(v) Find the length of BM. **2**

$AC:\ x-y+1=0 \qquad B(1,\ 6)$

$$BM=\frac{|1(1)-1(6)+1|}{\sqrt{1^2+(-1)^2}}$$

$\boldsymbol{BM=\frac{4}{\sqrt{2}}}$ OR $\boldsymbol{2\sqrt{2}}$

(vi) A line l is drawn through B and parallel to AC. The line l crosses the y-axis at D. Find the area of ΔADC **2**

ΔADC has the same area as ΔABC since it has the same base length AC, and the same perpendicular height (since $BD \parallel AC$).

So EITHER:

$$\begin{aligned} AC^2 &= (4+1)^2+(5-0)^2 \\ &= 50 \\ AC &= 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}\times AC\times BM \\ &= \frac{1}{2}\times 5\sqrt{2}\times 2\sqrt{2} \end{aligned}$$

$\boldsymbol{\text{Area}=10\ \text{units}^2}$

OR:

$$\begin{aligned} BC^2 &= (4-1)^2+(5-6)^2 \\ &= 10 \\ BC &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}\times AB\times BC \quad (\text{since } AB \parallel BC) \\ &= \frac{1}{2}\times 2\sqrt{10}\times\sqrt{10} \\ &= 10\ \text{units}^2 \end{aligned}$$

(b) (i) Find the domain of the function $y=\frac{1}{\sqrt{5-x}}$. **1**

$5-x>0$

$\boldsymbol{x<5}$

(ii) Find the domain of the function $y=\frac{1}{\sqrt{5-x}}-\frac{1}{\sqrt{x-2}}$. **1**

Also:

$$\begin{aligned} x-2&>0 \\ x&>2 \end{aligned}$$

$\therefore$ $\boldsymbol{2<x<5}$

(Can be written $x>2$ AND $x<5$, but REQUIRES the AND or $\cap$)

Question 10

(a) (i) Solve $8^x = 3$, giving your answer correct to three significant figures. **1**

$$8^x = 3$$
$$x = \log_8 3$$
$$= \frac{\log_{10} 3}{\log_{10} 8}$$

$x = 0.528$

(ii) Write $2 + \frac{1}{2}\log_3 x$ as a single logarithm. **1**

$$2 + \frac{1}{2}\log_3 x = \log_3 9 + \log_3 x^{\frac{1}{2}}$$

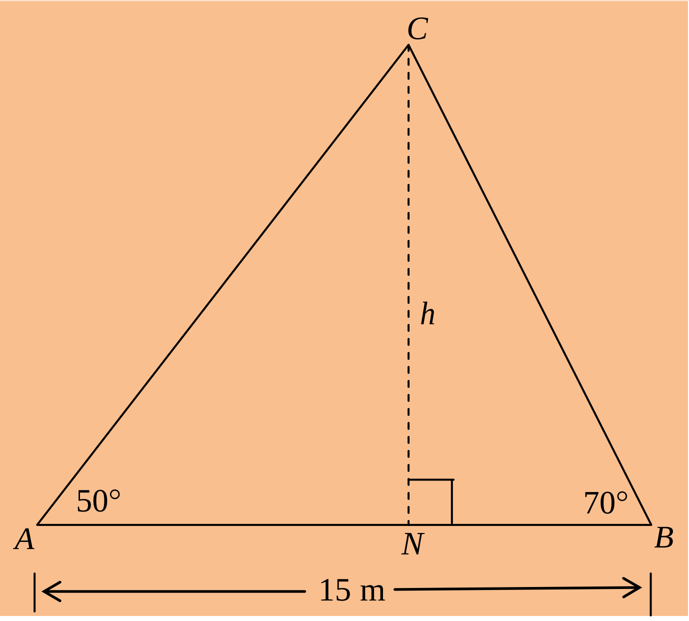
$= \log_3 9\sqrt{x}$

(b) Solve the equation $\cos 2x = -\frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$. **3**

$$\cos 2x = -\frac{\sqrt{3}}{2}$$
$$2x = 180-30,\ 180+30,\ 540-30,\ 540+30$$
$$= 150,\ 210,\ 510,\ 570$$

$x = 75,\ 105,\ 255,\ 285$

(c) In the following diagram, NC is a vertical pole of height h standing on level ground. N lies on AB, a horizontal line of length 15 metres.



(i) Show that $AN = h\tan 40°$. **1**

$$\angle ACN + 50 + 90 = 180 \quad \text{(angle sum of triangle)}$$
$$\angle ACN = 40$$

$$\frac{AN}{h} = \tan 40°$$
$$AN = h\tan 40°$$

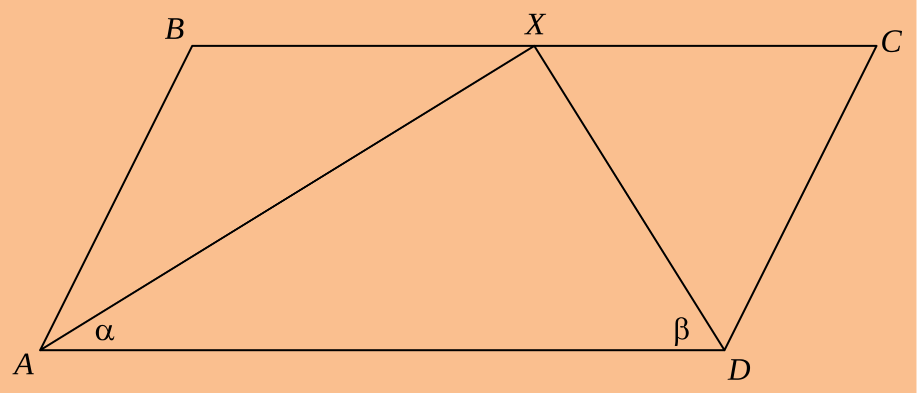
(ii) By finding a similar expression for BN or otherwise, find the height of the pole. **2**

$BN = h\tan 20°$

$$AN + BN = 15$$
$$h\tan 40° + h\tan 20° = 15$$
$$h(\tan 40° + \tan 20°) = \frac{15}{\tan 40° + \tan 20°}$$

$h = 12.47$ m

(d) $ABCD$ is a parallelogram. AX bisects $\angle BAD$ and DX bisects $\angle CDA$.
Let $\angle DAX = \alpha$ and $\angle ADX = \beta$.



(i) Prove that $AX \perp DX$. **2**

$$\angle BAD + \angle CDA = 180° \quad (\text{co-interior angles, } AB \parallel CD\text{, opposite sides of parallelogram})$$
$$2\alpha + 2\beta = 180° \quad (\angle BAX = \angle XAD = \alpha,\ \angle ADX = \angle CDX = \beta)$$
$$\alpha + \beta = 90°$$

$$\angle AXD = 180 - (\alpha + \beta) \quad (\text{angle sum of triangle})$$
$$= 90°$$

ie. $AX \perp DX$

(ii) Prove also that $BC = 2AB$. **3**

$$\angle BAX = \angle DAX \quad (AX \text{ bisects } \angle BAD)$$
$$= \angle XAD \quad (\text{alternate angles, } BC \parallel AD\text{, opposite sides of parallelogram})$$

Similarly $\angle CDX = \angle CDX$

$\therefore$ $BX = AB$ and $CX = CD$ (equal sides opposite equal angles in isosceles triangles)

$$BC = BX + CX$$
$$= AB + CD$$
$$= 2AB \quad (\text{opposite sides of parallelogram equal})$$

Question 11

(a) Differentiate:

(i) $9x^2 + x + 2$ **1**

$$\frac{d}{dx}\left(9x^2 + x + 2\right) \qquad = \boxed{18x+1}$$

(ii) $\dfrac{3}{\sqrt{1-2x}}$ **2**

$$\frac{d}{dx}\left(\frac{3}{\sqrt{1-2x}}\right) = \frac{d}{dx}\left[3(1-2x)^{-\frac{1}{2}}\right]$$

$$= -\frac{3}{2}(1-2x)^{-\frac{3}{2}} \qquad = \boxed{-\frac{3}{2\sqrt{(1-2x)^3}}}$$

(iii) $\dfrac{2x}{1+x^2}$ **2**

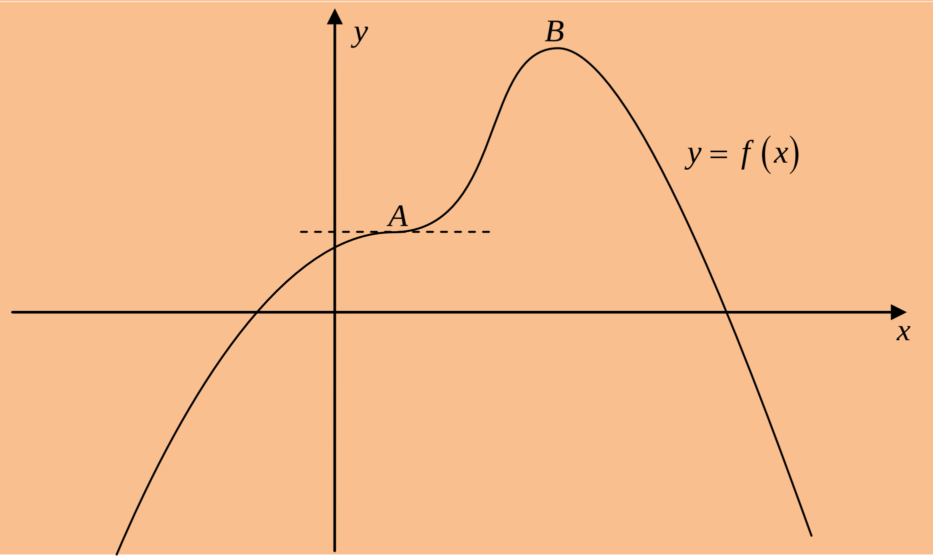
$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = \frac{(1+x^2)\cdot 2 - 2x\cdot 2x}{(1+x^2)^2}$$

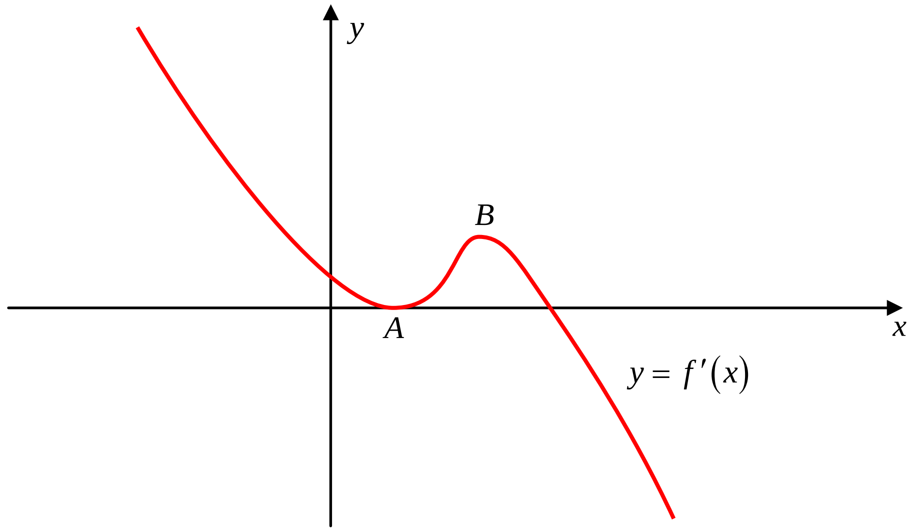
$$= \frac{2+2x^2-4x^2}{(1+x^2)^2} \qquad = \boxed{\frac{2(1-x^2)}{(1+x^2)^2}}$$

(b) Copy the following curve $y = f(x)$ into your answer booklet. **2**

Below this sketch, draw a sketch of the gradient function $y = f'(x)$.

Be sure to line up the features of the two graphs





(c) Find the equation of the normal to the curve $y=\left(\frac{x}{3}-1\right)^3$ at the point where $x=9$. **3**

$$y=\left(\frac{x}{3}-1\right)^3$$

$$\frac{dy}{dx}=3\left(\frac{x}{3}-1\right)^2\cdot\frac{1}{3}$$

$$=\left(\frac{x}{3}-1\right)^2$$

$(x=9)$

$$m_T=\left(\frac{9}{3}-1\right)^2$$

$$=4$$

$$m_N=-\frac{1}{4}$$

$$y=\left(\frac{9}{3}-1\right)^3$$

$$=8$$

(Normal) $y-8=-\frac{1}{4}(x-9)$

$$4y-32=-x+9$$

$$\boldsymbol{x+4y-41=0}$$

(d) (i) If $f(x)=x^2-3x$, write down an expression for $f(x+h)-f(x)$. **1**

$$f(x+h)-f(x)=\boldsymbol{(x+h)^2-3(x+h)-(x^2-3x)}$$

(ii) Hence differentiate $f(x)=x^2-3x$ by first principles. **2**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$$

$$=\boldsymbol{2x-3}$$

Question 12

(a) For what value(s) of x is the tangent to the curve $y = 2x + \frac{1}{x}$ perpendicular to the line $4x + 7y = 0$? **3**

$$y = 2x + \frac{1}{x}$$
$$= 2x + x^{-1}$$

$$\frac{dy}{dx} = 2 - x^{-2}$$
$$= 2 - \frac{1}{x^2}$$

$$4x + 7y = 0$$
$$y = -\frac{4}{7}x$$
$$m = -\frac{4}{7}$$

$$2 - \frac{1}{x^2} = \frac{7}{4}$$
$$\frac{1}{x^2} = \frac{1}{4}$$
$$x = \pm 2$$

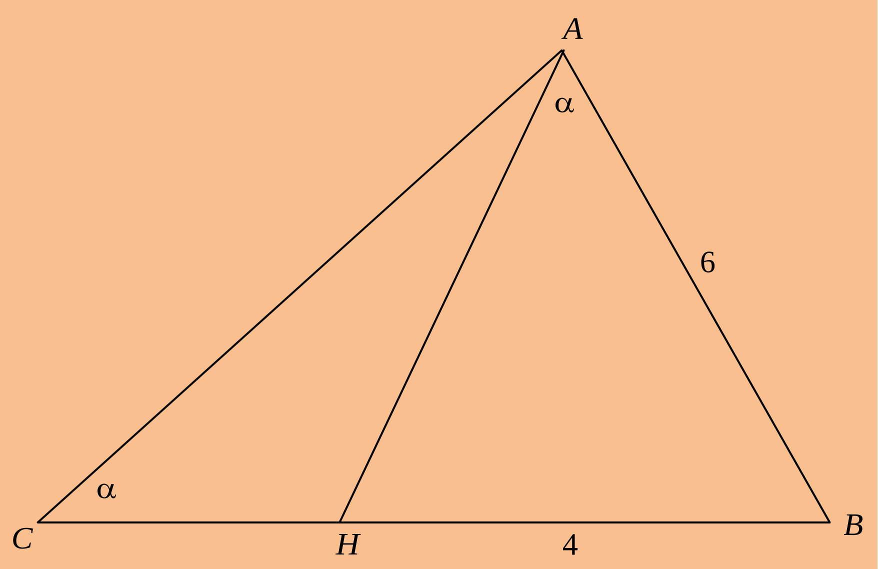
(b) Find the equation of the line that passes through the point of intersection of the lines $7x + 2y - 3 = 0$ and $5x + 9y - 1 = 0$, and also passes through the point $(-3, 4)$. **3**

$$7x + 2y - 3 + k(5x + 9y - 1) = 0$$
$$(-3, 4) \quad -21 + 8 - 3 + k(-15 + 36 - 1) = 0$$
$$-16 + 20k = 0$$
$$k = \frac{4}{5}$$

$$7x + 2y - 3 + \frac{4}{5}(5x + 9y - 1) = 0$$
$$5(7x + 2y - 3) + 4(5x + 9y - 1) = 0$$
$$35x + 10y - 15 + 20x + 36y - 4 = 0$$

$$\boxed{55x + 46y - 19 = 0}$$

(c) In the diagram, $\angle BCA = \angle BAH = \alpha$, $AB = 6$ and $BH = 4$.



(i) Show that $\Delta ABC \parallel\!| \Delta HBA$. **2**

$\angle ACB = \angle HAB = \alpha$ (given)
$\angle ABC = \angle HBA$ (common)
$\therefore\ \Delta ABC \parallel\!| \Delta HBA$ (equiangular)

(ii) Hence or otherwise find the length of HC. **2**

$$\frac{BC}{BA} = \frac{BA}{BH}$$ (corresponding angles of similar triangles)

$$\frac{BC}{6} = \frac{6}{4}$$

$$BC = 9$$

$HC = 5$

(d) If $f(x) = (x+1)^2$, solve the equation $f\left[f(x)\right] = 100$. **3**

$$f\left[f(x)\right] = 100$$

$$\left[(x+1)^2 + 1\right]^2 = 100$$

$$(x+1)^2 + 1 = \pm 10$$

$$(x+1)^2 = 9,\ \cancel{-11}$$

$$x + 1 = \pm 3$$

$$x = -4,\ 2$$

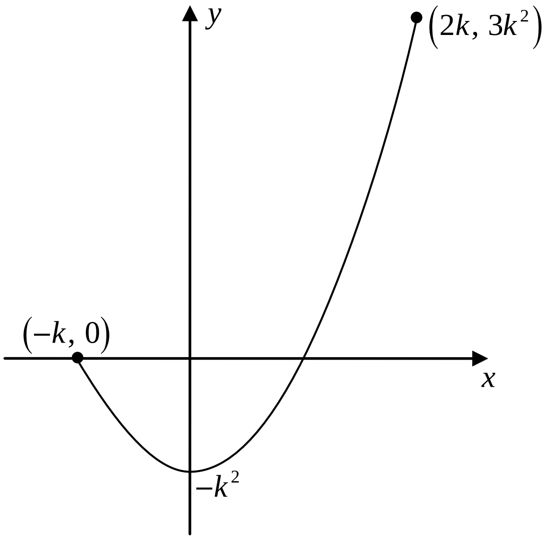
Question 13

(a) The function $f(x)$ is defined by $f(x) = x^2 - k^2$ on the domain $-k \le x \le 2k$. **2**

Find the range of $f(x)$ in terms of k over this domain.

$f(0) = -k^2$ $\quad$ $f(-k) = 0$ $\quad$ $f(2k) = (2k)^2 - k^2$

$= 3k^2$



$\therefore$ Range: $-k^2 \le f(x) \le 3k^2$

(b) (i) Prove that $\dfrac{\cos\theta}{\sin\theta+1} - \dfrac{\cos\theta}{\sin\theta-1} = 2\sec\theta$. **2**

$$\begin{aligned}
\text{LHS} &= \frac{\cos\theta}{\sin\theta+1} - \frac{\cos\theta}{\sin\theta-1} \\
&= \frac{\cos\theta(\sin\theta-1) - \cos\theta(\sin\theta+1)}{(\sin\theta+1)(\sin\theta-1)} \\
&= \frac{\sin\theta\cos\theta - \cos\theta - \sin\theta\cos\theta - \cos\theta}{\sin^2\theta - 1} \\
&= \frac{-2\cos\theta}{-\cos^2\theta} \\
&= \frac{2}{\cos\theta} \\
&= 2\sec\theta \\
&= \text{RHS}
\end{aligned}$$

(ii) Hence or otherwise solve the equation $\dfrac{\cos\theta}{\sin\theta+1} = 4 + \dfrac{\cos\theta}{\sin\theta-1}$ for $0 \le \theta \le 360°$. **2**

$$\frac{\cos\theta}{\sin\theta+1} = 4 + \frac{\cos\theta}{\sin\theta-1}$$

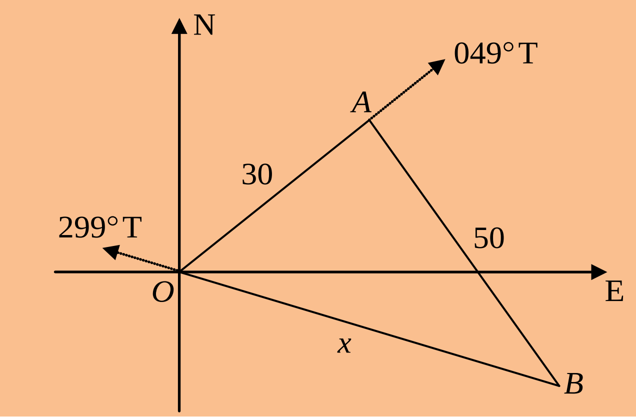
$$\frac{\cos\theta}{\sin\theta+1} - \frac{\cos\theta}{\sin\theta-1} = 4$$

$$2\sec\theta = 4$$

$$\cos\theta = \frac{1}{2}$$

$\theta = 60°,\ 300°$

(c) A ship sails for 30 nautical miles from O to A on a bearing of $049°T$.
It then turns and sails to a point B, 50 nautical miles away.
From B, the starting point O is observed on a bearing of $299°T$.



(i) Show that $\angle AOB = 70°$. **1**

Bearing of O from B is $299°$ T
$\therefore$ bearing of B from O is $299 - 180 = 119°$ T
$\therefore$ $\angle AOB = 119 - 49$
$= 70°$

(ii) Show that x satisfies the quadratic equation $x^2 - (60\cos 70°)x - 1600 = 0$. **2**

Cosine Rule:

$$50^2 = 30^2 + x^2 - 2(30)(x)\cos 70$$
$$2500 = 900 + x^2 - (60\cos 70°)x$$
$$x^2 - (60\cos 70°)x - 1600 = 0$$

(iii) Hence find the distance of B from O, giving your answer in nautical miles correct to one decimal place. **2**

Quadratic formula:
$$x = \frac{60\cos 70° \pm \sqrt{(60\cos 70°)^2 - 4(1)(-1600)}}{2}$$
$$x = -31.03,\ 51.56$$

Discarding negative distance: **$BO = 51.6$ nm**

(iv) By how many degrees did the ship turn at A? **2**

$$\frac{\sin\angle OAB}{51.56} = \frac{\sin 70°}{50}$$
$$\sin\angle OAB = \frac{51.56\sin 70°}{50}$$
$$= 0.968929$$
$$\angle OAB = 75.68°$$

$\therefore$ ship turned by $180 - 75.68$ = **104.32°**