NORTH SYDNEY GIRLS HIGH SCHOOL

2015 YEARLY EXAMINATION

Preliminary Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 8−13, show relevant mathematical reasoning and/or calculations

Total Marks – 85

Section I Pages 1–3

7 marks

- Attempt Questions 1−7
- Allow about 10 minutes for this section

Section II Pages 4−9

78 marks

- Attempt Questions 8−13
- Allow about 1 hour and 50 minutes for this section

Student Name:______________________________

Student Number:______________________________

Teacher:

- ○ Mr Trenwith
- ○ Ms Lee
- ○ Mr Moon
- ○ Mrs Juhn
- ○ Ms Everingham
- ○ Ms Narayanan
- ○ Mrs Kennedy
- ○ Mrs Thill

QUESTION	MARK
1−7	/7
8	/13
9	/13
10	/13
11	/13
12	/13
13	/13
TOTAL	/85

Section I

7 marks
Attempt Questions 1–7
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–7.

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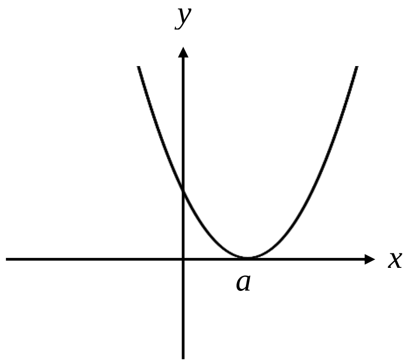
1 What is the value of $\sqrt{\dfrac{5\pi}{7}}$, correct to 2 significant figures?

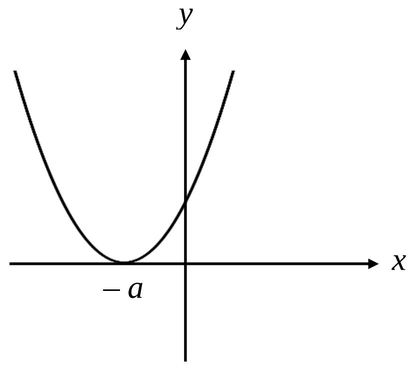
(A) 1.5
(B) 1.4
(C) 1.49
(D) 1.50

2 Given that $a > 0$, which graph best represents $y = (x - a)^2$?

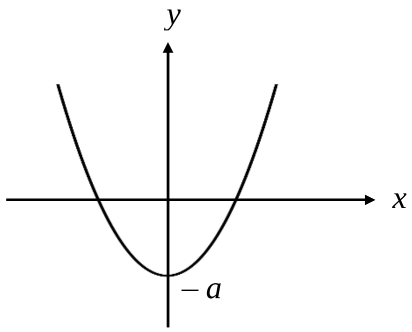
(A)



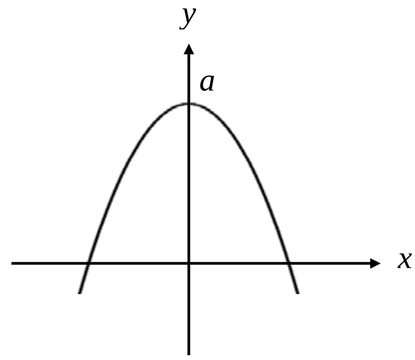
(B)



(C)



(D)



3 Which of the following is sufficient information to prove that a quadrilateral is a square?

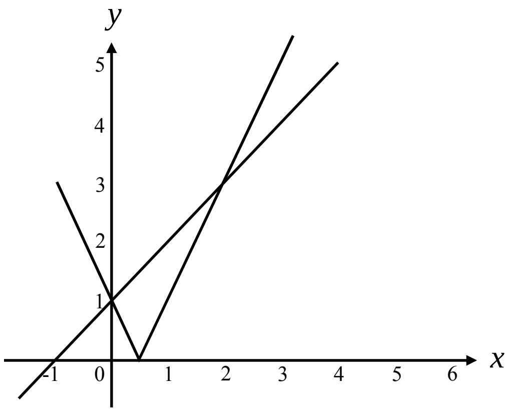
(A) Opposite sides are parallel.

(B) Diagonals are perpendicular and opposite sides are parallel.

(C) Diagonals bisect each other at right angles.

(D) All vertex angles are 90° and diagonals are perpendicular.

4 The graphs of $y = |1 - 2x|$ and $y = x + 1$ are shown below.



What are the solutions to $x + 1 - |1 - 2x| < 0$?

(A) $0 < x < 2$

(B) $x < 0$ or $x > 2$

(C) $1 < x < 3$

(D) $x < 1$ or $x > 3$

5 Which of the following is the derivative of $\sqrt{7x}$?

(A) $\dfrac{1}{2\sqrt{7x}}$

(B) $\sqrt{7}x$

(C) $\dfrac{\sqrt{7x}}{2}$

(D) $\dfrac{1}{2}\sqrt{\dfrac{7}{x}}$

6 The parabola $y = x^2$ is shifted left by 2 units, shifted up by 1 unit and then reflected about the y axis. What is the equation of the parabola now?

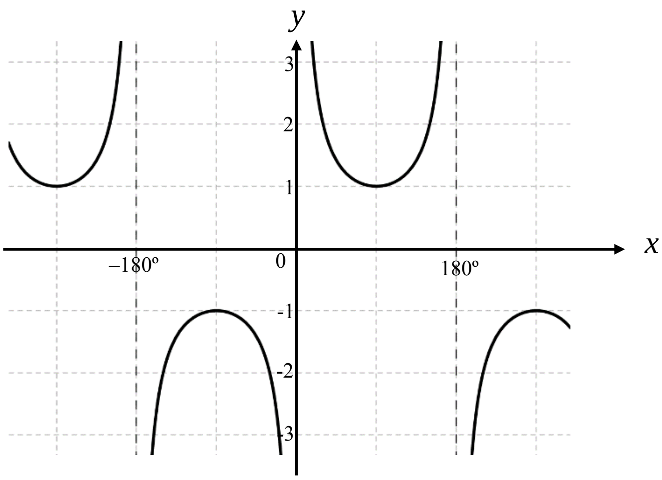
(A) $(x-2)^2 = y-1$

(B) $(x+2)^2 = y+1$

(C) $(x-2)^2 = y+1$

(D) $(x+2)^2 = y-1$

7 Which of the following equations best describes the graph below?



(A) $y = \operatorname{cosec} x$

(B) $y = \sec x$

(C) $y = \cot x$

(D) $y = -\sec x$

Section II

78 marks
Attempt Questions 8–13
Allow about 1 hour and 50 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing papers are available.

In Questions 8–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (13 marks) Use a SEPARATE writing booklet.

(a) Factorise $8m^3 - 1$. **2**

(b) Rationalise the denominator of $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$. **2**

(c) Find the points of intersection of the line $x - y + 3 = 0$ and the hyperbola $xy = 10$. **3**

(d) Find the centre and radius of the circle $x^2 - 6x + y^2 + 4y + 8 = 0$. **3**

(e) For what values of m does the equation $6x^2 - 12mx - 7m + 3 = 0$ have equal roots? **3**

End of Question 8

Question 9 (13 marks) Use a SEPARATE writing booklet.

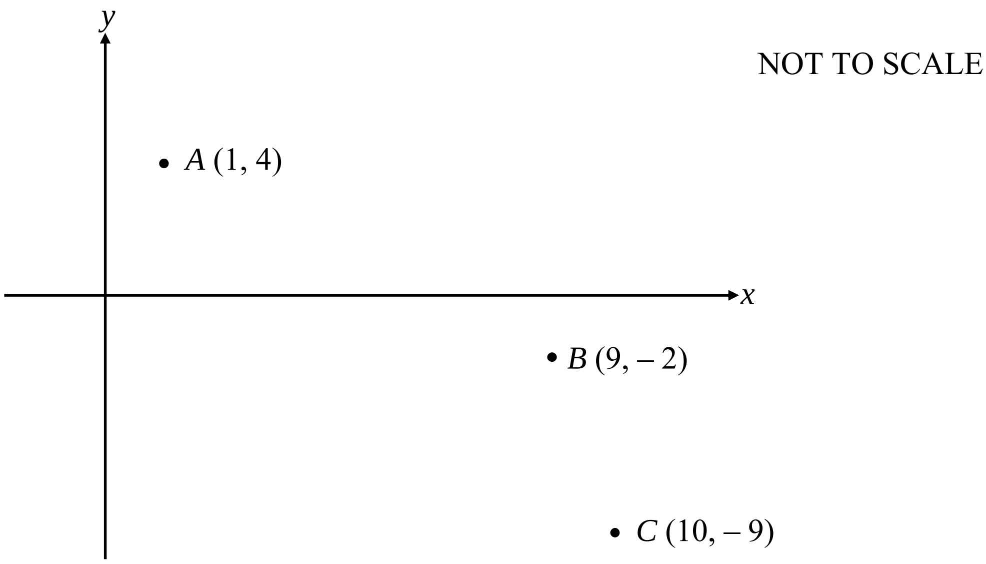
(a) Differentiate:

(i) $y = 2x^3 + 8x$ **1**

(ii) $y = \dfrac{x^2}{1-x}$ **2**

(iii) $y = 10x(3x-1)^5$ **3**

(b) It is given that points A, B and C have coordinates $(1, 4)$, $(9, -2)$ and $(10, -9)$ respectively.



(i) Find the distance between A and B. **1**

(ii) Find the equation of the line that is parallel to AB and passes through C. **2**

(iii) Hence, show that the perpendicular distance between C and the line AB is 5 units. **1**

(iv) Point D is placed so that $AB || DC$. Given that quadrilateral $ABCD$ has area of $50\ \text{u}^2$, show that the length of CD is 10 units. **2**

(v) Find the coordinates of D. **1**

End of Question 9

Question 10 (13 marks) Use a SEPARATE writing booklet.

(a) The gradient of the tangent to the curve $y = ax^3 + bx^2$ at (1, 4) is 5. **3**
Find the values of a and b.

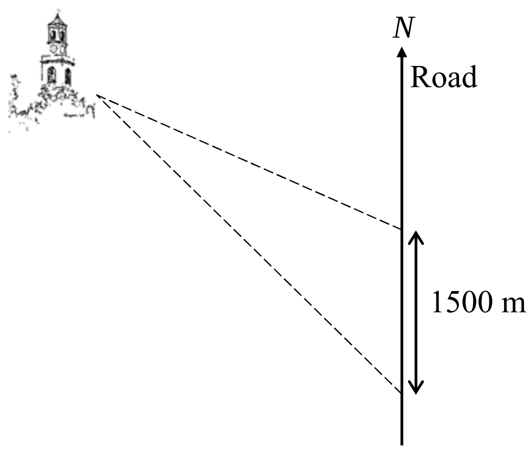
(b) Find all solutions of $\sin 2x = \frac{1}{2}$, where $0° \le x \le 360°$. **3**

(c) Find values of a, b and c, for which $-x^2 + 4x - 3 \equiv a(x+3)^2 + b(x+3) + c$. **2**

(d) Solve for x: $2^x < 5$. **2**

(e) A man walking due north along a level road observes a church spire on a bearing of $342°$ T. After walking 1500 metres, he observes it on a bearing of $337°$ T. **3**

Copy or trace the diagram into your writing booklet.



Find the perpendicular distance from the church to the road.

End of Question 10

Question 11 (13 marks) Use a SEPARATE writing booklet.

(a) If α and β are the roots of the equation $2x^2 - 3x + 5 = 0$, find the value of the following:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 2)(\beta + 2)$ **2**

(b) Prove that $(\sec\theta - \cos\theta)^2 = \tan^2\theta - \sin^2\theta$. **3**

(c) (i) Derive the equation of the locus of all points $P(x, y)$ which are equidistant from the origin O and the line $y = 4$. **2**

(ii) Describe the locus geometrically. **1**

(d) Find the values of k for which the expression $kx^2 + (k-1)x + k$ is positive definite. **3**

End of Question 11

Question 12 (13 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x \to 5} \dfrac{6x^2 - 29x - 5}{x - 5}$ **2**

(b) Sketch $y = |x^2 - 5| + 1$, showing all important features. **2**

(c) Consider the function $f(x) = x^2 - 5x + 2$.

(i) Differentiate $f(x)$ from first principles. **2**

(ii) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$. **2**

(d) A parabola whose axis is parallel to the x-axis has vertex (2, 3) and passes through (–1, 9). **2**
Find the equation of the parabola.

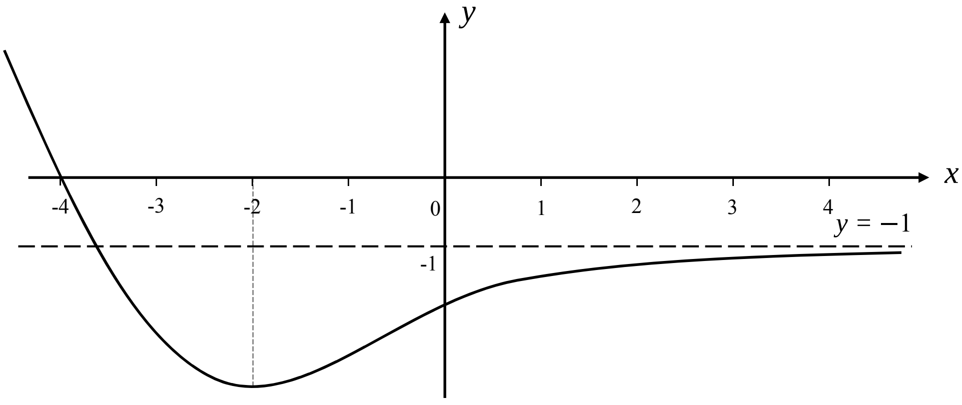
(e) Two cars leave a point P at the same time. They travel away from each other on different straight roads. One car travels at 10 km/h more than twice the speed of the other car. The angle between the roads is 120°. **3**

After 3 hours, if the distance between the cars is 500 km, find the speed of the slower car, correct to one decimal place.

End of Question 12

Question 13 (13 marks) Use a SEPARATE writing booklet.

(a) The graph of $y = f(x)$ is shown below. On a half-page diagram, sketch the graph of $y = f'(x)$, showing all important features. **2**



(b) The lines $l_1 : x - 2y - 5 = 0$ and $l_2 : 3x - y + 2 = 0$ intersect at *P*.

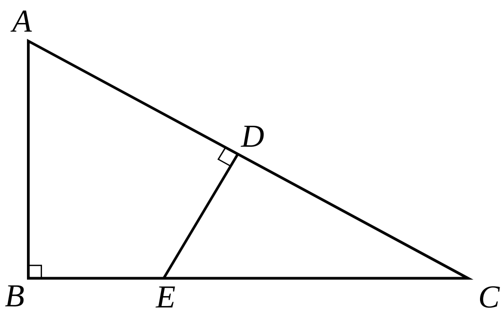
(i) Show that the family of lines through *P* can be written in the form **1**

$$x(1+3k) - y(2+k) + (2k-5) = 0$$

(i) Hence find the equation of the line through *P*, parallel to the line $4x - y - 1 = 0$. **2**

(c) Find all solutions of $2\cos^2 x + 3\sin x = 3$, where $0° \le x \le 360°$. **3**

(d) ΔABC is right-angled at *B* and *DE* is perpendicular to *AC*.



NOT TO SCALE

(i) Prove that ΔABC and ΔEDC are similar. **2**

(ii) Explain why $BC \times CE = AC \times CD$. **1**

(iii) Prove that $DE^2 = AD \times DC - BE \times EC$. **2**

End of Paper

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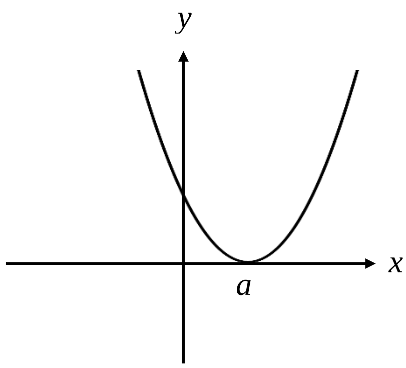

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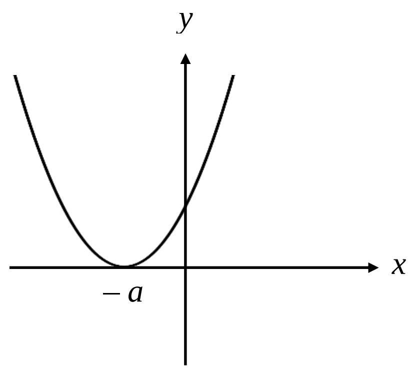

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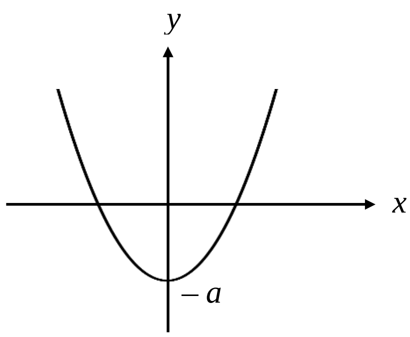

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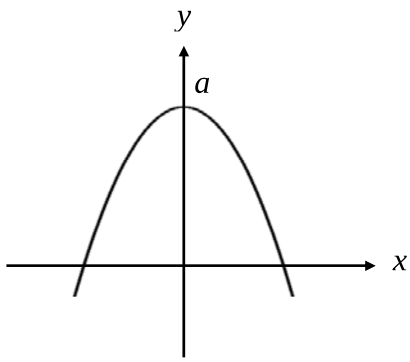

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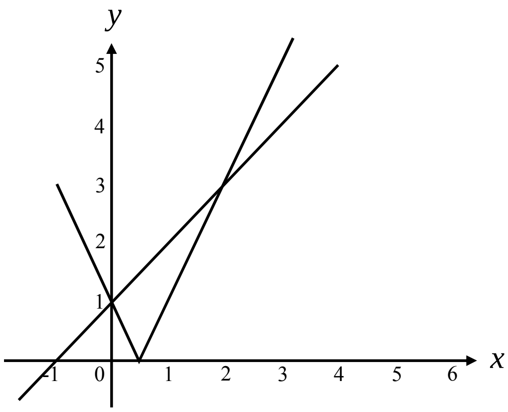

What are the solutions to $x + 1 - |1 - 2x| < 0$?

(A) $0 < x < 2$

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(C) $1 < x < 3$

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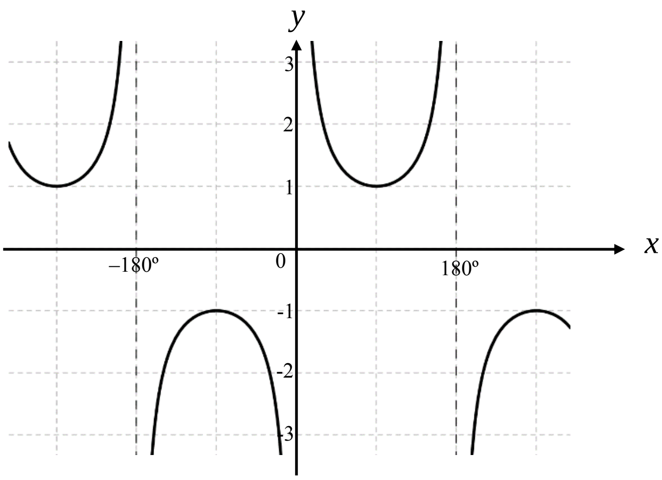


(A) $y = \operatorname{cosec} x$

(B) $y = \sec x$

(C) $y = \cot x$

(D) $y = -\sec x$

Question 8 (13 marks) Use a SEPARATE writing booklet.

(a) Factorise $8m^3 - 1$. **2**

$(2m-1)(4m^2+2m+1)$

(b) Rationalise the denominator of $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$. **2**

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}\right)^2 - 2\times\sqrt{3}\times\sqrt{2} + \left(\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= 5 - 2\sqrt{6}$$

(c) Find the points of intersection of the line $x - y + 3 = 0$ and the hyperbola $xy = 10$. **3**

$x - y + 3 = 0 \quad \cdots\cdots\ (1)$

$xy = 10 \quad \cdots\cdots\ (2)$

From (1),

$y = x + 3 \quad \cdots\cdots\ (3)$

Substitute (3) into (2)

$x(x+3) = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\therefore x = -5$ or $x = 2$

$\therefore$ points of intersection are $(-5,-2)$ and $(2,5)$

(d) Find the centre and radius of the circle $x^2 - 6x + y^2 + 4y + 8 = 0$. **3**

$x^2 - 6x + y^2 + 4y + 8 = 0$

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -8 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 5$

$\therefore$ The radius is $\sqrt{5}$ and the centre is $(3,\ -2)$

(e) For what values of m does the equation $6x^2 - 12mx - 7m + 3 = 0$ have equal roots? **3**

$6x^2 - 12mx - 7m + 3$

$\Delta = (-12m)^2 - 4(6)(-7m + 3)$

$= 144m^2 + 168m - 72 = 0$

Equal roots occur when $\Delta = 0$

$144m^2 + 168m - 72 = 0$

$6m^2 + 7m - 3 = 0$

$(3m - 1)(2m + 3) = 0$

$\therefore m = \frac{1}{3}$ or $m = -\frac{3}{2}$

End of Question 8

Question 9 (13 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $y = 2x^3 + 8x$ **1**

$y = 2x^3 + 8x$

$\frac{dy}{dx} = 6x^2 + 8$

(ii) $y = \frac{x^2}{1-x}$ **2**

$y = \frac{x^2}{1-x}$

$\frac{dy}{dx} = \frac{(2x)(1-x)-(-1)(x^2)}{(1-x)^2}$

$= \frac{2x - 2x^2 + x^2}{(1-x)^2}$

$= \frac{2x - x^2}{(1-x)^2}$

(iii) $y = 10x(3x-1)^5$ **3**

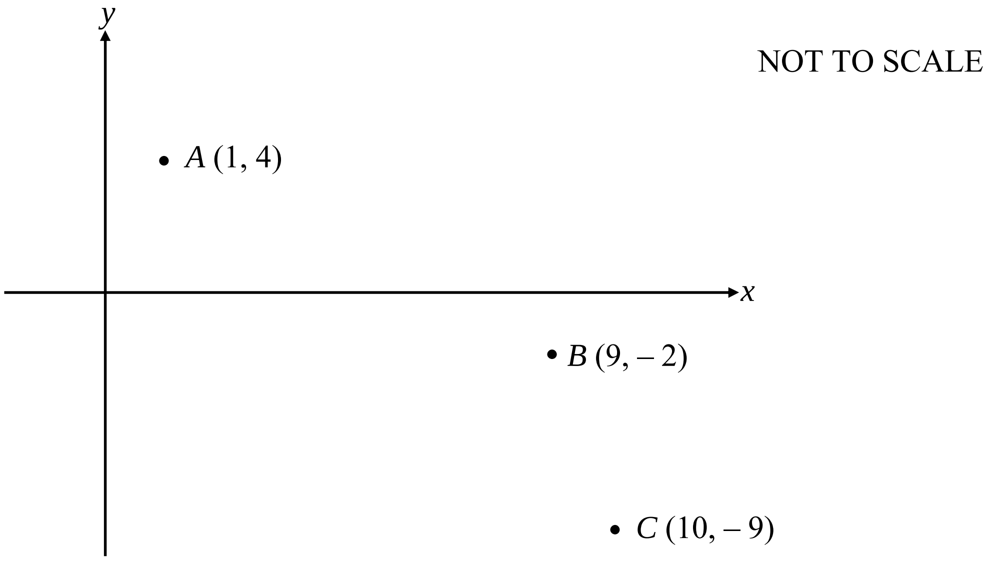
$y = 10x(3x-1)^5$

$\frac{dy}{dx} = 10(3x-1)^5 + 10x \times 5 \times (3x-1)^4 \times 3$

$= 10(3x-1)^4((3x-1)+15x)$

$= 10(3x-1)^4(18x-1)$

(b) It is given that points A, B and C have coordinates $(1,4)$, $(9,-2)$ and $(10,-9)$ respectively.



(i) Find the distance between A and B. **1**

$$d_{AB} = \sqrt{(1-9)^2 + (4-(-2))^2}$$
$$= \sqrt{64+36}$$
$$= 10 \text{ units}$$

(ii) Find the equation of the line that is parallel to AB and passes through C. **2**

$$m_{AB} = \frac{-2-4}{9-1}$$
$$= \frac{-6}{8} = -\frac{3}{4}$$

$$y-(-9) = -\frac{3}{4}(x-10)$$
$$4y+36 = -3x+30$$
$$3x+4y+6=0$$

(iii) Hence, show that the perpendicular distance between C and the line AB is 5 units. **1**

$$\perp d = \frac{|3(1)+(4)(4)+(6)|}{\sqrt{(3)^2+(4)^2}}$$
$$= \frac{25}{5} = 5$$

(iv) Point D is placed so that $AB||DC$. Given that quadrilateral $ABCD$ has area of 50 u^2, show that the length of CD is 10 units. **2**

$$50 = \frac{5}{2} \times (AB + CD)$$
$$10 + CD = 20$$
$$\therefore CD = 10$$

(v) Find the coordinates of D. **1**

$AB = CD$ and $AB \parallel CD$

$\therefore$ $ABCD$ is a parallelogram since there is one pair of opposite sides that are equal and parallel

Diagonals AC and BD bisect each other.

$$M_{AC} = \left(\frac{1+10}{2}, \frac{4+(-9)}{2}\right)$$
$$= \left(\frac{11}{2}, -\frac{5}{2}\right)$$

$$M_{BD} = \left(\frac{11}{2}, -\frac{5}{2}\right)$$

let D have coordinates (a, b)

$$\frac{9+a}{2} = \frac{11}{2} \text{ and } \frac{-2+b}{2} = -\frac{5}{2}$$
$$\therefore a = 2 \text{ and } b = -3$$

$\therefore$ D has coordinates $(2, -3)$

End of Question 9

Question 10 (13 marks) Use a SEPARATE writing booklet.

(a) The gradient of the tangent to the curve $y = ax^3 + bx^2$ at (1, 4) is 5. **3**
Find the values of a and b.

$y = ax^3 + bx^2$

$\frac{dy}{dx} = 3ax^2 + 2bx$

when $x = 1$, $\frac{dy}{dx} = 5$

also, when $x = 1, y = 4$

$5 = 3a + 2b$

$4 = a + b$

solving simultaneously,

$a = -3$ and $b = 7$

(b) Find all solutions of $\sin 2x = \frac{1}{2}$, where $0° \le x \le 360°$. **3**

$\sin 2x = \frac{1}{2}$

$2x = 30°, 150°, 390°, 510°$

$x = 15°, 75°, 195°, 255°$

(c) Find values of a, b and c, for which $-x^2 + 4x - 3 \equiv a(x+3)^2 + b(x+3) + c$. **2**

$-x^2 + 4x - 3 \equiv a(x+3)^2 + b(x+3) + c$

RHS $= a(x+3)^2 + b(x+3) + c$

$= a(x^2 + 6x + 9) + bx + 3b + c$

$= ax^2 + 6ax + 9a + bx + 3b + c$

$= ax^2 + (6a + b)x + (9a + 3b + c)$

Equating coefficients,

$a = -1$

$6a + b = 4$

$\therefore b = 10$

$9a + 3b + c = -3$

$\therefore c = -24$

$\therefore -x^2 + 4x - 3 \equiv -(x+3)^2 + 10(x+3) - 24$

Alternatively,

Equate coefficients, $a = -1$

Substitute $x = -3$,

$-9 - 12 - 3 = c$

$\therefore c = -24$

Substitute $x = 0$,

$-3 = -9 + 3b - 24$

$3b = 30$

$\therefore b = 10$

$a = -1, b = 10$ and c=-24

(d) Solve for x: $2^x < 5$. **2**

$2^x < 5$

$\log(2^x) < \log 5$

$x \times \log 2 < \log 5$

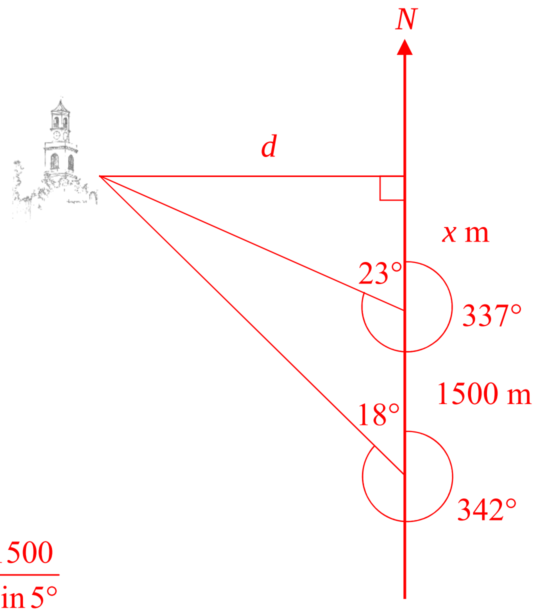
$x < \dfrac{\log 5}{\log 2}$

$x < 2.321928095...$

$x < 2.32$ (2 decimal places)

(e) A man walking due north along a level road observes a church spire on a bearing of $342°$ T. After walking 1500 metres, he observes on a bearing of $337°$ T. **3**

Find the perpendicular distance from the church to the road.



$\dfrac{y}{\sin 18°} = \dfrac{1500}{\sin 5°}$

$y = \dfrac{1500 \sin 18°}{\sin 5°}$

$\sin 23° = \dfrac{d}{y}$

$d = \dfrac{\sin 23° \times 1500 \sin 18°}{\sin 5°}$

$= 2078$ m (nearest metre)

End of Question 10

Question 11 (13 marks) Use a SEPARATE writing booklet.

(a) If α and β are the roots of the equation $2x^2 - 3x + 5 = 0$, find the value of the following:

(i) $\alpha + \beta$ **1**

$$\alpha + \beta = \frac{3}{2}$$

(ii) $\alpha\beta$ **1**

$$\alpha\beta = \frac{5}{2}$$

(i) $(\alpha + 2)(\beta + 2)$ **2**

$$\begin{aligned}(\alpha + 2)(\beta + 2) &= \alpha\beta + 2(\alpha + \beta) + 4 \\ &= \frac{5}{2} + 2\left(\frac{3}{2}\right) + 4 \\ &= \frac{19}{2}\end{aligned}$$

(b) Prove that $(\sec\theta - \cos\theta)^2 = \tan^2\theta - \sin^2\theta$. **3**

$$\begin{aligned}\text{LHS} &= (\sec\theta - \cos\theta)^2 \\ &= \sec^2\theta - 2\sec\theta\cos\theta + \cos^2\theta \\ &= (1 + \tan^2\theta) - \frac{2\cos\theta}{\cos\theta} + (1 - \sin^2\theta) \\ &= 1 + \tan^2\theta - 2 + 1 - \sin^2\theta \\ &= \tan^2\theta - \sin^2\theta = \text{RHS}\end{aligned}$$

(c) (i) Derive the equation of the locus of all points $P(x, y)$ which are equidistant from the origin O and the line $y = 4$. **2**

$$P(x,y) \quad l:(x,4) \quad O(0,0)$$

$$d_{Pl} = \sqrt{(x-x)^2 + (y-4)^2}$$
$$= y - 4$$

$$d_{PO} = \sqrt{(x-0)^2 + (y-0)^2}$$
$$= \sqrt{x^2 + y^2}$$

$$d_{Pl} = d_{PO}$$
$$y - 4 = \sqrt{x^2 + y^2}$$
$$y^2 - 8y + 16 = x^2 + y^2$$
$$x^2 = -8y + 16$$
$$x^2 = -8(y-2)$$

(ii) Describe the locus geometrically. **1**

Concave down parabola with focal length 2, and vertex (0, 2).

(d) Find the values of k for which the expression $kx^2 + (k-1)x + k$ is positive definite. **3**

$$\Delta = b^2 - 4ac$$
$$= (k-1)^2 - 4(k)(k)$$
$$= k^2 - 2k + 1 - 4k^2 = -3k^2 - 2k + 1$$

$$\Delta < 0,$$
$$-3k^2 - 2k + 1 < 0$$
$$(-3k+1)(k+1) < 0$$
$$k < -1 \text{ or } k > \frac{1}{3}$$
$$k > 0, \text{ since the quadratic is positive definite}$$
$$\therefore k > \frac{1}{3}$$

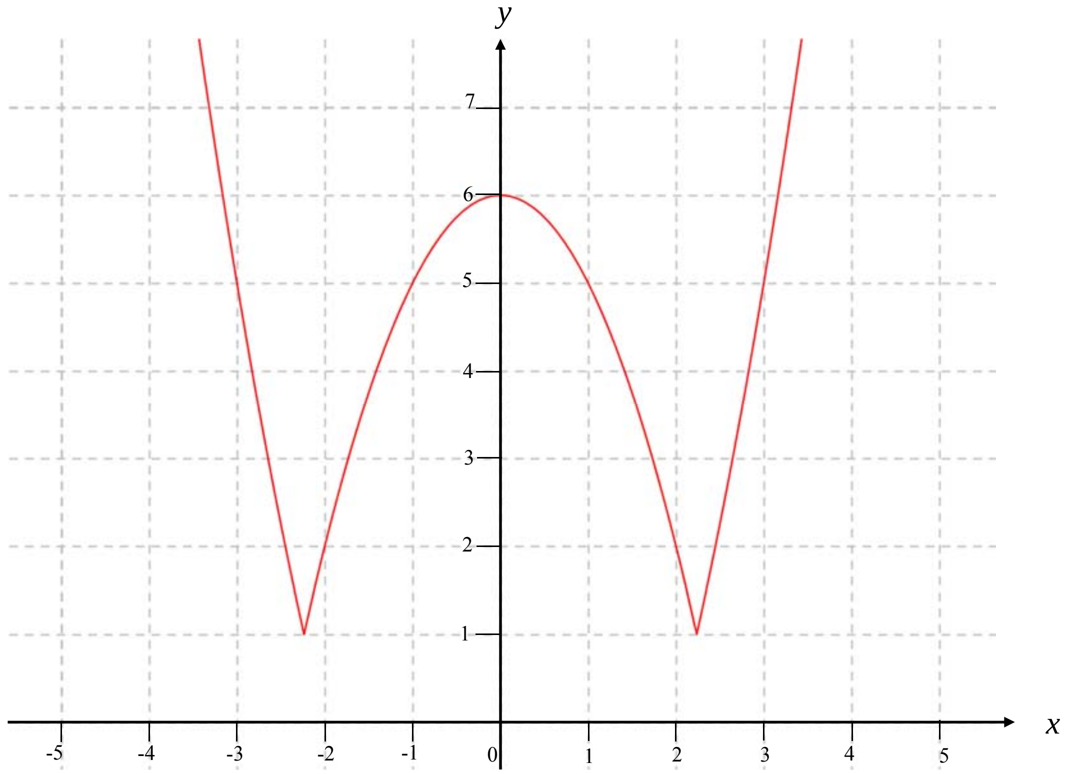
End of Question 11

Question 12 (13 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x \to 5} \frac{6x^2 - 29x - 5}{x - 5}$ **2**

$$\lim_{x \to 5} \frac{6x^2 - 29x - 5}{x - 5} = \lim_{x \to 5} \frac{(x-5)(6x+1)}{x-5}$$
$$= \lim_{x \to 5} 6x + 1$$
$$= 31$$

(b) Sketch $y = |x^2 - 5| + 1$, showing all important features. **2**



(c) Consider the function $f(x) = x^2 - 5x + 2$.

(i) Differentiate $f(x)$ from first principles. **2**

$$f(x) = x^2 - 5x + 2$$
$$f(x+h) = (x+h)^2 - 5(x+h) + 2$$
$$= x^2 + 2xh + h^2 - 5x - 5h + 2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h + 2 - x^2 + 5x - 2}{h}$$
$$= \lim_{h \to 0} \frac{h(2x - 5 + h)}{h}$$
$$= \lim_{h \to 0} 2x - 5 + h$$
$$= 2x - 5$$

(ii) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$. **2**

At $x = 1$, $m_T = 2(1) - 5 = -3$

$\therefore m_N = \frac{1}{3}$

At $x = 1, y = 1^2 - 5(1) + 2 = -2$

$y - (-2) = \frac{1}{3}(x - 1)$

$x - 3y - 7 = 0$

(d) A parabola whose axis is parallel to the x-axis has vertex (2, 3) and passes through (–1, 9). **2**

Find the equation of the parabola.

$(y-3)^2 = 4a(x-2)$

let $x = -1$ and $y = 9$

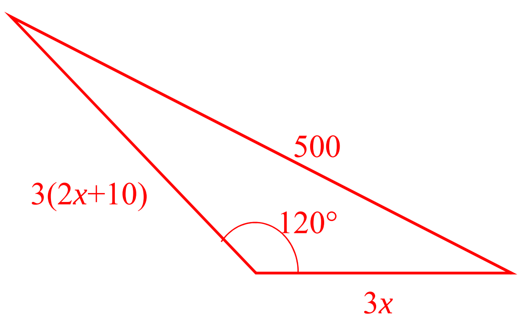
$36 = -12a$

$\therefore a = -3$

$\therefore (y-3)^2 = -12(x-2)$

(e) Two cars leave a point P at the same time. They travel away from each other on different straight roads. One car travels at 10 km/h more than twice the speed of the other car. The angle between the roads is 120°. **3**

After 3 hours, if the distance between the cars is 500 km, find the speed of the slower car, correct to one decimal place.



$\cos 120° = \frac{(3x)^2 + 3^2(2x+10)^2 - 500^2}{2 \times 3x \times 3(2x+10)}$

$-\frac{1}{2} = \frac{9x^2 + 9(4x^2 + 40x + 100) - 250000}{18x(2x+10)}$

$-\frac{1}{2} = \frac{9x^2 + 36x^2 + 360x + 900 - 250000}{18x(2x+10)}$

$-9x(2x+10) = 45x^2 + 360x - 249100$

$-18x^2 - 90x = 45x^2 + 360x - 249100$

$63x^2 + 450x - 249100 = 0$

$x = \frac{-450 \pm \sqrt{450^2 - (4)(63)(-249100)}}{2 \times 63}$

since $x > 0$,

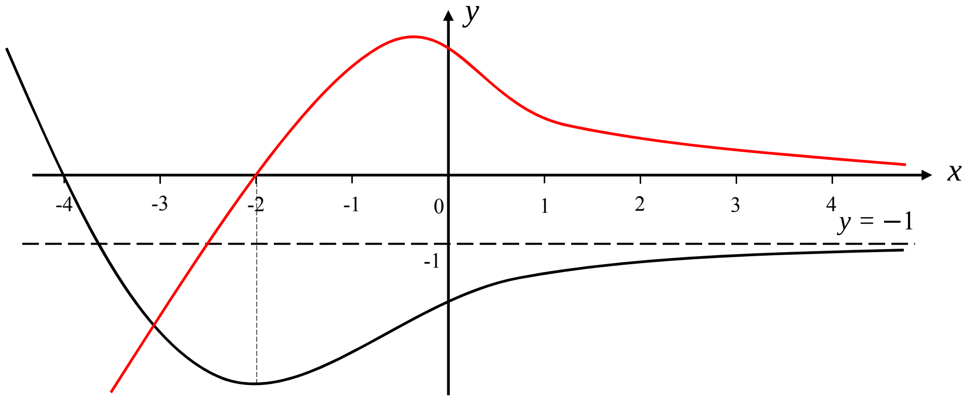
$x = 59.410...$

$= 59.4\,\text{km}$ (1 decimal place)

End of Question 12

Question 13 (13 marks) Use a SEPARATE writing booklet.

(a) The graph of $y = f(x)$ is shown below. On a half –page diagram, sketch the graph $y = f'(x)$, showing all important features. **2**



(b) The lines $l_1 : x - 2y - 5 = 0$ and $l_2 : 3x - y + 2 = 0$ intersect at *P*.

(i) Show that the family of lines through *P* can be written in the form **1**

$$x(1+3k) - y(2+k) + (2k-5) = 0$$

$x - 2y - 5 + k(3x - y + 2) = 0$

$x - 2y - 5 + 3kx - ky + 2k = 0$

$x + 3kx - 2y - ky - 5 + 2k = 0$

$x(1+3k) - y(2+k) + (2k-5) = 0$ as required

(ii) Hence find the equation of the line *l* through *P*, parallel to the line $4x - y - 1 = 0$. **2**

the line parallel to $4x - y - 1 = 0$ has gradient 4

the gradient of $x(1+3k) - y(2+k) + (2k-5) = 0$ is $\dfrac{1+3k}{2+k}$

$\dfrac{1+3k}{2+k} = 4$

$1 + 3k = 8 + 4k$

$k = -7$

$\therefore x(1-21) - y(2-7) + (-14-5) = 0$

$-20x + 5y - 19 = 0$

(c) Find all solutions of $2\cos^2 x + 3\sin x = 3$, where $0 \le x \le 360°$. **3**

$2(1-\sin^2 x) + 3\sin x = 3$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

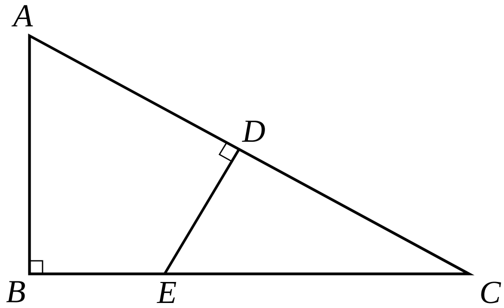
$(2\sin x - 1)(\sin x - 1) = 0$

$\therefore \sin x = \frac{1}{2}$ or $\sin x = 1$

in the domain $0° \le x \le 360°$

$x = 30°, 150°$ or $90°$

(d) ΔABC is right-angled at B and DE is perpendicular to AC.



NOT TO SCALE

(i) Prove that ΔABC and ΔEDC are similar. **2**

In ΔEDC and ΔABC,

$\angle BCA$ is common

$\angle ABC = 90°$ (ΔABC is a right-angled triangle)

$\angle EDC = 90°$ ($DE \perp AC$)

$\angle ABC = \angle EDC = 90°$

$\therefore \Delta EDC \ ||| \ \Delta ABC$ (equiangular)

(ii) Explain why $BC \times CE = AC \times CD$. **1**

$\frac{CE}{AC} = \frac{CD}{BC}$ (corresponding sides of similar triangles)

$\therefore BC \times CE = AC \times CD$

(iii) Prove that $DE^2 = AD \times DC - BE \times EC$. **2**

$AC = AD + DC$

$BC = BE + EC$

from (i),

$BC \times CE = AC \times CD$

$CE(BE + EC) = CD(AD + DC)$

$CE \times BE + CE^2 = AD \times CD + DC^2$

$EC^2 - DC^2 = AD \times DC - BE \times EC$

by Pythagoras theorm,

$DE^2 = EC^2 - DC^2$

$\therefore DE^2 = AD \times DC - BE \times EC$ as required

End of Paper