Student Number: _____

ROSEVILLE COLLEGE

MATHEMATICS

2003

PRELIMINARY EXAMINATION

General Instructions

Time Allowed – 3 hours, plus 5 minutes reading time

Directions to Candidates:

- Attempt all questions.
- Begin each question on a new page, with your number clearly written on every page.
- There are 10 questions, each worth 12 marks.
- Individual marks for each question are indicated on the left hand side of each question
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

Marks

Question 1 **Basic Arithmetic** (*Start work on a new page)*

(a) Simplify $|-5|-|8|$ **1**

(b) Calculate correct to two decimal places **2**

$$\frac{34.2}{16.3\times 2.7}$$

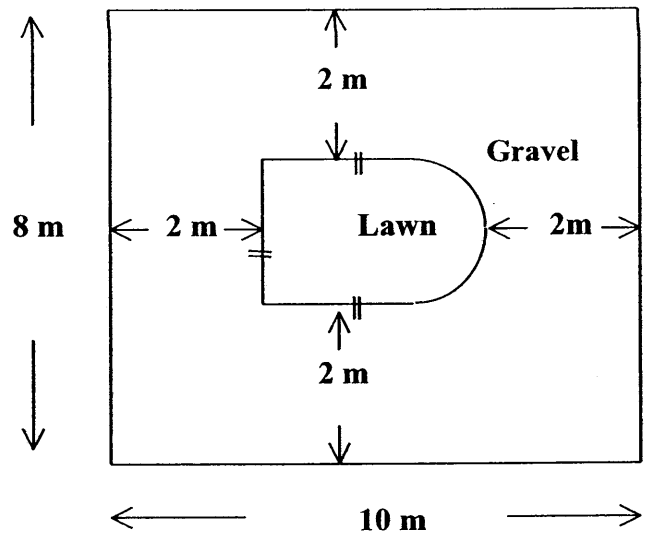
(c) A skirt is priced at \$82.50, including GST of 10%. What was the price of the skirt prior to adding GST? **1**

(d) Evaluate $(5.9 \times 10^{55})^2$ in scientific notation, correct to 2 significant figures **2**

(e) Find the exact value of $81^{-\frac{1}{4}} \div 2^{-2}$ **1**

(f) Express $0.\dot{1}0\dot{2}$ as a fraction in it's simplest form. **2**

(g) The Backyard Force TV team is renovating a backyard to the plan shown below. Calculate the area of lawn in the backyard (correct to 1 decimal place) **3**



Question 2 **Algebra and Surds** *(Start work on a new page)* **Marks**

(a) Simplify $4 - 6(m - 2)$ **1**

(b) If $S = \frac{a}{1-r}$, find S when $a = 0.8$ and $r = \frac{1}{2}$ **1**

(c) Factorise completely $4a^2 - b^2 + 2a - b$ **2**

(d) Simplify $\sqrt{75} + \sqrt{48}$ **2**

(e) Express $\frac{2\sqrt{3} - 1}{5 - \sqrt{3}}$ with a rational denominator. **2**

(f) Simplify $\frac{2}{x(x-3)} - \frac{1}{x}$ **2**

(g) Find the exact value of $\frac{t^3}{5} - t^2 + 1$ when $t = -3$ **2**

Marks

Question 3 **Equations** *(Start work on a new page)*

(a) Solve:

i. $3(2x-3)=4(x-2)$ **1**

ii. $\frac{5x}{2}-\frac{2x}{3}=2$ **2**

iii $x^2-3=3x+1$ **2**

(b) Mark on the number line the values of x for which $|x-1|\leq 2$ **2**

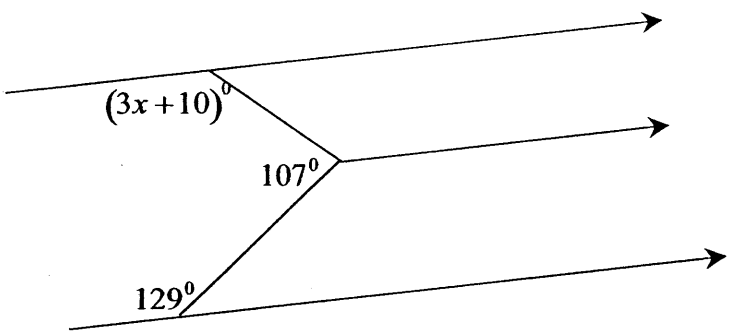
(c) Find the values of a and b such that :

$$\begin{cases} 2a-3b=7 \\ a-4b=1 \end{cases} \text{ and}$$ **2**

(d) Solve for x: $(x+1)^2=6$, leaving your answer in exact form. **3**

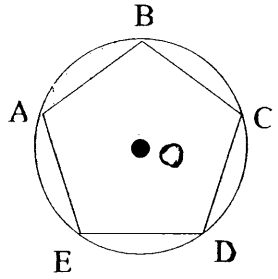
Question 4 **Geometry** *(Start work on a new page)*

(a) Find the value of x in the following diagram. **2**



Marks

(b) ABCDE is a regular pentagon where ABCDE lie on the circumference of a circle of diameter 32 cm. O is the centre of the circle. This is shown below.



(i) Find the length of OA. **1**

(ii) Find the angle AOB. **1**

(iii) Find the angle OAB. **1**

(iv) Find the length of the side of the pentagon, giving your answer to one decimal place. **2**

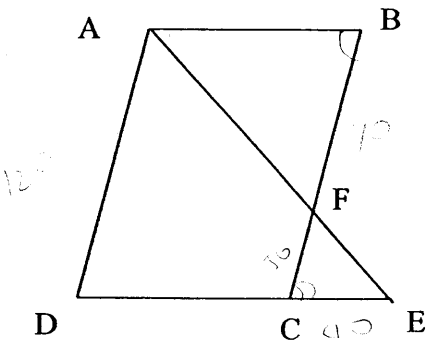
(v) Find the area of the pentagon, giving your answer to one decimal place. **1**

(vi) Find the area of the sector AOB, giving your answer to one decimal place. **1**

(c) ABCD is a parallelogram with DC produced to E, where AD = 120mm, CE = 40mm and BF = 70mm. **3**

(i) Show that ΔABF is similar to ΔECF

(ii) Find the length of AB



Question 5 **Functions and Graphs** *(Start on a new page)* **Marks**

(a) A function $f(x)$ is defined as: $f(x) = \begin{cases} 2-x & \text{for } x<-3 \\ 5 & \text{for } -3 \le x<0 \\ x^2-1 & \text{for } x \ge 0 \end{cases}$

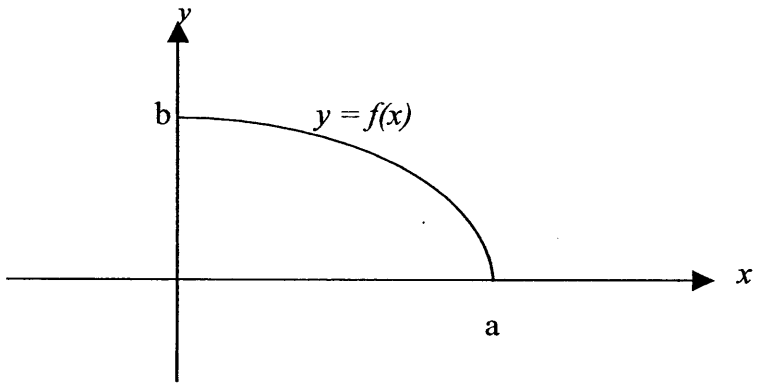
(i) Calculate the value of (a) $f(-10)$ **1**

(β) $f(-3)$ **1**

(γ) $f(a^2)$ **1**

(ii) Sketch the graph of $y=f(x)$ over the domain $-5 \le x \le 2$, showing all essential features including any intercepts with the x and y axes. **2**

(b) The diagram below shows part of the graph of the function $y=f(x)$. You are told that it is an odd function. Copy the diagram and complete the graph of the function. **1**



(c) Solve simultaneously: **2**

$$\begin{cases} y=4x+3 \\ y=x^3+3 \end{cases}$$

(d) (i) Sketch the curve $y=x^2-2x$, clearly showing all intercepts. **1**

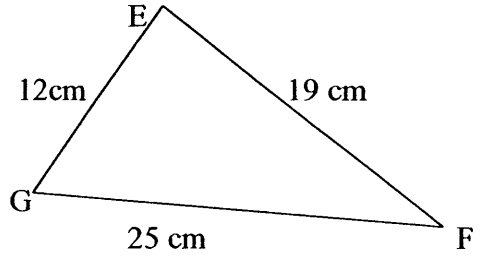
(ii) What is the minimum value of this function? **1**

(e) Sketch the graph of the function $y=\dfrac{1}{x+1}$ and state the domain and the range of the function. **2**

Marks

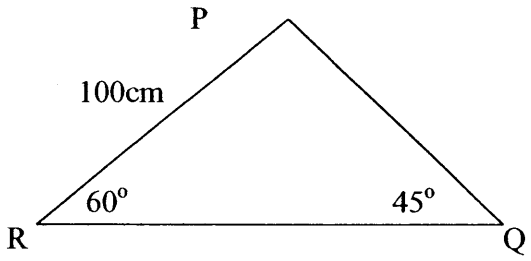
Question 6 **Trigonometry** *(Start work on a new page)*

(a) Find the size of the largest angle in the triangle below, correct to the nearest degree. **2**



NOT TO SCALE

(b) In $\triangle$ PQR below:



(i) Show that the length of PQ is $50\sqrt{6}$ cm. **2**

(ii) Hence or otherwise, find the area of the triangle PQR. (correct to 1 decimal place) **2**

(c) Find all values of θ such that $\cos\theta = \frac{-1}{2}$ *and* $0° \leq \theta \leq 360°$ **2**

(d) (i) Show that: $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$ **1**

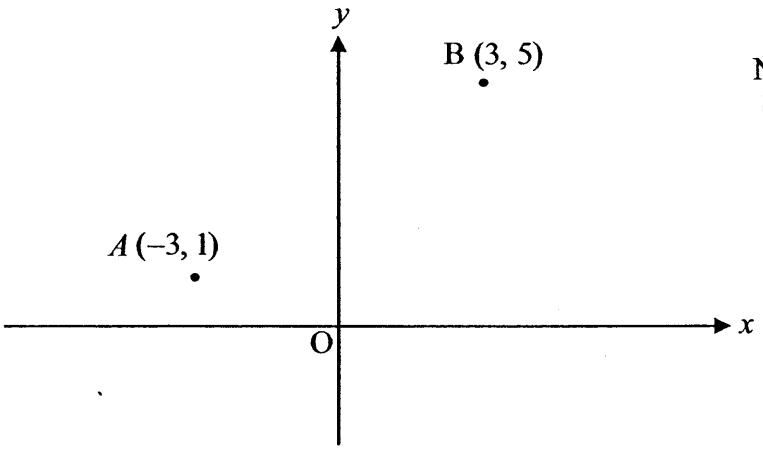
(ii) Hence or otherwise, solve: **3**

$$\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta+\cot\theta} = 1, \quad 0° \leq \theta \leq 360°$$

Marks

Question 7 **Straight Line Graphs** *(Start work on a new page)*

(a) Sketch the graph of $y = -x + 2$ on a set of axes, showing any x and y intercepts. **2**



(b) The diagram shows the points $A(-3,1)$ and $B(3,5)$ on the Cartesian number plane.

Copy or trace this diagram onto your writing page.

(i) Show that the equation of AB is $2x - 3y + 9 = 0$. **2**

(ii) Show that the point C, which is the midpoint of AB is the y-intercept of AB. **1**

(iii) Calculate the perpendicular distance from the point $D(2,0)$ to the line AB and mark the point D on your diagram. **2**

(iv) The point E, lies on the line $y = -1$ and the line BE is perpendicular to the line AB. Show that E has the coordinates $(7,-1)$ and mark point E on your diagram. **2**

(v) Show that $BCDE$ is a trapezium. **1**

(vi) Find the area of $BCDE$. **2**

Marks

Question 8 **Introductory Calculus** *(Start work on a new page)*

(a) Differentiate with respect to x.

(i) $2x^4 + 7x^2 + 7$ **1**

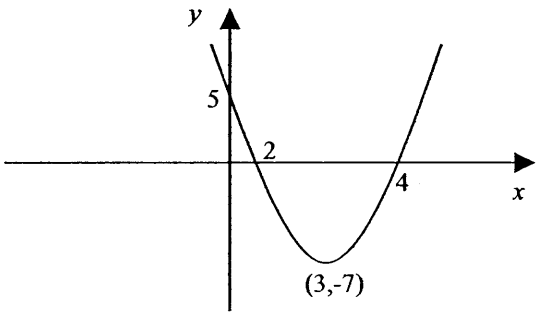
(ii) $x^{\frac{1}{3}}$ **1**

(iii) $\sqrt{x^2+4}$ **1**

(iv) $\dfrac{6x^2-7}{3-2x}$ **2**

(b) The graph shown below represents $f(x)$

Sketch a graph to show the general shape of $f'(x)$. **1**



(c) Find the equation to the normal to the curve $y = x^2 + x$ at the point where $x = 1$. **2**

(d) A tangent is drawn to the parabola $y = x^2 - 4x$ at the point P. The tangent has a gradient of 6.

(i) Show that the coordinates of P are (5, 5). **2**

(ii) Find the equation of the tangent at P. **1**

(iii) Find the gradient of the normal at P. **1**

Question 9 **Quadratic Functions** *(Start work on a new page)* **Marks**

(a) Find the value of k for which $x^2-(k-1)x-k=0$ has equal roots **2**

(b) Solve for x: $x^6-7x^3-8=0$ **2**

(c) Find the values of P, Q and R if $3x^2+5x-1\equiv P(x+1)^2+Q(x+1)+R$ **3**

(d) For what values of x is $x^2+5x>0$? **2**

(e) For the quadratic expression: $2x^2-x+3$ which has zeros when $x=\alpha$ and $x=\beta$.

(i) Show that the expression is positive definite. **1**

(ii) Find the value of $\alpha\beta$. **1**

(iii) Find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$. **1**

Question 10 **Locus and the Parabola** *(Start work on a new page)*

Marks

(a) For the parabola $16y = x^2$ write down the coordinates of the focus and the equation of the directrix. **2**

(b) The points P and Q have the coordinates $(-1, 0)$ and $(3, 3)$ respectively. If PR is perpendicular to QR, show that the locus of R is $x^2 + y^2 - 2x - 3y - 3 = 0$ **2**

(c) Find the equation of the parabola which has its vertex at (2,0) and its directrix is given by $x = 5$. **2**

(d) By expressing $y = \frac{x^2}{-16} + \frac{6x}{-16} - \frac{7}{-16}$ in the form $(x-h)^2 = 4a(y-k)$, find: **6**

(i) The focal length

(ii) The coordinates of the vertex

(iii) The equation of the axis

(iv) The equation of the directrix

(v) Sketch the graph

End of paper

Suggested Answers

Task Mathematics Preliminary 2003

Question 1

1a. -3

b. 0·78

c. \$75

d. 34.81×10^{110} (without calc!)
$= 3.481 \times 10^{111}$
$= 3.5 \times 10^{111}$ (2 s.f.)

e. $\frac{4}{3}$

f. $x = 0.102102\ldots$
$1000x = 102.102\ldots$
$999x = 102$
$x = \frac{102}{999}$
$= \frac{34}{333}$

g. Area $= 4 \times 4 + \frac{1}{2} \times \pi \times 2^2$
$= 16 + 2\pi$
$\doteqdot 22.3 m^2$

Question 2

a. $4 - 6(m-2)$
$= 4 - 6m + 12$
$= 16 - 6m$

b. $S = \frac{0.8}{1 - \frac{1}{2}}$
$= 1.6$

c. $(2a-b)(2a+b) + (2a-b)$
$= (2a-b)(2a+b+1)$

d. $\sqrt{75} + \sqrt{48}$
$= 5\sqrt{3} + 4\sqrt{3}$
$= 9\sqrt{3}$

e) $\frac{2\sqrt{3}-1}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$

2 continued

e. $\frac{(2\sqrt{3}-1)(5+\sqrt{3})}{25-3}$
$= \frac{10\sqrt{3} - 5 - \sqrt{3} + 6}{22}$
$= \frac{9\sqrt{3}+1}{22}$

f. $\frac{2}{x(x-3)} - \frac{1}{x}$
$= \frac{2 - (x-3)}{x(x-3)}$
$= \frac{5-x}{x(x-3)}$

g. $\frac{t^3}{5} - t^2 + 1$
$= \frac{(-3)^3}{5} - 9 + 1$
$= -13\frac{2}{5}$.

Question 3

ai) $3(2x-3) = 4(x-2)$
$6x - 9 = 4x - 8$
$2x = 1$
$x = \frac{1}{2}$

ii) $\frac{5x}{2} - \frac{2x}{3} = 2$ ($\times 6$)
$15x - 4x = 12$
$11x = 12$
$x = \frac{12}{11}$

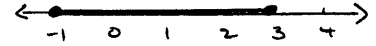
iii) $x^2 - 3 = 3x + 1$
$x^2 - 3x - 4 = 0$
$(x-4)(x+1) = 0$
$x = 4, -1$

Task

Quest 2 cont.

b. $x - 1 = 2$ (i) $-(x-1) = 2$
$x = 3$ $-x + 1 = 2$
$x = -1$

$|x-1| \leq 2$
test $x = 0$
$|-1| \leq 2$ ✓ true



c. $2a - 3b = 7$
$2a - 8b = 2$
$5b = 5$
$b = 1$

$2a - 8 = 2$
$2a = 10$
$a = 5$

d. $x + 1 = \pm\sqrt{6}$
$x = -1 \pm \sqrt{6}$

OR

$x^2 + 2x + 1 = 6$
$x^2 + 2x - 5 = 0$
$x = \frac{-2 \pm \sqrt{4 + 4 \times 1 \times 5}}{2}$
$= \frac{-2 \pm \sqrt{24}}{2}$
$= \frac{2(-1 \pm \sqrt{6})}{2}$
$= -1 \pm \sqrt{6}$.

Question 4

a) $3x + 10 + 107 + 129 = 360°$
$3x = 114$
$x = 38°$

bi) $OA = 16cm$

ii) $\frac{360}{5} = 72°$
$\angle AOB = 72°$

iii) $\angle OAB = 54°$

iv) $x^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 72°$
$x = 18.8 cm$.

v) Area $= \frac{1}{2} \times 5 \times 16 \times 16 \sin 72°$
$= 121.7 cm^2$
$\therefore$ Pentagon $= 608.7 cm^2$

vi) $A = \frac{\pi}{5} \times 16^2$
$= 160.8 cm^2$

c)i $\angle ABF = \angle FCE$ (alt $\angle$s // lines)
$\angle AFB = \angle CFE$ (vert. opp)
$\therefore \triangle ABF \,|||\, \triangle CFE$ (equiangular)

ii) $\frac{AB}{40} = \frac{70}{50}$
$\therefore AB = 56mm$

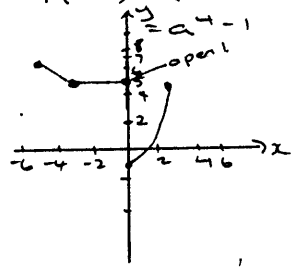
Question 5

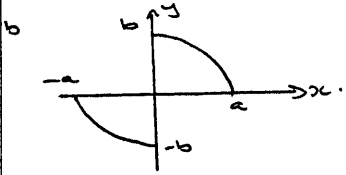
i. a) $f(-10) = 2 - (-10)$
$= 12$

b) $f(-3) = 5$

c) $f(a^2) = (a^2)^2 - 1$

ii)





Task Maths Prelim. Suggested Answers 2008

Question 6

c. $y = 4x + 3$

$y = \frac{1}{2}x^3 + 3$

$x^3 + 3 = 4x + 3$

$x^3 - 4x = 0$

$x(x+2)(x-2) = 0$

$x = 0, \pm 2$

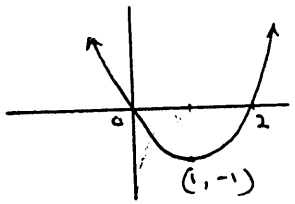
$x = -2,\ y = -5$

$x = 0,\ y = 3$

$x = 2,\ y = 11.$

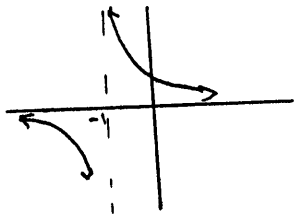
di $y = x^2 - 2x$

$= x(x-2)$



min value is -1

e)



Domain all x, x except $x \neq -1$

Range all y, except $y \neq 0$

Question 6

a. $\cos\theta = \frac{12^2 + 19^2 - 25^2}{2 \times 12 \times 19}$

$\theta = 105°$

di) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$ 1

2 ii) $\left(1 + \frac{\cos\theta}{\sin\theta}\right) \div \frac{1}{\sin\theta} - \frac{1}{\cos\theta} \times \frac{\sin\theta\cos\theta}{1} = 1$ 1

$= \frac{\sin\theta + \cos\theta}{\sin\theta} \times \frac{\sin\theta}{1} - \sin\theta = 1$ 1

$= \sin\theta + \cos\theta - \sin\theta = 1$

$\therefore \cos\theta = 1$

$\theta = 0, 360°$ 1

1 6b sorry about order!

$\frac{x}{\sin 60} = \frac{100}{\sin 45°}$ 1

1 $\frac{x}{\sqrt{3}/2} = \frac{100}{1/\sqrt{2}}$ 1

$2x = 100\sqrt{6}$

$x = 50\sqrt{6}$

ii) $A = \frac{1}{2} \times 100 \times 50\sqrt{6} \sin 75°$

$\div 5915.1 \text{ cm}^2$

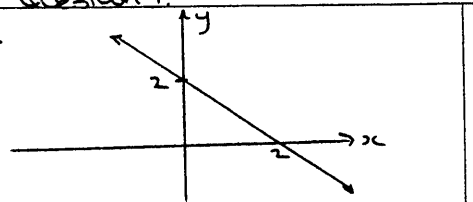
c) $\theta = 60°$ in the 2nd / 3rd quad

$\therefore \theta = 120°, 240°.$

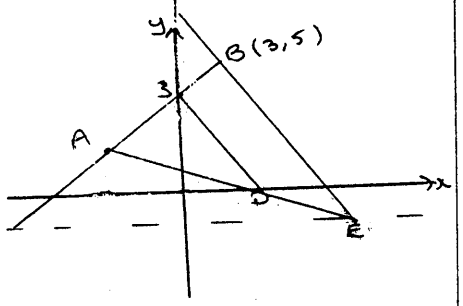
Task _____

Question 7.

a.



b.



i) $m = \frac{2}{3}$

$y - 1 = \frac{2}{3}(x+3)$

$3y - 3 = 2x + 6$

$2x - 3y + 9 = 0$

ii) $m(xy) = \left(\frac{0}{2}, \frac{6}{3}\right) = (0, 3)$

$2x - 3y + 9 = 0$

at $x = 0$

$0 - 3y + 9 = 0$

$\therefore y = 3.$

$\therefore$ the midpt of AB is also the y-int. of the line AB

iii) $\perp d = \left|\frac{2 \times 2 - 3 \times 0 + 9}{\sqrt{4+9}}\right|$

$= \left|\frac{13}{\sqrt{13}}\right| \times \frac{\sqrt{13}}{\sqrt{13}}$

$= \sqrt{13}$ units

iv) $m_2 = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x-3)$

$2y - 10 = -3x + 9$

$3x + 2y - 19 = 0$

at $y = -1$, $3x - 2 - 19 = 0$

$3x = 21$, $x = 7$

$\therefore E(7, -1)$

v) m of CD $= -\frac{3}{2}$

m of BE $= -\frac{3}{2}$

$\therefore$ CD // BE

$\therefore$ BCDE is a quad. $\because$ 1 pair of parallel lines & $\therefore$ a trapezium 1

vi) $BE = \sqrt{16 + 36} = \sqrt{52} = \sqrt{4}\sqrt{13} = 2\sqrt{13}$

$BC = \sqrt{13}$ $\quad \therefore CD = \sqrt{13}$

$\therefore A = \frac{1}{2} \times \sqrt{13}(\sqrt{13} + 2\sqrt{13})$

$= \frac{\sqrt{13}}{2} \times 3\sqrt{13}$

$= \frac{39}{2} u^2.$ 2.

Question 8.

i) $8x^3 + 14x^{-2/3}$

2 ii) $\frac{1}{3}x$

$= \frac{1}{3\sqrt{x^3}}$

1 iii) $\frac{1}{2}(x^2+4)^{-1/2} \times 2x$

$= \frac{x}{\sqrt{x^2+4}}$

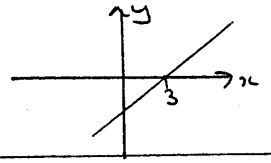
iv) $f(x) = \frac{6x^2 - 7}{3 - 2x}$

$f'(x) = \frac{(3-2x).12x - 2(6x^2-7)}{(3-2x)^2}$

2 $= \frac{36x - 24x^2 + 12x^2 - 14}{(3-2x)^2}$

$= \frac{-12x^2 + 36x - 14}{(3-2x)^2}$

b).



2

Question 8 cont.

c. $y = x^2 + x$

$y' = 2x + 1$

at $x = 1$, $y = 2$

$\therefore m_1 = 3$

$m_2 = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x - 1)$

$3y - 6 = -x + 1$

$\therefore x + 3y - 7 = 0.$

or $y = -\frac{1}{3}x + \frac{7}{3}.$

di) $y = x^2 - 4x$

$y' = 2x - 4$

at $y' = 6$

$2x - 4 = 6$

$2x = 10$

$x = 5, \quad y = 25 - 20 = 5$

$\therefore P$ is $(5, 5)$

$y - 5 = 6(x - 5)$

$y - 5 = 6x - 30$

$y = 6x - 25$

iii) $m_2 = -\frac{1}{6}.$

Question 9

a) $x^2 - (k-1)x - k = 0$

$\Delta = 0$ 1

$(k-1)^2 + 4k = 0.$

$k^2 - 2k + 1 + 4k = 0$

$k^2 + 2k + 1 = 0$

$(k+1)^2 = 0$ 1

$\therefore k = -1.$

b. $x^6 - 7x^3 - 8 = 0$

$(x^3 - 8)(x^3 + 1) = 0$

$\therefore x^3 = 8, \; -1$ 1

$x = 2, \; x = -1$ 1

c. $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$

$\equiv Px^2 + 2Px + P + Qx + Q + R$

$\equiv Px^2 + (2P + Q)x + P + Q + R$ 1

$\therefore P = 3 \quad 2P + Q = 5 \quad P + Q + R = -1$ 1

$Q = -1 \quad \therefore R = -3$ 1

d. $x^2 + 5x > 0$

$x(x + 5) > 0$ 1

$x < -5, \text{ or } x > 0$ 1

e) $\Delta = 1 - 11, \; a = 2$

$= -10$

since $\Delta < 0$ & $a > 0$,

$\Rightarrow$ +ve definite

ii) $\alpha\beta = \frac{c}{a}$ iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 4

$= \frac{3}{2}$ $= \frac{\alpha + \beta}{\alpha\beta}$ $\frac{1}{2}$

$= \left(\frac{1}{2}\right) \div \frac{3}{2} = \frac{1}{3}$ $\frac{1}{2}$.

Question 10

a) focus $(0, 4)$

directrix $y = -4$

b) $PR = \frac{y - 0}{x + 1}$ $(\frac{1}{2})$ $QR = \frac{y - 3}{x - 3}$ $(\frac{1}{2})$

$\frac{y}{x+1} = \frac{-(x-3)}{y-3}$

$y(y-3) = -(x-3)(x+1)$

$y^2 - 3y = -(x^2 - 3x + x - 3)$

$y^2 - 3y = -x^2 + 2x + 3$

$\therefore x^2 + y^2 - 2x - 3y - 3 = 0.$

Question 10 cont.

c) $(y-k)^2 = 4a(x-h)$ $\frac{1}{2}$ -ve $\frac{1}{2}$ subst $\frac{1}{2}$ vertex $h = 2$.

$(y-0)^2 = -4 \times 3(x-2)$

$y^2 = -12(x-2)$

d) $-16y = x^2 + 6x - 7$

$x^2 + 6x + 9 = -16y + 7 + 9$

$(x+3)^2 = -16(y-1)$ 1

i) $a = 4 \quad (\text{not } -4)$ 1

ii) $(-3, 1)$ 1

iii) $x = -3$ 1

iv) $y = 5$ 1