Student No: ....

ROSEVILLE COLLEGE

2007

PRELIMINARY MATHEMATICS

YEARLY EXAMINATION

Time allowed: **Three hours (plus 5 minutes reading time)**

Directions to Candidates:

- Attempt all questions
- All questions are of equal value
- All necessary working must be shown. Marks may not be awarded for answers unsupported by working
- Start each question on a new page
- Your rough work is to be attached to the back of this question paper

1	Basic Arithmetic		
2	Algebra and Surds		
3	Equations		
4	Geometry		
5	Functions and Graphs		
6	Trigonometry		
7	Linear expression and the Straight Line		
8	Introductory Calculus		
9	The Quadratic Function		
10	Locus and the Parabola		
	Total		

Question 1 Basic Arithmetic Show all Working **12 marks**

(a) Evaluate $|-5|-|-8--5|$ (2)

(b) Find , correct to 2 decimal places, the value of: $\dfrac{14\cdot73+8\cdot96}{\sqrt{(5\cdot86)^2-2\cdot78}}$ (2)

(c) Express $0\cdot2\dot{3}$ as a fraction in lowest terms (2)

(d) At a mobile phone sale offering 40% discounts, Julie paid \$135 for her phone. What was its original price? (2)

(e) If $a=-3$ and $b=-2$, find the exact value of $a^3(a-b)$ (2)

(f) Express $3\cdot2^{25}\div0\cdot014$ in scientific notation correct to 3 significant figures (2)

Question 2 Algebra and Surds (Start a new page) **12 marks**

(a) Simplify: $2(a-3)-2(2a-3)$ (2)

(b) Simplify giving your answer in simplest surd form: (2)

$$\sqrt{8}+\sqrt{50}-3\sqrt{18}$$

(c) Express $\dfrac{2\sqrt{3}}{5-\sqrt{3}}$ with a rational denominator (2)

(d) Simplify this algebraic fraction:

$$\frac{a+1}{3}-\frac{3-a}{2}$$ (2)

(e) Fully factorise the expression $3x^3-24$ (2)

(f) Heron's formula for the area of a triangle with sides a, b , and c is: (2)

$$A=\sqrt{S(S-a)(S-b)(S-c)} \text{ where } S=\frac{a+b+c}{2}$$

Find the area of a triangle with sides 5 cm, 6 cm and 7 cm.

(Give your answer in simplest surd form)

Question 3 Equations Start a new page 12 marks

(a) Solve for x: $4(x + 5) = -6(3 - x)$ (2)

(b) Solve this inequality and graph the solution on a number line: (3)

$$|2x-3| \leq 5$$

(c) Solve the following simultaneous equations: (3)

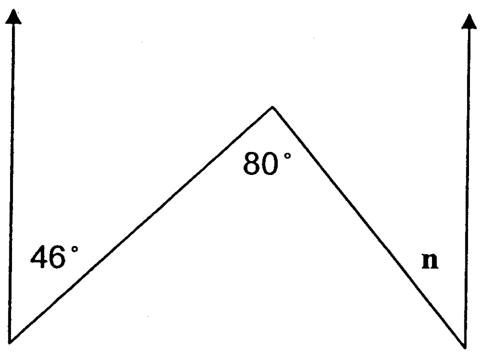
$$2x+y=12$$
$$3x+2y=22$$

(d) Use the quadratic formula or otherwise to solve $2x^2 - 3x - 1 = 0$, and give your answers correct to 1 decimal place. (2)

(e) Solve the equation $4^{3-x} = 8^x$ (2)

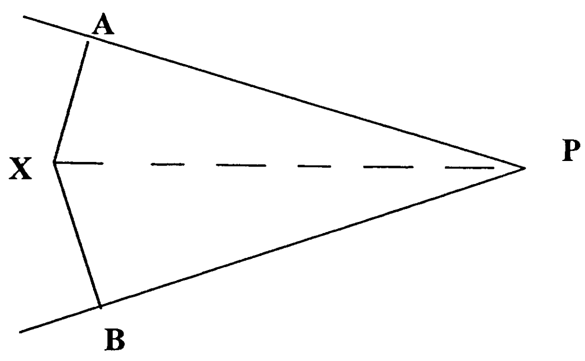
Question 4 Geometry Start a new page 12 marks

(a) Find the value of n. You do not need to give reasons. (2)



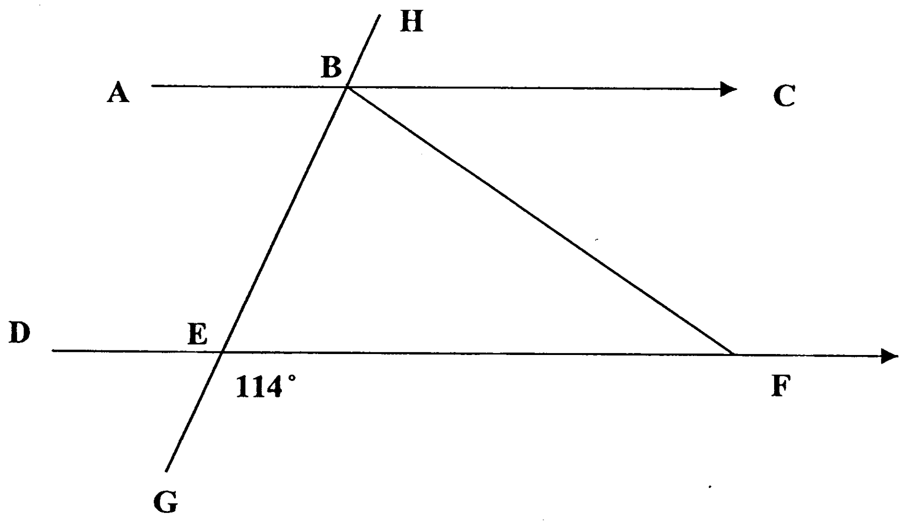
(b) In the diagram, AX = BX, $\angle PAX = \angle PBX = 90°$ Mark this on your diagram.

Using congruent triangles, prove that PX bisects $\angle APB$ (3)



Question 4 (continued)

(c)



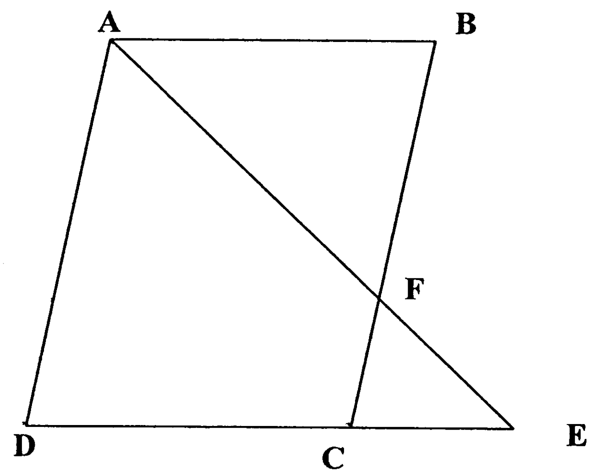
In the diagram, AC ∥ DF, EF = BF, ∠GEF = 114°, and ∠CBF = x°. (3)

(i) Copy or trace the diagram, and mark the information on it.
(ii) Find the value of x. giving complete reasons.

(d) ABCD is a parallelogram with DC produced to E, where AD = 120 mm, CE = 40 mm and BF = 70 mm. (4)

(i) Show that ΔABF is similar to ΔECF

(ii) Find the length of AB



Question 5 Functions and Graphs Start a new page **12 marks**

(a) Draw clear sketches of each of the following showing the intercepts on the axes wherever appropriate. (8)

(i) $x^2 + y^2 = 9$

(ii) $2x + 3y = 6$

(iii) $y = 2^x + 1$

(iv) $y = |x - 2|$

(b) Find $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ (2)

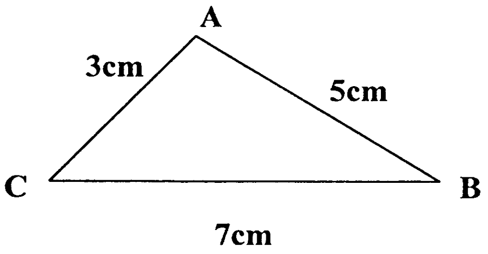
(c) For the function $y = 4 - x^2$, find : (2)

(i) its Domain;

(ii) its Range

Question 6 Trigonometry Start a new page 12 Marks

(a) Find the size of the largest angle in the triangle ABC (3)



(b) Find the exact value of : (3)

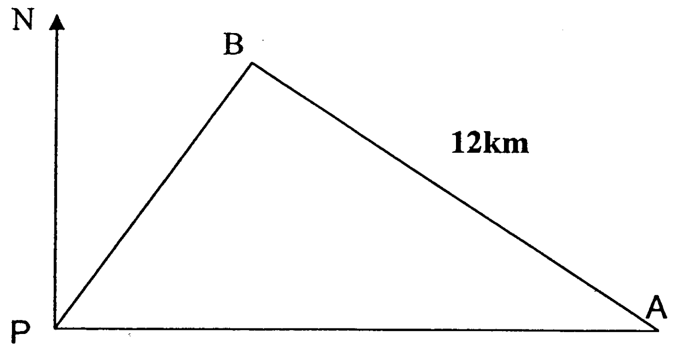
(i) $\sin 60^\circ$

(ii) $\cos 225^\circ$

(iii) $\cot(-30^\circ)$

(c) On the sketch, two towns A and B are 12km apart on a straight road running northeast from A to B. A surveyor at P , due West of A, observes that the bearing of B is 037°.

(i) Write down the size of $\angle BAP$ and $\angle APB$ (2)



(ii) Calculate the distance of P from B, correct to 3 significant figures. (2)

(d) Find all values of θ such that $\sqrt{3}\tan\theta = 1$ *and* $0^\circ \le \theta \le 360^\circ$ (2)

Question 7 Linear Expression and the straight Line Start a new page 12 marks

(a) On a neat sketch, mark the origin O and A(5,0) B(8,4) and C(0,10) (1)
Join A to B, B to C and C to A

(b) Show that the line AB has equation $4x - 3y - 20 = 0$ (2)

(c) Show that AB is perpendicular to BC (2)

(d) Show that AO = AB (2)

(e) Find the area of the quadrilateral ABCO (2)

(f) If D is the point (8,0), calculate the perpendicular distance of D from AB (2)

(g) What is the equation of the circle with centre A (1)
and which passes through both O and B ?

Question 8 Introduction to Calculus Start a new page 12 marks

(a) Differentiate the following:

(i) $y=(3x-2)^5$ (2)

(ii) $V=\frac{4}{3}\pi r^3$ (1)

(b) Given that $f(t) = \frac{2t-1}{3t+4}$, find $f'(-1)$ (2)

(c) Find the equation of the tangent to the curve $y = x^2 - 3x$ (3)
at the point where $x = 2$.

(d) Find the gradient of the normal to the curve to (2)
the curve $f(x)=2x^3-4x-5$ at (-1,-3)

(e) Find the size of the angle that the line $8x - 5y = 9$ makes (2)
with the positive x axis.

Question 9 The Quadratic Function Start a new page 12 marks

(a) The quadratic equation $2x^2 - 5x + 8 = 0$ has roots α and β (3)

Evaluate:

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^3\beta^2 + \alpha^2\beta^3$

(b) Without solving, show that the quadratic equation $3x^2 + 2x - 5 = 0$ has 2 different rational roots. (1)

(c) If the minimum value of the expression $x^2 - 4x + k$ is 5, find the value of k (2)

(d) Find A , B and C if $6x^2 - 2x + 9 \equiv Ax(x-1) + B(x-1) + C$ (3)

(e) For what values of k does the equation $x + \frac{1}{x} = k$ have real roots? (3)

Question 10 Locus and the parabola Start a new page 12 Marks

(a) What is the equation of the locus of a point P which moves so that Its distance from the origin is always less than or equal to 4 units (2)

(b) By completing the squares, find the centre and radius of the circle whose equation is $x^2+y^2-2x+6y-15=0$ (3)

(c) A parabola has its vertex at the point (3,1) and its focus at the point (3,3). (3)

(i) What is its focal length?

(ii) What is the equation of the directrix?

(iii) What is the equation of the parabola?

(d) A point P (x, y) moves so that the line PA is perpendicular to the line PB, where A = (1,5) and B = (-5,-3). Find the equation of this locus and describe it. (4)

SOLUTIONS + [MARKS] 2007 YR 11 Maths PRELIMINARY.

Q1

(a) $5-3 = 2$ [2]

(b) 4.22 [2]

(c) $a = 0.2\dot{3}$

$10a = 2.\dot{3}$

$100a = 23.\dot{3}$

$90a = 21$

$a = \frac{21}{90} = \frac{7}{30}$ [2]

(d) 60% of $x = 135$

20% of $x = 45$

$x = \$225$

[or unitary method] [2]

(e) $(-3)^3(-3--2)$

$= -27 \times -1 = 27$ [2]

(f) 3.04×10^{14} [2]

Q2

(a) $2a - 6 - 4a + 6 = -2a$ [2]

(b) $\sqrt{8} = 2\sqrt{2}$

$\sqrt{50} = 5\sqrt{2}$

$3\sqrt{18} = 3 \times 3\sqrt{2}$

$7\sqrt{2} - 9\sqrt{2} = -2\sqrt{2}$ [2]

(c) $\frac{2\sqrt{3}(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})} = \frac{10\sqrt{3}+6}{25-3}$

$= \frac{2(5\sqrt{3}+3)}{22}$ [2]

OR $\rightarrow = \frac{(5\sqrt{3}+3)}{11}$

(d) $\frac{a+1}{3} - \frac{3-a}{2} = \frac{2a+2-9+3a}{6}$

$= \frac{5a-7}{6}$ [2]

(e) $3(x^3-8) = 3(x-2)(x^2+2x+4)$ [2]

(f) $s = \frac{5+6+7}{2} = 9$

$A = \sqrt{9(4)(3)(2)} = \sqrt{36 \times 6} = 6\sqrt{6}$

[1] for $\sqrt{216}$ [2]

Q3

(a) $4x + 20 = -18 + 6x$

$38 = 2x$

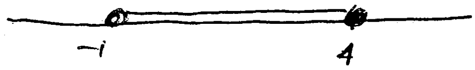
$x = 19$ [2]

(b) $|2x-3| \leq 5$

$2x - 3 \leq 5$ or $2x - 3 \geq -5$

$2x \leq 8$ $\quad$ $2x \geq -2$

$x \leq 4$ [1] $\quad$ $x \geq -1$ [1]



[1]

(c) $2x + y = 12$ — ①

$3x + 2y = 22$ — ②

①×2 $4x + 2y = 24$ — ③

③−② $x = 2$

Sub $y = 8$

Check $6 + 16 = 22$

$\therefore x = 2$, $y = 8$ [3]

(d) $2x^2 - 3x - 1 = 0$ $\quad$ $a = 2$, $b = -3$, $c = -1$

$x = \frac{3 \pm \sqrt{9 - 4(2)(-1)}}{4}$ [3]

$= \frac{3 \pm \sqrt{17}}{4} = 1.8$ or -0.3

(e) $4^{3-x} = 8^x$

$(2^2)^{3-x} = (2^3)^x$

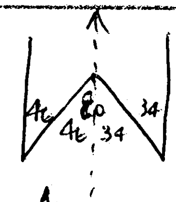
$2^{6-2x} = 2^{3x}$

$6 - 2x = 3x$

$6 = 5x$

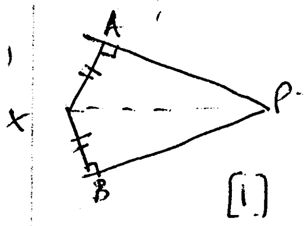
$\therefore x = \frac{6}{5}$ [2]

4. (a)



$n = 34$ [2]

(b)



In Δ's APX, BPX,

AX = BX (given)

[1] $P\hat{A}X = P\hat{B}X = 90°$ (given)

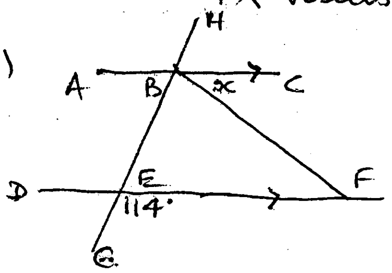
PX is common

∴ the Δ's are congruent (RHS) [1]

[1] { ∴ $A\hat{P}X = B\hat{P}X$ (corresp. angles in congruent Δ's)

So PX bisects $A\hat{P}B$. }

(c)



[1]

$B\hat{E}F = 66°$ (Suppl. adj ∠'s) } [1]

Δ BFE is isosceles, ∴ base ∠'s equal

So $E\hat{F}B = 180 - 2 \times 66 = 48°$

[1] Then $x = 48°$ equal alternate ∠'s with AC ∥ DF.

(d) In Δ's ABF and CEF,

$B\hat{A}F = C\hat{E}F$ (equal alt. ∠'s AB ∥ DE)

$A\hat{B}F = F\hat{C}E$ (" " ")

∴ $A\hat{F}B = C\hat{F}E$ (remaining ∠ in a Δ (angle sum theorem))

∴ Δ's are Similar (A.A.A.) [2]

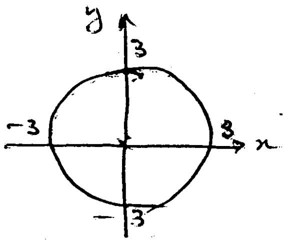
So $\frac{AB}{40} = \frac{70}{50} = \frac{7}{5}$

∴ $5AB = 280$

$AB = 56$ cm. [2]

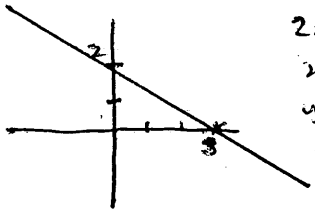
Q5

(a) (i)



[2]

(ii)



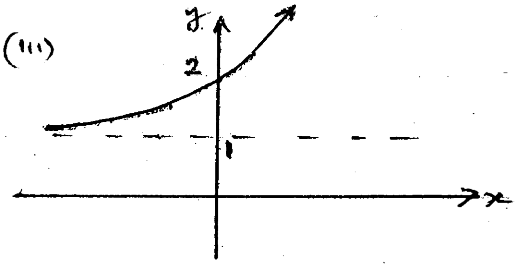
$2x + 3y = 6$

$x = 0, y = 2$

$y = 0, x = 3$

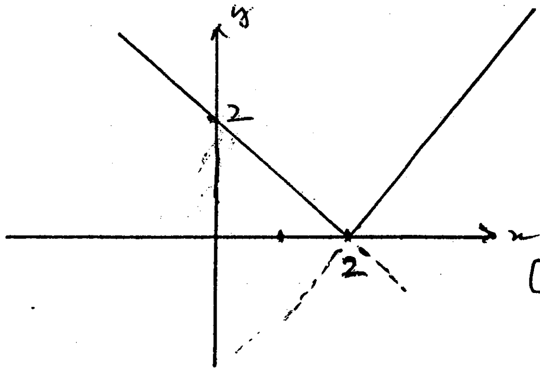
[2]

(iii)



[2]

(iv)



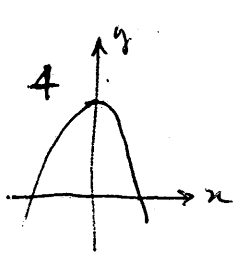
[2]

(b) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

$= \lim_{x \to 3} x + 3 = 6.$ [2]

(c) $y = 4 - x^2$



D: $x \in R$ [1]

R: $y \leq 4$ [1]

Q6 (a)

$\cos A = \frac{3^2+5^2-7^2}{2\times3\times5} = \frac{9+25-49}{30}$

$= \frac{-15}{30} = -\frac{1}{2}$

$\therefore A = 120^\circ$ [3]

(b) (i) $\sin 60 = \frac{\sqrt{3}}{2}$ [1]

(ii) $\cos 225 = \cos(180+45)$

$= -\cos 45 =$

$= -\frac{1}{\sqrt{2}}$ [1]

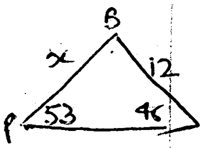
(iii) $\cot(-30) = \frac{1}{\tan -30}$

$= \frac{1}{-\frac{1}{\sqrt{3}}}$

$= -\sqrt{3}$ [1]

(c) (i) $B\hat{A}P = 45^\circ$ [1]

$A\hat{P}B = 53^\circ$ [1]



(ii) $\frac{PB}{\sin 45} = \frac{12}{\sin 53}$

$PB = \frac{12\sin 45}{\sin 53}$ [1]

$= 10.6$ Km [1]

(d) $\sqrt{3}\tan\theta = 1$

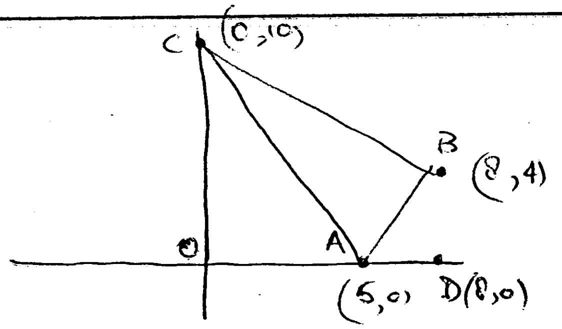
$\therefore \tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$ and $180+30$

$= 30^\circ$ and 210° [2]

Q7

(a)



(b) grad. of AB $= \frac{4}{3}$

eqn. is $y-0 = \frac{4}{3}(x-5)$

$3y = 4x-20$

$4x-3y-20 = 0$ [2]

(c) gradient of BC $= \frac{6}{-8} = -\frac{3}{4}$

" " AB $= \frac{4}{3}$ [2

Since $-\frac{3}{4}\times\frac{4}{3} = -1$, $BC \perp AB$.

(d) AO = 5 units AB $= \sqrt{(4-0)^2+(8-5)^2}$

$= \sqrt{25} = 5$

$\therefore AO = AB$. (2)

(e) Area of $\triangle OCA = \frac{1}{2}\times5\times10 = 25$ u

" " $\triangle ABC = 25$ units^2

$\therefore$ Area of quad. OABC $= 50$ units^2 [2

(f) $d = \left|\frac{8\times4-3\times0-20}{\sqrt{4^2+3^2}}\right|$

$= \frac{32-20}{5} = 2\frac{2}{5}$ units [2

(g) Centre is (5,0) Radius is 5

$\therefore$ eqn is $(x-5)^2+(y-0)^2 = 5^2$

$(x-5)^2+y^2 = 25$ [1]

Q8

(a)(i) $y = (3x-2)^5$

$y' = 5(3x-2)^4 \times 3$

$= 15(3x-2)^4$ [2]

(ii) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{12}{3}\pi r^2 = 4\pi r^2$ [1]

(b) $f(t) = \frac{2t-1}{3t+4}$

$f'(t) = \frac{(3t+4).2 - 3(2t-1)}{(3t+4)^2}$

$= \frac{6t+8-6t+3}{(3t+4)^2}$

$= \frac{11}{(3t+4)^2}$ [1]

So $f'(-1) = \frac{11}{(-3+4)^2} = 11$ [1]

(c) $y = x^2 - 3x$

$y' = 2x - 3$

if $x = 2$, $y = 4 - 6 = -2$ $(2,-2)$

if $x = 2$, $y' = 4 - 3 = 1$

$\therefore m = 1$, $(x_1, y_1) = (2, -2)$

So eqn is $y + 2 = 1(x - 2)$

$y + 2 = x - 2$

$y = x - 4$

(d) $f(x) = 2x^3 - 4x - 5$

$f'(x) = 6x^2 - 4$

$f'(-1) = 6(-1)^2 - 4 = 2$ [1]

$\therefore$ Normal has gradient $-\frac{1}{2}$ [1]

(e) $8x - 5y = 9 \Rightarrow y = \frac{8}{5}x - \frac{9}{5}$

$\tan\theta = \frac{8}{5}$, $\therefore \theta = 58°$

Q9 $2x^2 - 5x + 8 = 0$

(a)(i) $\alpha + \beta = \frac{-b}{a} = \frac{5}{2}$ [1]

(ii) $\alpha\beta = \frac{c}{a} = \frac{8}{2} = 4$ [1]

(iii) $\alpha^2\beta^2(\alpha+\beta) = 4^2 \times \frac{5}{2}$

$= 40$ [1]

(b) $3x^2 + 2x - 5 = 0$

$\Delta = 4 - 4(3)(-5) = 4 + 60 = 64$

Since 64 is a perfect square, roots are RATIONAL and DIFFERENT [1

(c) $x^2 - 4x + K$

Min. value is when $x = -\frac{b}{2a} = \frac{4}{2} =$

if $x = 2$, $2^2 - 4 \times 2 + K = 5$

$4 - 8 + K = 5$,

$\therefore K = 9$ [2]

(d) $Ax(x-1) + B(x-1) + C = 6x^2 - 2x +$

Equating coefficients: $A = 6$. [1]

if $x = 1$, $C = 6 - 2 + 9 = 13$

if $x = 2$, $12(1) + B(1) + 13 = 24 - 4 + 9$

$B + 25 = 29$

$B = 4$ [1]

ie $A = 6$, $B = 4$, $C = 13$

(e) $x + \frac{1}{x} = K \rightarrow x^2 + 1 = Kx$

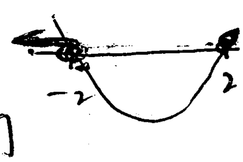
ie $x^2 - Kx + 1 = 0$ [1]

For real roots, $\Delta \geq 0$

ie $K^2 - 4(1)(1) \geq 0$

$K^2 - 4 \geq 0$ [1]

$(K-2)(K+2) \geq 0$



$K \leq -2$ and $K \geq 2$ [1]

Q10

(a) $x^2 + y^2 \leqslant 4^2$

i.e. $x^2 + y^2 \leqslant 16$ [2]

[1] for $= 16$, [1] for $<$

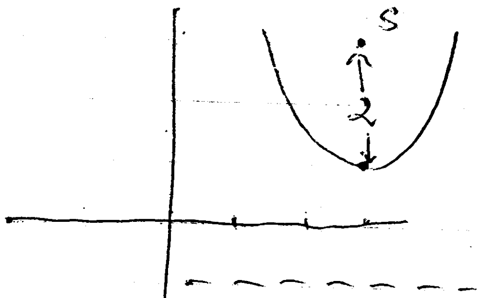
(b) $x^2 + y^2 - 2x + 6y - 15 = 0$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 15 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 25$

Centre is $(1, -3)$ radius $= 5$. [3]

(c)



(i) $a = 2$ [1]

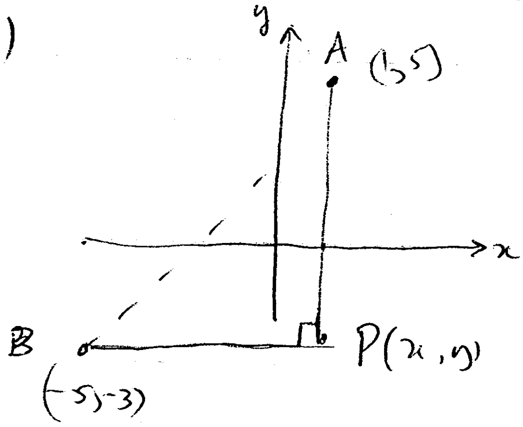
(ii) directrix is $y = -1$ [1]

(iii) Parabola is

$$(x-3)^2 = 4 \times 2(y-1)$$

i.e. $(x-3)^2 = 8(y-1)$ [2].

(d)



$PA \perp PB$

gradient of $PA = \dfrac{y-5}{x-1}$

gradient of $PB = \dfrac{y+3}{x+5}$

So $\dfrac{y-5}{x-1} \times \dfrac{y+3}{x+5} = -1$ [4]

$y^2 - 2y - 15 = -x^2 - 4x + 5$

$x^2 + y^2 + 4x - 2y - 20 = 0$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 20 + 4 + 1$

$(x+2)^2 + (y-1)^2 = 25$

Circle, centre $(-2, 1)$ radius 5.