**QUESTION 1**

a) Simplify $\frac{x}{2} - \frac{4-2x}{3}$ (2)

b) Solve $x^2 - 3x - 10 > 0$ (2)

c) Find a primitive function of (2)

i) $4x - 2$

ii) $\frac{1}{x^3}$

d) Solve $x^3 - x^2 - x + 1 \geq 0$ (3)

e) Find $\lim_{x \to 3} \frac{4x^3 - 108}{3 - x}$ (3)

**QUESTION 2** **Start a New Page**

a) Consider the curve $y = 1 + 3x - x^3$ for $-2 \le x \le 3$

i) Find the co-ordinates of the stationery points and determine their nature (3)

ii) Find any point(s) of inflexion (2)

iii) Sketch the curve for $-2 \le x \le 3$ (2)

iv) What is the minimum value of $y$ for $-2 \le x \le 3$ (1)

v) For $-2 \le x \le 3$, for what values of $x$ is the curve concave up? (1)

b) Find the values of $m$ for which the roots of the quadratic equation

$2x^2 + x + 5 - m = 0$ are

i) real and equal (2)

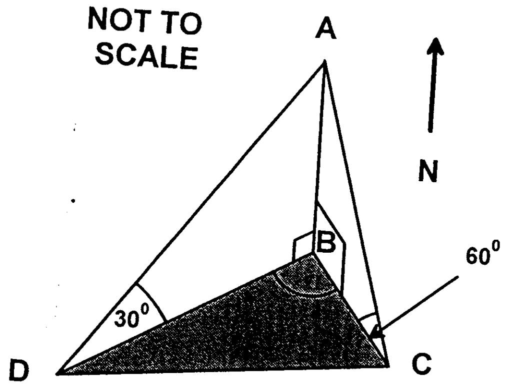
ii) real and reciprocals of each other (1)

**QUESTION 3** **Start a New Page**

a) Find the co-ordinates of the point which divides the interval AB with A(-6, 2) and B(4,7) <u>externally</u> in the ratio 3 : 2 (2)

b) Solve for $x$: $\dfrac{3}{x+5} \leq 1$ (4)

c)




The bearing of $C$ from $B$ is $160^0$ and the bearing of $D$ from $B$ is $220^0$.
The angle of elevation from $D$ to vertical tower $AB$ is $30^0$ and similarly the angle of elevation of the tower $AB$ from $C$ is $60^0$. The distance from $D$ to $C$ is 14 *metres*.

Let $AB = h$ *metres*

(i) Explain why $\angle DBC = 60^0$. (1)

(ii) Considering $\Delta ABC$, show that $BC = \dfrac{h}{\tan 60^0}$. (1)

(iii) Find a similar expression for $BD$. (1)

(iv) By using the cosine rule in $\Delta BCD$, show that the height of tower $AB$ is $2\sqrt{21}$ metres exactly. (3)

**QUESTION 4** **Start a new page**

a) The gradient function of a curve is $2x - 3$ and the curve passes through $(1, 5)$. Find the equation of the curve. (2)

b) Find the equation of the parabola with focus $(2, -5)$ and directrix $y = 3$ (3)

expand?

c) Given the function $y = \dfrac{2}{1+x^2}$

(i) Show that $y = \dfrac{2}{1+x^2}$ is an even function and describe the graphical significance of an even function. (2)

(ii) Find $\lim\limits_{x \to \infty} \dfrac{2}{1+x^2}$ (1)

(iii) Calculate the turning point of $y = \dfrac{2}{1+x^2}$ and determine its nature. (3)

(iv) Sketch the curve $y = \dfrac{2}{1+x^2}$ (1)

**QUESTION 5** **Start a New Page**

a) Find the domain of $y = \dfrac{1}{\sqrt{(9 - x^2)}}$ (2)

b) The volume, V, of water in a dam at time $t$ was monitored over a period of time. When monitoring began, the dam was 75% full. During the monitoring period, the volume decreased at an increasing rate due to a long period of drought.

i) What does this tell us about $V'$ and $V''$ ? (2)

ii) Sketch a graph of V against $t$. (1)

c) Given the series

$$2 + 4 + 6 + \dots\dots\dots + 1000, \text{ find}$$

(i) the 21st term (1)

(ii) the sum of the series (2)

d) Use Mathematical Induction to prove that, for all positive integers n:

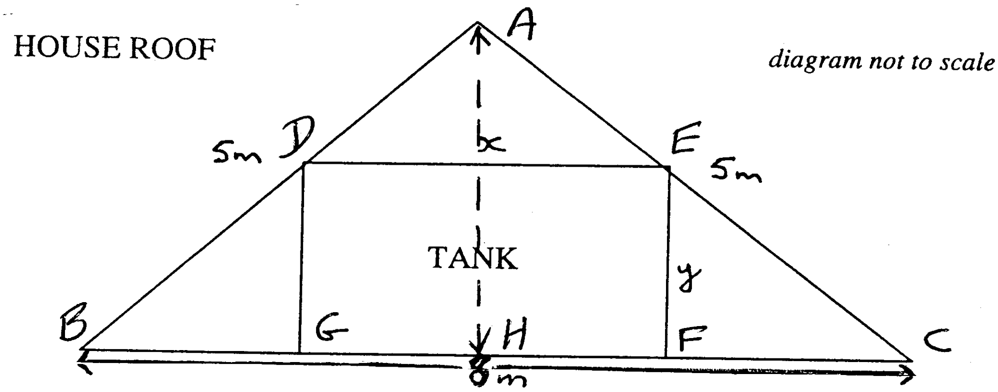
$$1^2 + 2^2 + 3^2 + 4^2 + \dots\dots\dots + n^2 = {}^1\!/_6\, n\,(n + 1)(2n + 1)$$ (4)

**QUESTION 6**

a) Solve $(2^x)^2 + 7(2^x) - 8 = 0$ (2)

b) Evaluate $\sum_{n=4}^{50}(3n+2)$ (2)

c) The diagram below show a rectangular water tank, $x$ metres wide, $y$ metres high and 1.5 metres long which fits exactly into the roof of a house. A cross section of the roof is in the shape of an isosceles triangle with base 8 metres and equal sides 5 metres in length.




(i) Copy the diagram onto your answer sheet.

(ii) Explain why the roof of the house is 3 metres high. (1)

(iii) Show that FC $= 4 - x/2$ (1)

(iv) Using the fact that $\Delta$EFC ||| $\Delta$AHC, show that $y = {}^1/_8(24 - 3x)$ (2)

(v) Show that the volume of the tank is given by:

$$V = {}^3/_{16}\, x(24 - 3x)$$ (1)

(vi) Calculate the maximum volume of the tank. (3)

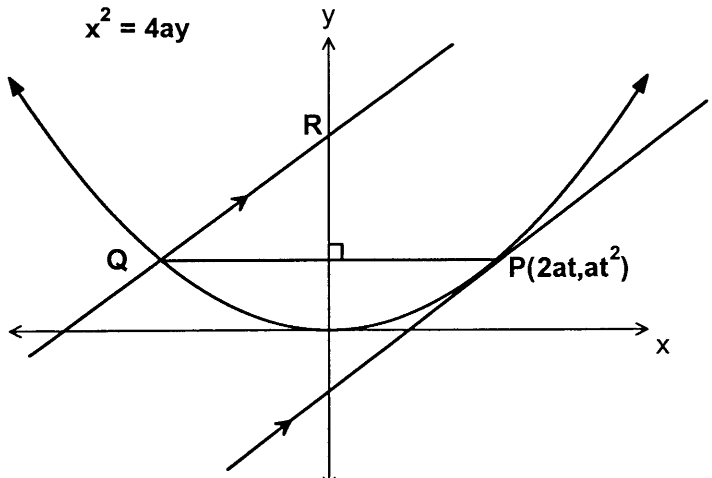
## QUESTION 7

(a) A stone dropped from the top of a 245m high cliff falls 5 metres in the first second, 15 metres in the second second, 25 metres in the third and so on.

(i) Find a formula for the distance fallen after n seconds (2)

(ii) How longs does it take before the stone hits the ground? (1)

(b) $P(2at, at^2)$ lies on $x^2 = 4ay$. $PQ$ is perpendicular to the axis, and the line through $Q$ parallel to the tangent at $P$ meets the $y$ axis in $R$.




**Question 7 continues over page**

(i) Show that the equation of the tangent at $P$ is $tx - y - at^2 = 0$. (2)

(ii) Write down the coordinates of $Q$ in terms of $t$. (1)

(iii) Find the equation of $QR$. (2)

(iv) Find the coordinates of $R$. (1)

(v) Hence find the cartesian equation of the locus of midpoints of $QR$ (3)

End of paper!

Year 11 Extension 1 Mathematics Yearly Examination 2008

Suggested Solutions | Comments

Question (1)

a) $\frac{x}{2} - \frac{4-2x}{3} = \frac{3x - 2(4-2x)}{6}$ (1)

$= \frac{3x - 8 + 4x}{6}$

$= \frac{7x-8}{6}$ (1)

b) $x^2 - 3x - 10 > 0$

$(x-5)(x+2) > 0$ (1)

$x < -2,\ x > 5$ (1)

c) i) $4\frac{x^2}{2} - 2x + c = 2x^2 - 2x + c$ 1

ii) $\frac{x^{-2}}{-2} + c = \frac{1}{-2x^2} + c$ 1

d) $x^3 - x^2 - x + 1 \geq 0$

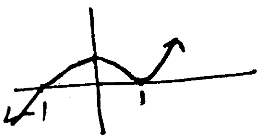
$x^2(x-1) - 1(x-1) \geq 0$ (1)

$(x-1)(x^2-1) \geq 0$ (1)

$(x-1)(x-1)(x+1) \geq 0$

$(x-1)^2(x+1) \geq 0$ (1)

$x \geq -1$




e) $\lim_{x\to 3} \frac{4x^3 - 108}{3-x} = \lim_{x\to 3} \frac{4(x^3-27)}{3-x}$ (1)

$= \lim_{x\to 3} \frac{4(x-3)(x^2+3x+9)}{3-x}$ (1)

$= \lim_{x\to 3} \frac{-4\cancel{(3-x)}(x^2+3x+9)}{\cancel{3-x}}$

$= -4 \times 27$ (1)

$= -108$

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Suggested Solutions — Comments

Question (2)

a) $y = 1 + 3x - x^3$

i) $y' = 3 - 3x^2$

Stationary points $y' = 0$

$0 = 3(1 - x^2)$

$= 3(1-x)(1+x)$

$x = 1, -1$

at $x = 1$, $y = 1 + 3(1) - (1)^3$

$= 3$

at $x = -1$, $y = 1 + 3(-1) - (-1)^3$

$= -1$

Stationary points are $(1,3)$ and $(-1,-1)$

$y'' = -6x$

at $x = 1$, $y'' = -6 < 0$ $\therefore$ Maximum turning point at $(1,3)$

at $x = -1$, $y'' = 6 > 0$ $\therefore$ Minimum turning point at $(-1,-1)$

Comments: 3 — 1 for finding points only

ii) Points of inflexion: $y'' = 0$

$-6x = 0$

$x = 0$

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $y''$ | $6$ | $0$ | $-6$ |

change in concavity.

Point of inflexion at $(0,1)$

Comments: 1 for $(0,1)$; 1 for testing concavity

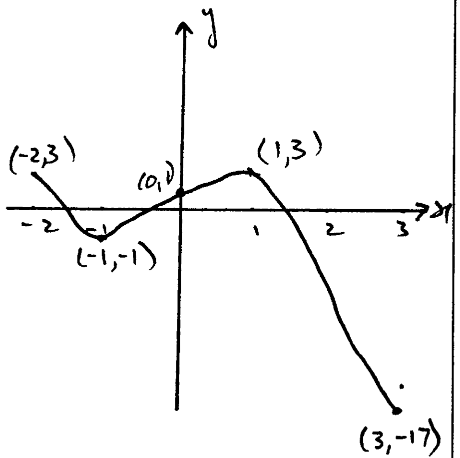
iii) At $x = -2$, $y = 1 + 3(-2) - (-2)^3$

$= 3$ $\quad (-2,3)$

at $x = 3$, $y = 1 + 3(3) - (3)^3$

$= -17$ $\quad (3,-17)$

Comments: 1 for $(-2,3)$ and $(3,-17)$




iv) Minimum value is $-17$ (1)

v) $-2 \le x < 0$ (1)

b) i) $\Delta = (1)^2 - 4 \times 2 \times (5 - m) = 0$

$1 - 40 + 8m = 0$

$8m = 39$

$m = \frac{39}{8}$ (2)

Comments: 1 for $\Delta = 0$

ii) $\alpha\beta = 1$

$\frac{c}{a} = 1$

$\frac{5-m}{2} = 1$

$5 - m = 2$

$m = 3$

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Suggested Solutions

Comments

Question ( 3 )

a) $\left(\frac{3\times 4-2\times -6}{1}, \frac{3\times 7-2\times 2}{1}\right) = (24, 17)$ 1 1

b) $\frac{3}{\cancel{x+5}}(x+5)^2 \le 1(x+5)^2$ 1 $x \neq -5$

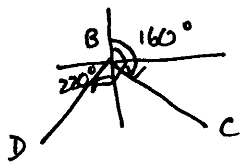
$0 \le (x+5)^2 - 3(x+5)$ 1

$(x+5)[(x+5)-3] \geq 0$

$(x+5)(x+2) \geq 0$ 1 $-5$ $-2$

$x < -5,\ x \geq -2$ 1

c) i)




$\therefore \angle DBC = 220° - 160°$
$= 60°$ 1

ii)


$\tan 60° = \frac{h}{BC}$

$BC = \frac{h}{\tan 60°}$ 1

iii) $BD = \frac{h}{\tan 30°}$ 1

iv) $14^2 = \left(\frac{h}{\tan 60}\right)^2 + \left(\frac{h}{\tan 30}\right)^2 - 2\left(\frac{h}{\tan 60}\right)\left(\frac{h}{\tan 30}\right)\cos 60°$ 1

$196 = \left(\frac{h}{\sqrt{3}}\right)^2 + \left(\frac{h}{\frac{1}{\sqrt{3}}}\right)^2 - 2\left(\frac{h}{\sqrt{3}}\right)(\sqrt{3}h)\cdot\frac{1}{2}$

$= \frac{h^2}{3} + 3h^2 - h^2$ 1

$196 = \frac{7h^2}{3}$

$84 = h^2$

$h = \sqrt{21\times 4}$

$= 2\sqrt{21}$ metres 1

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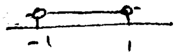
Suggested Solutions

Comments

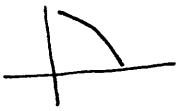
Question (5)

a) $y = \dfrac{1}{\sqrt{9-x^2}}$

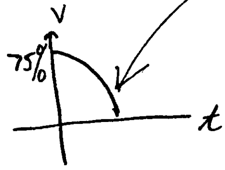
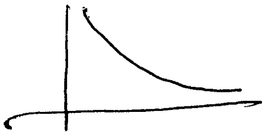
$= \dfrac{1}{\sqrt{(3-x)(3+x)}}$ $x \neq 3, -3$ (1)

$(3-x)(3+x) > 0$  (1)

$-3 < x < 3$

b) i)  $V' < 0$ (✗)

$V'' < 0$ (1) $V'' > 0$

ii)  (1) 

c) i) $a = 2$, $d = 2$. $T_n = a + (n-1)d$

$T_{21} = 2 + 20 \times 2$

$= 42$ (1)

ii) $1000 = 2 + (n-1)2$

$= 2n$ (1)

$n = 500$

$S_n = \frac{n}{2}(a + l)$

$S_{500} = \frac{500}{2}(2 + 1000)$ (1)

$= 250\,500$

d) Step 1: To prove true for $n=1$

LHS $= 1^2$ RHS $= \frac{1}{6} \times 1(1+1)(2\times 1+1)$ (1)

$= 1$ $= \frac{1}{6} \times 2 \times 3$

$= 1$

$=$ LHS

$\therefore$ True for $n=1$

Step 2: Assume true for $n=k$

i.e. $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{1}{6}k(k+1)(2k+1)$ (1)

To prove true for $n = k+1$

i.e. $1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$

LHS $= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$

$= \frac{1}{6}(k+1)[2k^2 + k + 6k + 6]$ (2)

$= \frac{1}{6}(k+1)[2k^2 + 7k + 6]$

$= \frac{1}{6}(k+1)[2k^2 + 4k + 3k + 6]$

$= \frac{1}{6}(k+1)[2k(k+2) + 3(k+2)]$

$= \frac{1}{6}(k+1)(k+2)(2k+3)$ Step 3

$=$ RHS

Year 11 Extension 1 Mathematics — Yearly Examination 2008

Suggested Solutions — Comments

Question (4)

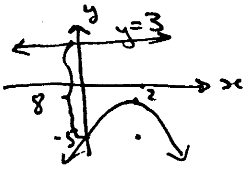
a) $y' = 2x - 3$

$y = x^2 - 3x + c$ ①

$x=1, y=5, \quad 5 = (1)^2 - 3(1) + c$

$7 = c$ ①

$\therefore y = x^2 - 3x + 7$

b) 


Vertex $(2,-1)$

$a = 4$

$(x-h)^2 = -4a(y-k)$

$(x-2)^2 = -16(y+1)$

1 for vertex
1 for a
1 for equation/format

c) i) $f(x) = \frac{2}{1+x^2}$

$f(-x) = \frac{2}{1+(-x)^2}$

$= \frac{2}{1+x^2}$ ①

$= f(x)$. $\therefore$ even function.

Symmetrical in the y-axis ①

ii) $\lim\limits_{x\to\infty} \frac{2}{1+x^2} = \lim\limits_{x\to\infty} \frac{\frac{2}{x^2}}{\frac{1}{x^2}+\frac{x^2}{x^2}}$

$= \frac{0}{0+1}$ ①

$= 0$

iii) $y = \frac{2}{1+x^2}$

$y' = \frac{(1+x^2).0 - 2.(2x)}{(1+x^2)^2}$

$= \frac{-4x}{(1+x^2)^2}$

stationary points $y' = 0$

$0 = \frac{-4x}{(1+x^2)^2}$

$x = 0$. $\quad (0,2)$

| $x$ | -1 | 0 | 1 |
|---|---|---|---|
| $y'$ | + | 0 | − |

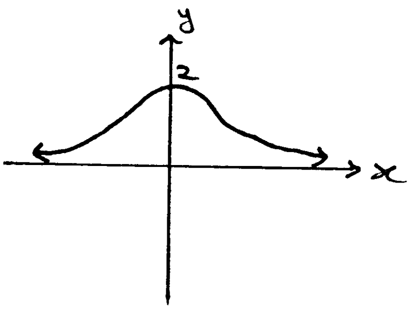
/ — \

$\therefore$ Maximum turning point at $(0,2)$

1 - quotient rule/product rule
1 (0,2)
1 nature

$y'' = \frac{-4(1+x^2)^2 - -4x[2(1+x^2)^1 . 2x]}{(1+x^2)^4}$

$= \frac{-4(1+x^2)^2 + 16x^2(1+x^2)}{(1+x^2)^4}$

iv) 


①

Comments

Question (6)

a) $(2^x)^2 + 7(2^x) - 8 = 0$

Let $u = 2^x$

$u^2 + 7u - 8 = 0$

$(u+8)(u-1) = 0$

$u = -8, 1$

$2^x = -8, 1$

$x = 0$ (no real solution for $2^x = -8$)

(2)

1 for $u = -8, 1$

1 for answer

b) $\sum_{n=4}^{50} (3n+2) = 14 + 17 + 20 + \dots + 152$

$S_n = \frac{n}{2}(a + l)$

$S_{47} = \frac{47}{2}(14 + 152)$

$= 3901$

(2)

$n = 47$

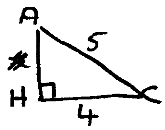
$a = 14$

$d = 3$

$l = 152$

$14 + 17 + \dots + 152$ = 0 marks. Needed to show some idea of sum of arithmetic series.

c) ii)




By Pythagoras' Rule,

$5^2 = AH^2 + 4^2$

$AH = 3$.

(1)

iii) $FC = HC - HF$

$= 4 - \frac{1}{2}x$

(1)

iv) $\frac{3}{y} = \frac{4}{4 - x/2}$

$4y = 3(4 - \frac{x}{2})$

$4y = 12 - \frac{3x}{2}$

$8y = 24 - 3x$

$y = \frac{1}{8}(24 - 3x)$

(2)

v) $V = x \cdot y \cdot 1.5$

$= 1.5x \frac{1}{8}(24 - 3x)$

$= \frac{3}{16}x(24 - 3x)$

(1)

vi) $V' = \frac{3}{16}(24 - 3x) + \frac{3}{16}x \cdot -3$

$= 4\frac{1}{2} - \frac{9}{16}x - \frac{9}{16}x$

$= 4\frac{1}{2} - \frac{9}{8}x$

Let $V' = 0$ for stationary points

$0 = 4\frac{1}{2} - \frac{9}{8}x$

$x = 4$

$V'' = -\frac{9}{8} < 0$ $\therefore$ Maximum.

$\therefore$ Maximum volume $= \frac{3}{16} \times 4 \times (24 - 3 \times 4)$

$= 9 m^3$

1 for $x = 4$

1 for showing a max

1 for $9m^3$

Question ( 7 )

a) $5 + 15 + 25 + \cdots$

i) $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{10 + (n-1).10\}$

$= \frac{n}{2}\{10 + 10n - 10\}$

$= 5n^2$

1 for a and d correct.

ii) $245 = 5n^2$

$n^2 = 49$

$n = 7$ $\therefore$ after 7 seconds (1)

If solve (i) = 245 correctly, 1 mark.

b) i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$

$= \frac{x}{2a}$

at $x = 2at$, $y' = \frac{2at}{2a}$

$= t$.

$y - y_1 = m(x - x_1)$

$y - at^2 = t(x - 2at)$ (2)

$y - at^2 = tx - 2at^2$

$tx - y + at^2 = 0$

ii) $Q = (-2at, at^2)$ (1)

iii) $m = t$ since lines parallel

1 for gradient $= t$

$y - at^2 = t(x + 2at)$

$y - at^2 = tx + 2at^2$ (2)

$tx - y + 3at^2 = 0$

1 if let $x = 0$ in (iii) & find $y$ correctly.

iv) let $x = 0$, $-y + 3at^2 = 0$

$y = 3at^2$ (1)

$R(0, 3at^2)$

v) midpoint of $QR = \left(\frac{-2at + 0}{2}, \frac{at^2 + 3at^2}{2}\right)$

1 for midpoint

$= (-at, 2at^2)$

$x = -at$

$t = -\frac{x}{a}$

$y = 2at^2$

$= 2a\left(-\frac{x}{a}\right)^2$

$= 2\cancel{a}\frac{x^2}{a^{\cancel{2}}}$

$y = \frac{2x^2}{a}$