Section I (10 marks)

Use the multiple-choice answer sheet for Questions 1-10

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1. Which of the following is equivalent to $\sqrt{243}+2\sqrt{75}$?

   (A) $19\sqrt{3}$ (B) $81\sqrt{3}$ (C) $106\sqrt{3}$ (D) $2\sqrt{318}$

2. The domain of the function $f(x)=\dfrac{1}{(x-3)(1-x)}$ is all real x but:

   (A) $x \neq -1$ or $x \neq -3$ (B) $x \neq -1$ or $x \neq 3$

   (C) $x \neq 1$ or $x \neq -3$ (D) $x \neq 1$ or $x \neq 3$

3. What is the value of $\dfrac{dy}{dx}$ if $y = x^4 + 5x^{-1}$?

   (A) $\dfrac{dy}{dx} = 4x^3 - 5x^0$ (B) $\dfrac{dy}{dx} = 4x^3 + 5x^0$

   (C) $\dfrac{dy}{dx} = 4x^3 - 5x^{-2}$ (D) $\dfrac{dy}{dx} = 4x^3 + 5x^{-2}$

4. Solve $|2-3x| \geq 5$

   (A) $x \leq -1$ or $x \geq 2\frac{1}{3}$ (B) $x \geq -1$ or $x \leq 2\frac{1}{3}$

   (C) $x \leq -2\frac{1}{3}$ or $x \geq 1$ (D) $x \geq -2\frac{1}{3}$ or $x \leq 1$

5. Which type of function is $f(x) = 2x^3 - x$?

   (A) Odd (B) Even (C) Neither odd or even (D) Zero

6. What is solution to the equation $2\sin\beta = -\sqrt{3}$ for $0^\circ \leq \beta \leq 360^\circ$?

   (A) $\beta = 60°, 300°$ (B) $\beta = 120°,\ 240°$

   (C) $\beta = 210°,\ 330°$ (D) $\beta = 240°,\ 300°$

7. What is the solution to the equation $x^2 + 2x - 7 = 0$?

   (A) $x = -1 \pm \sqrt{2}$ (B) $x = -2 \pm \sqrt{2}$ (C) $x = -2 \pm 2\sqrt{2}$ (D) $x = -1 \pm 2\sqrt{2}$

9. Which of the following is true for the equation $x^2 + 8x + 16 = 0$?

   (A) No real roots (B) Equal roots

   (C) Two real distinct roots (D) Three real roots

Question 12 (15 marks) Start a new answer booklet **Marks**

(a) Draw neat sketches of the following equations on a separate set of axes. Use a ruler to draw axes and mark scales
Show clearly the essential features of each graph.

i) $y = 4 - x^2$ **2**

ii) $y = \sqrt{4 - x^2}$ **2**

iii) $y = \dfrac{4}{x}$ **2**

(d) Find $\lim_{x \to 4} \dfrac{x^2 - x - 12}{x - 4}$ **2**

(e) Rationalise the denominator of $\dfrac{1}{\sqrt{5} - 1}$ **1**

Section II

Answer each question in a new answer booklet.

Question 11 (15 marks) **Marks**

(a) Express $0.1\dot{5}$ as a fraction in its simplest form. **2**

(b) Solve $\dfrac{1}{2}(y-3) - \dfrac{1}{3}(y-2) = 3$ **3**

(c) Simplify $\dfrac{a^2 + b^2 + 2ab}{-b^2 + a^2}$ **2**

(d) Find the exact value of:

(i) $\tan 240°$ **1**

(ii) $\sec 510°$ **2**

Question 13 (15 marks) Start a new answer booklet **Mark s**

(b) Prove $\tan\theta - \tan\theta \sin^2\theta = \sin\theta\cos\theta$ **2**

(c) Solve the following pair of simultaneous equations. **2**

$$2x + y - 1 = 0$$
$$3x - y - 4 = 0$$

(d) Differentiate with respect to *x*.

i) $\frac{1}{x^3}$ **1**

ii) $\sqrt{x}$ **2**

(e) Find the equation of the tangent to $y = x^3 - 4x$ at the point (1,-3). **2**

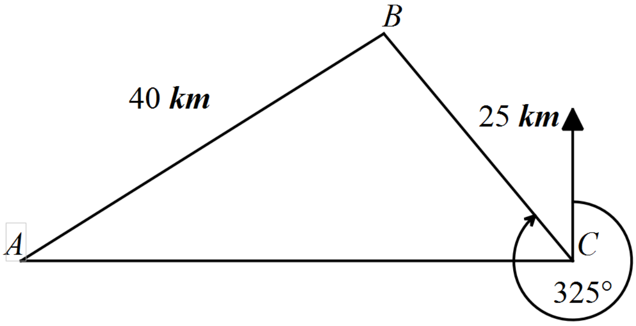
(f) For what values of *k* does $x^2 - kx + 4 = 0$ have no real roots. **2**

Question 14 (15 marks) Start a new answer booklet

Marks

End of paper

(a) Point C is due east of A. Point B is 40 km from A and 25 km from C.
The bearing of B from C is 325°.



i) Show that $\angle ACB = 55^\circ$ **1**

ii) What is the bearing of B from A? **3**

(c) Find the quadratic equation with roots $(1+\sqrt{3})$ and $(1-\sqrt{3})$. **2**

(d) The curve $y^2 = x+9$ and the straight line $x-3y+9=0$ intersect at A and B.

i) Find the coordinates of points A and B. **2**

ii) Sketch $y^2 = x+9$ and $x-3y+9=0$ on the same number plane. **2**

2018 Adjusted Prelim

1. $\sqrt{243} + 2\sqrt{75}$

$= \sqrt{3^5} + 2\sqrt{5^2 \times 3}$

$= \sqrt{3^4 . 3} + 2\sqrt{5^2 \times 3}$

$= 9\sqrt{3} + 10\sqrt{3}$

$= 19\sqrt{3}$ (A)

Simplest method: evaluate each on your calculator!

2 $f(x) = \dfrac{1}{(x-3)(1-x)}$

so $(x-3)(1-x) \neq 0$

ie $x \neq 3,\ x \neq 1$ (D)

Cannot divide by zero

3. $y = x^4 + 5x^{-1}$

$\dfrac{dy}{dx} = 4x^3 + 5.(-1)x^{-2}$

$= 4x^3 - 5x^{-2}$ (C)

4. $|2-3x| \geq 5$

$2 - 3x \geq 5$ $\quad -(2-3x) \geq 5$

$-3x \geq 3$ $\quad -2 + 3x \geq 5$

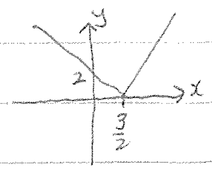
$x \leq -1$ $\quad 3x \geq 7$

$x \geq \dfrac{7}{3}$

ie $x \leq -1$ or $x \geq \dfrac{7}{3}$ (A)

Two steps: the positive and the negative.

Check by sketching $y = |2-3x|$



5. $f(x) = 2x^3 - x$

If even $f(-x) = f(x)$

$f(-x) = 2(-x)^3 - (-x)$

$= -2x^3 + x$

$= -(2x^3 - x)$

$= -f(x)$

$\therefore$ not even

Odd if $f(-x) = -f(x)$

$\therefore$ is odd (A)

Substitute "$-x$" for x and test.

6. $2\sin\beta = -\sqrt{3}$ $\quad 0^\circ \leq \beta \leq 360^\circ$

$\sin\beta = \dfrac{-\sqrt{3}}{2}$

S A
T✓ ✓C

Related angle $\sin\beta = \dfrac{\sqrt{3}}{2}$ $\quad \beta = 60^\circ$

$\therefore \beta = 180 + 60,\ 360 - 60$

$= 240^\circ, 300^\circ$ (D)

Determine related angle (first quadrant θ, ratio is positive and check/evaluate for the correct quadrant

7. $x^2 + 2x - 7 = 0$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4.1.(-7)}}{2}$

$= \dfrac{-2 \pm \sqrt{32}}{2}$ $\quad \sqrt{32} = \sqrt{16 \times 2}$

$= -1 \pm 2\sqrt{2}$ (D)

Quadratic formula or complete the square

9. $x^2+8x+16=0$

$\Delta = b^2-4ac$

$= 64-4\times 16$

$= 0$

$\therefore$ the quadratic has equal roots (B)

Consider the quadratic formula

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

If $\Delta < 0$ then <u>no</u> real roots

If $\Delta > 0$

* <u>and</u> a perfect square, then two rational roots
* <u>and</u> not a perfect square then two real (irrational) roots.

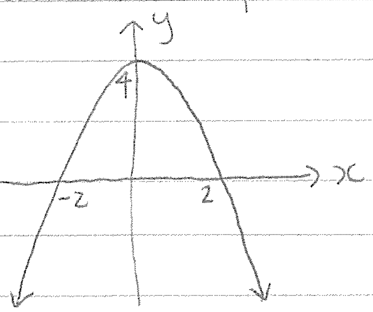
QUESTION 12

a) Sorry - no ruler here!

i) $y = 4-x^2$

$= (2-x)(2+x)$

Concave down parabola



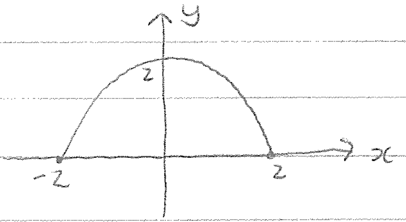
ii) $y=\sqrt{4-x^2}$

[Aside: square both sides

$y^2 = 4-x^2$

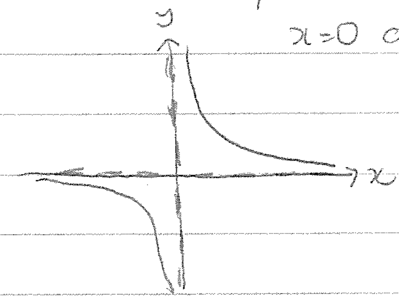
ie $x^2+y^2=4$ circle]

Semicircle $y \geq 0$, $-2 \leq x \leq 2$



iii) $y=\dfrac{4}{x}$ hyperbola

asymptotes of $x=0$ and $y=0$



d) $\displaystyle\lim_{x\to 4} \frac{x^2-x-12}{x-4}$

$= \displaystyle\lim_{x\to 4} \frac{(x-4)(x+3)}{x-4}$

$= \displaystyle\lim_{x\to 4} x+3$

$= 4+3$

$= 7$

Do not get rid of the lim notation until there is no longer a variable

QUESTION 11

a) let $x = 0.1\dot{5}$

$= 0.155555\ldots$

$100x = 15.555555\ldots$

$99x = 15.4$

$x = \frac{15.4}{99}$

$= \frac{154}{990}$

Multiply by sufficient power of 10 to eliminate the tail by subtraction. Do not leave a decimal in the answer

b) $^{6\times}\frac{1}{2}(y-3) - ^{6\times}\frac{1}{3}(y-2) = 3^{\times 6}$

$\times 6$: $3(y-3) - 2(y-2) = 18$

$3y - 9 - 2y + 4 = 18$

$y - 5 = 18$

$y = 23$

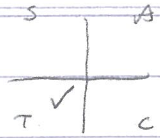
Check by substituting answer back in - use your calculator.

c) $\frac{a^2+b^2+2ab}{-b^2+a^2}$

$= \frac{(a+b)^2}{(a-b)(a+b)}$

$= \frac{a+b}{a-b}$

d) i) $\tan 240$



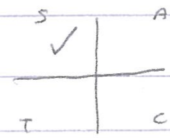
$= \tan(180+60)$

$= \tan 60$ (related angle)

$= \sqrt{3}$

Check with calculator!

ii) $\sec 510$



$= \frac{1}{\cos(360+150)}$

$= \frac{1}{\cos(180-30)}$

$= \frac{1}{-\cos 30}$

$= \frac{1}{-\sqrt{3}/2}$

$= \frac{-2}{\sqrt{3}}$

QUESTION 14

b) RTP $\tan\theta - \tan\theta \sin^2\theta = \sin\theta\cos\theta$

LHS $= \tan\theta - \tan\theta\sin^2\theta$

$= \tan\theta(1-\sin^2\theta)$

but $\cos^2\theta + \sin^2\theta = 1$ Pythagoras

$= \frac{\sin\theta}{\cos\theta} \cdot \cos^2\theta$

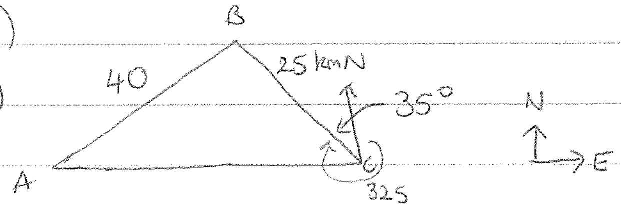
$= \sin\theta \cdot \cos\theta$

$=$ RHS as required

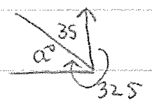
QUESTION 14

a)

i)

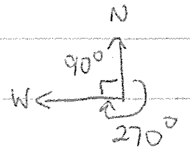


At C:

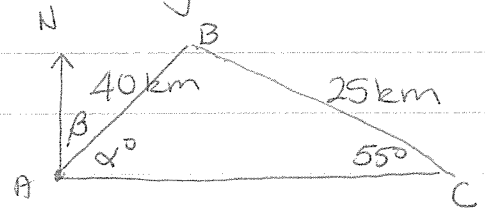


$a^\circ = 325^\circ - 270^\circ$

$= 55^\circ$



ii) bearing B from A



Using the sine rule:

$$\frac{\sin\alpha}{25} = \frac{\sin 55}{40}$$

$$\sin\alpha = \frac{25\sin 55}{40}$$

$= 0.511970027$ (calc)

$\alpha = 30^\circ\ 47'\ 42.51$

$\therefore \beta = 90 - \alpha$

$= 59^\circ\ 12'\ 17.49''$

$\therefore$ the bearing of B from A is 059°T.

c) $\alpha = 1 + \sqrt{3}$ $\quad \beta = 1 - \sqrt{3}$

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$\alpha + \beta = 1 + \sqrt{3} + 1 - \sqrt{3}$

$= 2$

$\alpha\beta = (1+\sqrt{3})(1-\sqrt{3})$

$= 1 - \sqrt{3} + \sqrt{3} - 3$

$= -2$

$\therefore$ The monic quadratic is

$x^2 - 2x - 2 = 0$

d) $y^2 = x + 9$ $\quad \ldots (1)$

$x - 3y + 9 = 0$ $\quad \ldots (2)$

(1) $\Rightarrow$ $x = y^2 - 9$

sub into (2)

$y^2 - 9 - 3y + 9 = 0$

$y^2 - 3y - 18 = 0$

$(y-6)(y+3) = 0$

$\therefore y = -3, 6$

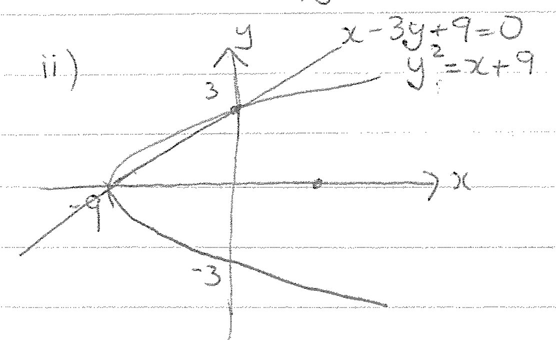
sub into (2)

When $y = -3$ $\qquad\qquad y = 6$

$x + 9 + 9 = 0$ $\qquad x - 18 + 9 = 0$

$x = -18$ $\qquad\qquad x = 9$

ii)



(Curves intersect at (−9, 0) and (0, 3))

c) $2x+y-1=0 \quad \ldots ①$

$3x-y-4=0 \quad \ldots ②$

Show what you are doing

$② + ① \quad 5x-5=0$

$5x=5$

$x=1$

sub into ①

$2+y-1=0$

$\therefore y=-1$

$\therefore x=1, y=-1$

Check by substitution.

d) i) $\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{d}{dx}\left(x^{-3}\right)$

$= -3x^{-4}$

$= \frac{-3}{x^4}$

Change to index form, return to orginal form
Subtract 1 from the power!
Becareful of negatives

ii) $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}\left(x^{1/2}\right)$

$= \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

e) $y = x^3 - 4x$

$\frac{dy}{dx} = 3x^2 - 4$

At $(1,-3)$ the gradient of the tangent is

$m = 3(1)^2 - 4$

$= -1$

Equation of the tangent

$y-(-3) = -1(x-1)$

$y+3 = -x+1$

$x+y+2=0$

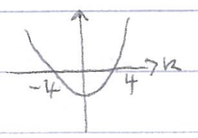
Remember point-gradient form of a line.
Answer here is in general form.

f) $x^2 - kx + 4 = 0$

No real roots $\Rightarrow \Delta < 0$

$\therefore (-k)^2 - 4\times1\times4 < 0$

$k^2 - 16 < 0$



$(k+4)(k-4) < 0$

$\therefore -4 < k < 4$