Centre Number

Student Number

SCEGGS Darlinghurst

2006

Preliminary Course
Semester 2 Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Write your Centre Number and Student Number at the top of each page
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used
- **Start each question on a new page**
- **Do not** attach all question together in one bundle

Total marks – 76

- Attempt Questions 1–6

Question	Comm	Calc	Reason	Total
1	/2		/2	/13
2				/12
3	/1	/4	/6	/13
4	/2		/2	/14
5		/3	/6	/12
6	/6	/2		/11
TOTAL	/11	/9	/16	/76

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Total marks – 76
Attempt Question 1 – 6

Attempt **all** questions on the pad paper provided
Write your Centre Number and Student Number at the top of each page
Show all necessary working
Marks may be deducted for careless or badly arranged work
Mathematical templates, geometrical equipment and scientific calculators may be used
Start each question on a new page

Marks

Question 1 (13 marks)

(a) Evaluate correct to 2 decimal places. **1**

$$\frac{3.65 - 2.3^2}{6.4 - 8.39}$$

(b) Simplify fully:

$$\sqrt{72} + 2\sqrt{18}$$ **2**

(c) Solve: $3x^2 - 15 = 0$ **2**

(d) Express with a rational denominator in simplest form. **2**

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

(e) Solve: $27^{2x} = 9$ **2**

(f) Factorise fully: $8x^3 + 125$ **2**

(g) Solve $|x+5| < 3$ and graph your solution on a number line **2**

Start a new page

Marks

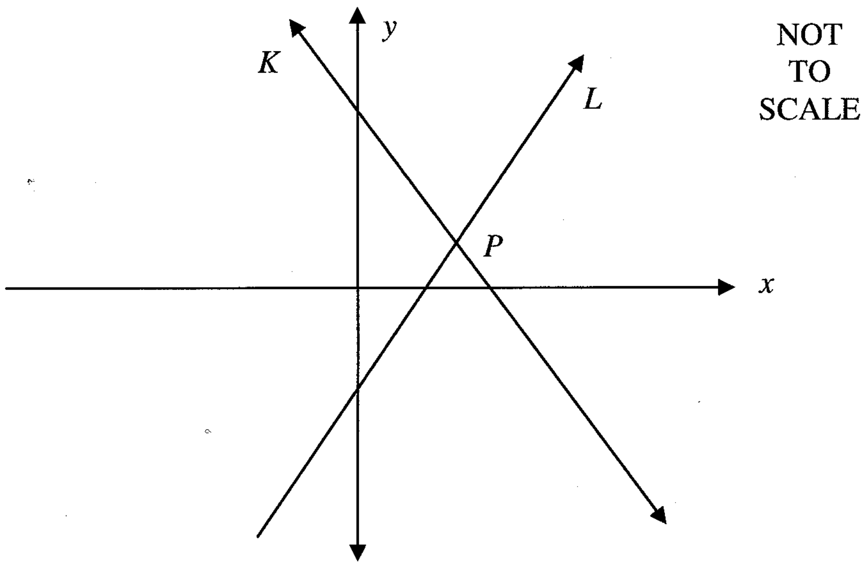
Question 2 (12 marks)

(a) Given $f(x) = x - \dfrac{1}{x}$

(i) find $f(-2)$ and $f\left(\dfrac{1}{2}\right)$ **2**

(ii) if $f(a) = 0$ find a **1**

(b)



(i) Two straight lines L and K with equations $3x - 2y = 4$ and $y = -2x + 5$ respectively, meet at the point P as shown in the diagram above. Find the co-ordinates of P. **2**

(ii) Write down the gradient and y-intercept of the line L. **2**

(iii) Calculate the area of the triangle formed by the lines L, K and $x = 0$. **1**

(iv) Find the equation of the line J which is parallel to L and which passes through the point $(0, 5)$. **2**

(v) Calculate the exact perpendicular distance between the parallel lines L and J. **2**

Start a new page

Marks

Question 3 (13 marks)

(a) The interior angle of a regular polygon is $170°$. Find the number of sides. **2**

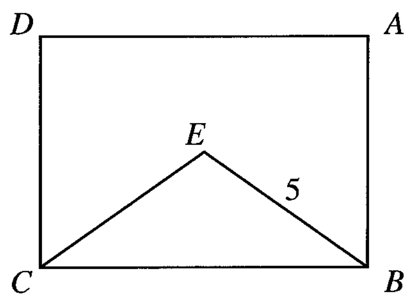
(b) Simplify $(\tan\theta + \sec\theta)(\sec\theta - \tan\theta)$ **2**

(c) Differentiate the following:

(i) $y = 3x - \dfrac{5}{x^2}$ **2**

(ii) $y = \dfrac{x^3 - 1}{x + 1}$ **2**

(d) In the diagram below *ABCD* is a square and *EC* and *EB* are the bisectors of $\angle DCB$ and $\angle ABC$ respectively.



NOT TO SCALE

(i) Copy the diagram onto your answer sheet and show all given information. **1**

(ii) Prove that ΔBCE is isosceles. **2**

(iii) Hence find the exact length of *BC*. **2**

Start a new page

Marks

Question 4 (14 marks)

(a) Find the quadratic equation whose roots are 1 and –4. **1**

(b) If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$, find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ **2**

(c) Given $\sin\theta = \frac{3}{5}$ and $\tan\theta < 0$, find $\sec\theta$. **2**

(d) Consider the function $y = \frac{2}{x+1}$

(i) State the Domain and Range of the function. **2**

(ii) Is this function continuous? Clearly explain your answer. **1**

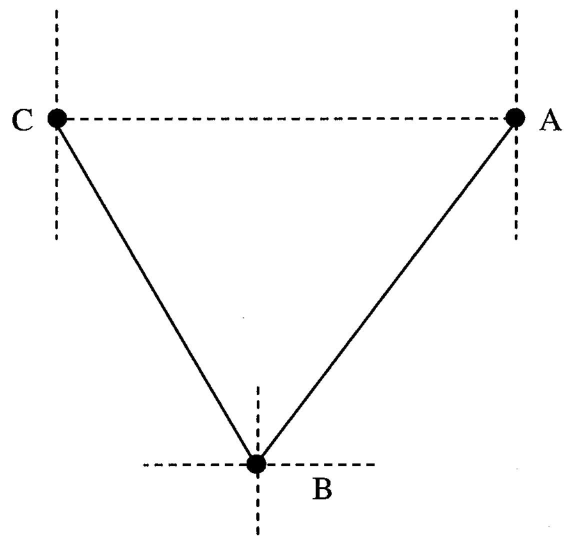
Question 4 continues on the next page

Marks

Question 4 (continued)

(e) A yacht sails from a port A on a course bearing of $212°$ for 2.8 nautical miles to B. It then turns and sails on a course bearing of $330°$ to a point C, due west of A.

(i) Copy the diagram below onto your answer sheet and show all given information. **1**



(ii) Find $\angle ACB$. **1**

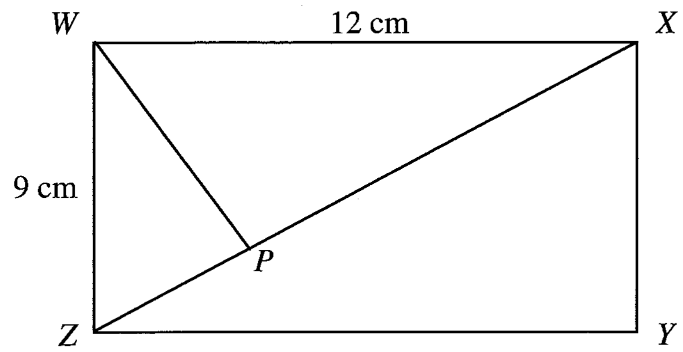
(iii) Hence, find to the nearest tenth of a nautical mile, the distance BC. **2**

Start a new page

Marks

Question 5 (12 marks)

(a)



Not to scale

In the diagram above, $WXYZ$ is a rectangle with $WX = 12$ cm and $WZ = 9$ cm. WP is perpendicular to XZ.

Copy the diagram onto your answer sheet.

(i) Find the length of XZ. **1**

(ii) Prove that ΔWXP is similar to ΔZXW. **3**

(iii) Hence find the length of XP, give reason(s). **2**

(b) Solve $4\cos^2\theta = 3$ if $0° \le \theta \le 360°$. **3**

(c) Find the equation of the normal to the curve $y = (2x-5)^4$ at $x = 2$ in general form. **3**

Start a new page

Marks

Question 6 (11 marks)

(a) Find the values of p for which the expression $3x^2 + px + 3$ is positive definite. **3**

(b) On a number plane shade the region(s) which represent the simultaneous solution of: **4**

$$y < x^2 \quad \text{and} \quad y \geq |x - 1|$$

(c) (i) Explain why the gradient of $y = \frac{1}{x}$ is always negative. **2**

(ii) Find the gradient of l, the tangent of $y = \frac{1}{x}$ at $x = \frac{1}{2}$. **1**

(iii) What angle to the nearest degree, does the line l make with the positive direction of the x-axis. **1**

End of paper

Yr 11 Preliminary Mathematics Semester 2 Exam 2006

Question 1 (13 Marks) MF

a) $0 \cdot 82$ ✓

Put a bracket around the denominator before dividing if you got this one wrong.

b) $6\sqrt{2} + 6\sqrt{2} = 12\sqrt{2}$ ✓ ✓

Note $2\sqrt{18} = 2 \times 3\sqrt{2} = 6\sqrt{2}$
↑ multiply don't add!

c)
$$3x^2 = 15$$
$$x^2 = 5 \quad ✓$$
$$x = \pm\sqrt{5} \quad ✓$$

If you solve this way using difference of 2 squares you get the same answers.

$$3x^2 - 15 = 0$$
$$3(x^2 - 5) = 0$$
$$3(x - \sqrt{5})(x + \sqrt{5}) = 0$$
$$x = \sqrt{5}, \quad x = -\sqrt{5}$$

↑ Note: This 3 is only a numerical factor it does not give a solution of 0.

d)
$$\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= \frac{3+\sqrt{3}+\sqrt{3}+1}{3-1}$$
$$= \frac{4+2\sqrt{3}}{2} \quad ✓$$
$$= 2+\sqrt{3} \quad ✓$$

An easy question!!
This part was not very well done.
More practice at expanding binomial products is recommended if you got this one wrong.
← Simplest form required.

e)
$$(3^3)^{2x} = 3^2$$
$$3^{6x} = 3^2 \quad ✓$$
$$6x = 2$$
$$x = \tfrac{1}{3} \quad ✓$$

(R 2)

⊛ Poorly done. Index Rules are often used to solve exponential equations.

f) $(2x+5)(4x^2 - 10x + 25)$ ✓ ✓

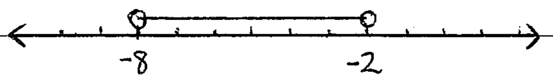
Learn the rule $a^3+b^3 = (a+b)(a^2-ab+b^2)$ and how to use it.

g)
$x+5 < 3$ $\qquad$ $-(x+5) < 3$

$x < -2$ $\qquad$ $x+5 > -3$ $\qquad$ ✓ both solutions

$\qquad\qquad$ $x > -8$

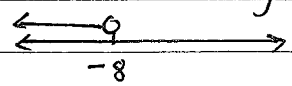
- First mark awarded for solving two cases correctly.



✓

(C 2)

If you get the solution $x < -8$, $x < -2$ your graph should show only $x < -8$.



Question 2 (12 Marks) AV

a i) $f(-2) = -2 - \frac{1}{-2}$

$= -1\frac{1}{2}$ ✓

$f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{\frac{1}{2}}$

$= \frac{1}{2} - 2$

$= -1\frac{1}{2}$ ✓

ii) $0 = a - \frac{1}{a}$

$\frac{1}{a} = a$

$1 = a^2$

$a = \pm 1$ ✓

b i) $L: 3x - 2y = 4 \quad K: y = -2x + 5$

$P: 3x - 2(-2x + 5) = 4$

$3x + 4x - 10 = 4$

$7x = 14$

$x = 2$ ✓

From K

$y = -2 \times 2 + 5$

$= 1$

$\therefore P(2, 1)$ ✓

ii) $3x - 2y = 4$

$-2y = 4 - 3x$

$y = -2 + \frac{3x}{2}$

$m = \frac{3}{2}$ ✓ $b = -2$ ✓

iii) $A = \frac{1}{2} \times 7 \times 2$

$= 7u^2$ ✓

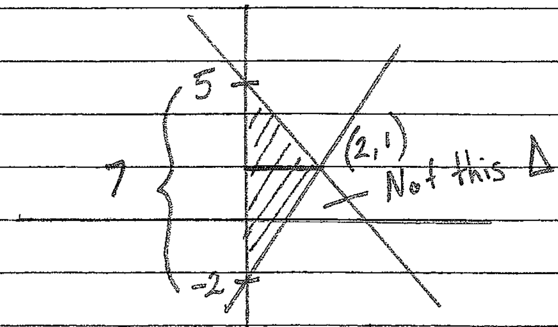
Generally well done, be careful with your negatives

* If you don't know $\frac{1}{2} - 2 = -1\frac{1}{2}$ then use your calculator!!

Not well done
$f(a) = 0$ does not mean substitute 0 for a

Generally well done

ii



Poorly done – Most did not read the question carefully and tried to find the area of the wrong Δ

* If a question is worth 1 mark it is not going to involve a page of working

iv) parallel to L $\therefore m = \frac{3}{2}$

through (0, 5) ✓

Eqn J : $y - 5 = \frac{3}{2}(x - 0)$

$2y - 10 = 3x$ ✓

$3x - 2y + 10 = 0$

Not perpendicular – read the question

Generally well done

v) L: $3x - 2y - 4 = 0$

through (0, 5)

$d = \frac{|3\times 0 + -2\times 5 + -4|}{\sqrt{3^2 + (-2)^2}}$ ✓

$d = \frac{14}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$

$d = \frac{14\sqrt{13}}{13}$ ✓

OR

J: $3x - 2y + 10 = 0$

through P(2, 1)

$d = \frac{|3\times 2 + -2\times 1 + 10|}{\sqrt{3^2 + (-2)^2}}$

$= \frac{|14|}{\sqrt{13}}$

OK.

Make sure you choose a point that is not on the line

- quite a few did this and ended up with a distance of 0
- if this happens check what line and point you have chosen.

Question 3 (13 Marks) TS

a) Ext ∠ = $10°$ ✓

$n = 360 \div 10$

$n = 36$ ✓

or $180(n-2) = 170n$ ✓

⋮

$n = 36$ ✓

Note: $170°$ is NOT the whole ∠ sum!

b) $\sec^2\theta - \tan^2\theta$

$= 1 + \tan^2\theta - \tan^2\theta$ ✓

$= 1$ ✓ (R 2)

c) i) $y = 3x - 5x^{-2}$

$y' = 3 + 10x^{-3}$ ✓✓

$\frac{5}{x^2}$ does NOT need the quotient rule to be differentiated

ii) $u = x^3 - 1 \qquad v = x + 1$

$u' = 3x^2 \qquad v' = 1$

(Ca 4)

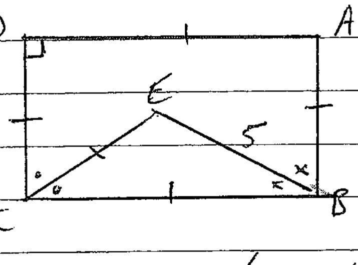
$y' = \frac{3x^2(x+1) - (x^3-1)}{(x+1)^2}$ ✓

$= \frac{3x^3 + 3x^2 - x^3 + 1}{(x+1)^2}$

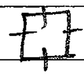
$= \frac{2x^3 + 3x^2 + 1}{(x+1)^2}$ ✓

Lots of people had the correct answer & then ruined it by bad algebra!

d) i)



✓ must show equal angles and square (C 1)



is NOT a square! It is only a rhombus.

ii) $\angle ABE = \angle CBE$ (EB bisects $\angle ABC$)

$= 45°$ ($\angle ABC = 90°$, vertex of a square)

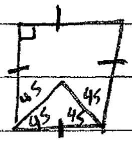
$\angle DCE = \angle BCE$ (EC bisects $\angle DCB$)

$= 45°$ ($\angle DCB = 90°$, vertex of a square)

$\therefore \angle ECB = \angle EBC$ ✓

$\therefore \triangle BCE$ is isosceles (2 equal angles)

✓ This was very badly done. And when you are told to mark the information given that is exactly what you need to do — do not deduce anything (R 2)



Is not correct!

ii) $\angle CEB = 90^\circ$ ($\angle$ sum $\Delta CEB = 180^\circ$) ✓

$CE = EB = 5$ (sides opp = $\angle$s of isos Δ =)

$BC^2 = 5^2 + 5^2$ (R 2)

$BC^2 = 50$

$BC = 5\sqrt{2}$ ✓

Must given reasons here
- why is $\angle CEB = 90^\circ$?
- why does $CE = EB = 5$.

It is NOT given!

Question 4 (14 Marks) SL

a) $(x-1)(x+4) = 0$

$x^2 - x + 4x - 4 = 0$

$x^2 + 3x - 4 = 0$ ✓

Please remember a quadratic equation EQUALS ZERO !!!

b i) $\alpha + \beta = 5$ ✓

ii) $\alpha\beta = 2$ ✓

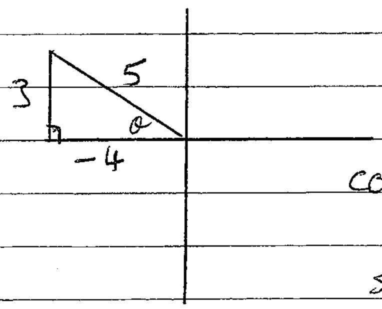
iii) $\dfrac{\beta^2 + \alpha^2}{(\alpha\beta)^2} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ ✓

$= \dfrac{5^2 - 2\times 2}{4}$

$= \dfrac{21}{4}$ ✓

$\alpha^2 + \beta^2 \neq (\alpha+\beta)^2$

c) sin +ve, tan -ve in Q2



$\cos\theta = \dfrac{-4}{5}$ ✓

$\sec\theta = \dfrac{5}{-4}$ ✓

(R 2)

Be careful with quadrants

2nd quadrant means $\cos\theta$ is negative

$\therefore \sec\theta$ is negative

d) i) D: all real x, $x \neq -1$ ✓

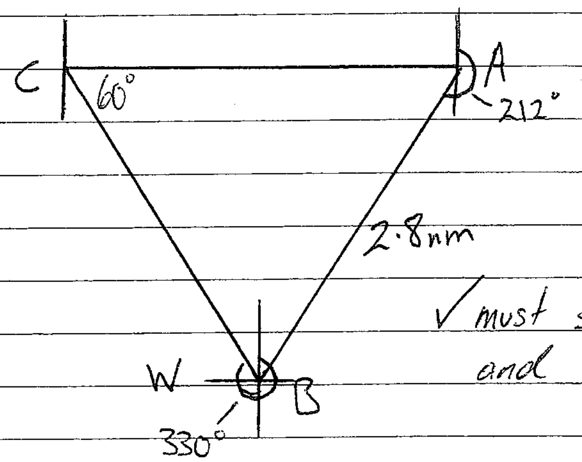
R: all real y, $y \neq 0$ ✓

You must say all real x & real y.

ii) Not continuous as there is a vertical asymptote at $x = -1$ ✓ (C 1)

Need to give mathematical reasons

e) i)



(C 1)

✓ must show bearing at A, B and length of AB

Done well ✓✓

ii) $\angle CBW = 330° - 270°$

$= 60°$

$\angle ACB = \angle CBW$ (alt. ∠s = on ∥ lines)

$= 60°$ ✓

iii) $\angle CAB = 270 - 212$

$= 58°$ ✓

$$\frac{BC}{\sin 58°} = \frac{2 \cdot 8}{\sin 60°}$$

$BC = 2 \cdot 7\,nm$ ✓

Question 5 (12 Marks) MF

a) i)

$XZ^2 = 12^2 + 9^2$ (By Pythagoras)

$XZ^2 = 225$

$XZ = 15$ ✓

Note that ΔXWZ is right-angled so Pythagoras is the most efficient method. Using Trig. is more difficult especially for only one mark.

ii)

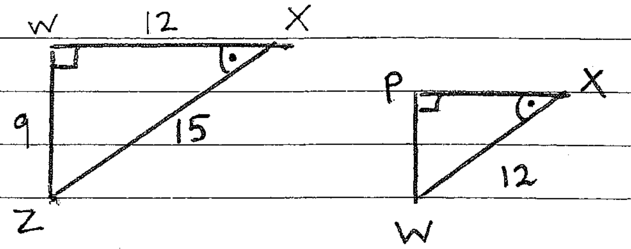
$\angle WXP$ is common ✓

$\angle ZWX = 90°$ (vertex of a rectangle) ✓

$\angle WPX = 90°$ ($WP \perp XZ$)

$\therefore \Delta WXP \parallel\!\mid \Delta ZXW$ (equiangular) ✓

R 3



Draw the triangles carefully in the same orientation. Then it's easy!

iii)

$\frac{XP}{12} = \frac{12}{15}$ (corr. sides of sim Δs in equal ratio) ✓

$XP = \frac{144}{15}$

$XP = 9.6$ cm ✓

- Make sure you use correct sides!
- You must put a reason for the second mark.

b)

$4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ ✓

R 3

$\therefore$ soln is Q 1, 2, 3, 4

acute $\angle = 30°$

$\therefore \theta = 30°, 150°, 210°, 330°$ ✓✓ (all 4 solutions)

- First mark for knowing there are two cases $\pm$
- Second mark for solving correctly $\theta = 30°$ & $330°$.

(*) This is a standard question. It was poorly done.

c)

$y = (2x-5)^4$

$y' = 8(2x-5)^3$ ✓ at $x = 2$ $y = 1$

$m_T = 8(4-5)^3$

$m_T = -8$ $\therefore m_N = \frac{1}{8}$ ✓

Ca 3

Eqn: $y - 1 = \frac{1}{8}(x-2)$

$8y - 8 = x - 2$

$x - 8y + 6 = 0$ ✓ general form

Note $(-1)^3 = -1$ many mistakes made with this negative sign.

This is an easy calculus question. Practise more of the same type if you got it wrong.

Question 6 (11 marks) AV

a) pos def. when $a > 0$

$\Delta < 0$

$a = 3 \therefore a > 0$

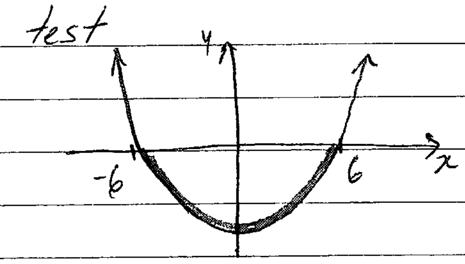
$\Delta < 0$

$p^2 - 4 \times 3 \times 3 < 0$

$p^2 - 36 < 0$

$p^2 < 36$ ✓

test



✓

$\therefore -6 < p < 6$ ✓

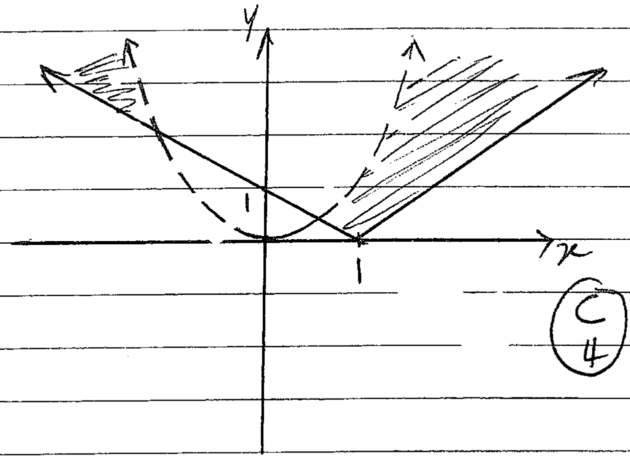
Not well done at all

most new to solve for $\Delta < 0$ and got down to

$p^2 - 36 < 0$

To solve from here you MUST sketch the quadratic inequality

b)



(C 4)

* If you can't sketch get a Mathaid!!! ...

✓ $y = x^2$
✓ $y = |x-1|$ } lines
✓✓ both regions

most could sketch $y = x^2$; $y = |x-1|$ was not well done

Make sure you sketch to a reasonable scale so that the two regions can be easily shown.

* label axes & x & y intercepts.

c i) $y = x^{-1}$

$y' = -x^{-2}$

$= -\frac{1}{x^2}$ ✓

y' represents the gradient function of y, since x^2 must be positive $-\frac{1}{x^2}$ must therefore be negative ✓

(C 2)

(OR sketch showing gradient is always negative)

Extremely poorly done

you must explain why a gradient function of $-\frac{1}{x^2}$ is always negative (ie x^2 is always +ve $\therefore -\frac{1}{x^2}$ is always -ve)

If you did a clearly detailed diagram showing that the gradient is -ve at any point by giving examples this was also acceptable.

ii) m_t at $x = \frac{1}{2}$

$y' = -\frac{1}{x^2}$

$m_t = -\frac{1}{(\frac{1}{2})^2}$

$= -1 \div \frac{1}{4}$

$= -4$ ✓

Ca 1

Ok, not great.

you must differentiate then substitute $x = \frac{1}{2}$

iii) $\tan\theta = m$

$\tan\theta = -4$

tan -ve in Q2

$\theta = 180 - 76°$

$\theta = 104°$ ✓

Ca 1

Not too bad
Remember if the gradient is negative the angle will be obtuse.