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Centre Number

Student Number

SCEGGS Darlinghurst

2007

Preliminary Course
Semester 2 Examination

Mathematics

Outcomes Assessed: P2 – P8
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt **all** questions and show all necessary working
- Answer all questions on the pad paper provided
- Write your Student Number at the top of each page
- **Begin each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 84

- Attempt Questions 1 – 7

Question	Reasoning	Comm	Calc	Marks
1	/2			/12
2	/6		/2	/12
3	/3	/1		/12
4	/3	/1	/6	/12
5	/4	/4		/12
6	/2	/6	/2	/12
7	/4	/3	/3	/12
TOTAL	**/24**	**/15**	**/13**	**/84**

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Marks

Question 1 (12 marks)

(a) Find the value of $\sqrt{\dfrac{15.26+31.98}{(4.75)^3\times 3.5}}$ correct to 3 significant figures. **2**

(b) Express $0.6\dot{4}\dot{3}$ in simplest fraction form **2**

(c) Find the values of a and b if: **2**

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} = a + b\sqrt{5}$$

(d) Simplify $\dfrac{x^2-144}{x^3+8} \div \dfrac{x+12}{x+2}$. **2**

(e) Find the exact value of $\cos 210°$. **2**

(f) Solve $|x-2|<3$ **2**

- Start a new page

Marks

Question 2 (12 marks)

(a) (i) Find the gradient of the curve $y = 6 + 4x - 2x^2$ at the point $P(1, 8)$. **2**

(ii) What is the maximum value of the parabola $y = 6 + 4x - 2x^2$. **1**

(b) A function is defined as:

$$f(x) = \begin{cases} 4 - x & \text{for} \quad x < -2 \\ 5 & \text{for} \quad -2 \le x < 0 \\ x^3 + 1 & \text{for} \quad x \ge 0 \end{cases}$$

Calculate the value of:

(i) $f(-10)$ **1**

(ii) $f(-2)$ **1**

(iii) $f(m^2)$ **1**

(c) If $\cos\theta = -\dfrac{2}{3}$ and $\tan\theta > 0$ find the exact value of $\operatorname{cosec}\theta$. **3**

(d) Find the values of k for which the equation $x^2 - (k+3)x + 4k = 0$ has:

(i) equal roots. **2**

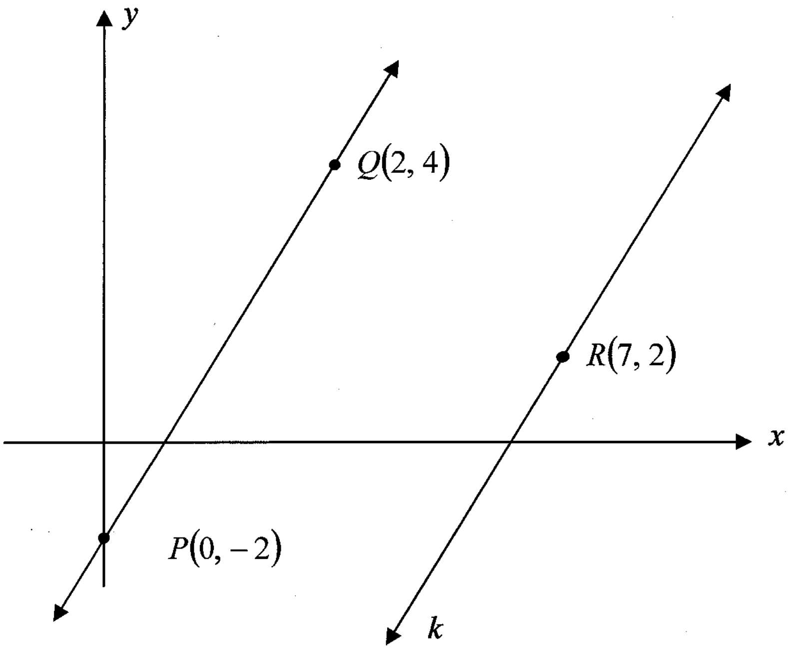
(ii) roots which are equal in magnitude but opposite in sign. **1**

- Start a new page

Marks

Question 3 (12 Marks)

(a)



NOT TO SCALE

In the diagram $P(0,-2)$, $Q(2,4)$ and $R(7,2)$ are points on the number plane.

Copy or trace the diagram onto your worksheet.

(i) Find the length of *PQ*. **1**

(ii) Find the equation of the line through *PQ*. **1**

(iii) Find the equation of the line *k* passing through *R* parallel to *PQ*. **2**

(iv) The point *S* lies on *k* such that *QR* is parallel to *PS*. **1**
What type of quadrilateral is *PQRS*. (Give reasons.)

(v) Find the co-ordinates of *S*. **2**

(vi) Find the perpendicular distance from *R* to *PQ*. **2**

(vii) Find the area of *PQRS*. **1**

(b) Solve $\sqrt{3}\tan\theta + 1 = 0$ for $0° \leq \theta \leq 360°$. **2**

- Start a new page

Marks

Question 4 (12 Marks)

(a) Differentiate the following:

(i) $y = 5x^3 - \dfrac{1}{3x^2}$ **2**

(ii) $y = x(4x+3)^5$ **2**

(iii) $y = \dfrac{6x-1}{4-2x}$ **2**

(b) Holly and Ellen go bushwalking. They leave their campsite and set out on different directions. Holly walks due East for 2.5km while Ellen walks 1.7km on a bearing of $250°T$

(i) Draw a diagram showing the above information. **1**

(ii) Calculate the distance between Holly and Ellen. **2**

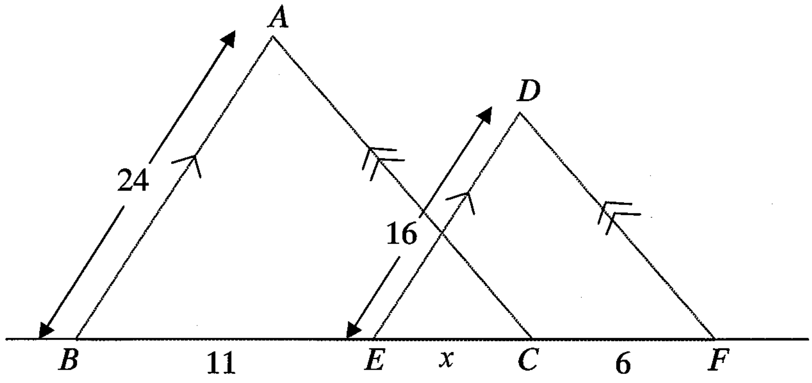
(iii) Calculate the bearing of Holly from Ellen. **3**

- Start a new page

Marks

Question 5 (12 marks)

(a)



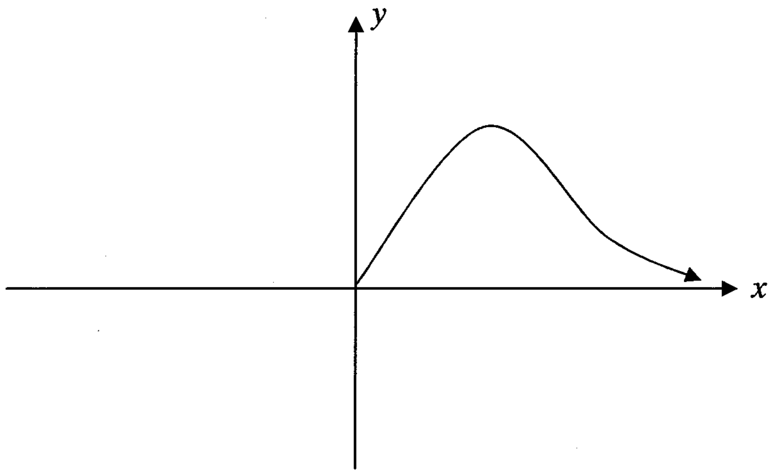
NOT TO SCALE

In the diagram $AB \parallel DE$ and $AC \parallel DF$. $AB = 24$, $DE = 16$, $BE = 11$ and $CF = 6$.

(i) Prove $\Delta ABC \parallel\!| \Delta DEF$ **3**

(ii) Find EC. **2**

(b) The diagram below shows part of the graph of $y = f(x)$. **1**
You are told that $f(x)$ is an odd function.



Copy the diagram and complete the graph of the function.

Question 5 continues on the next page

Marks

Question 5 (continued)

(c) If α and β are the roots of the equation $x^2 - 2x + 4 = 0$ find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) $\left(\alpha - \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ **2**

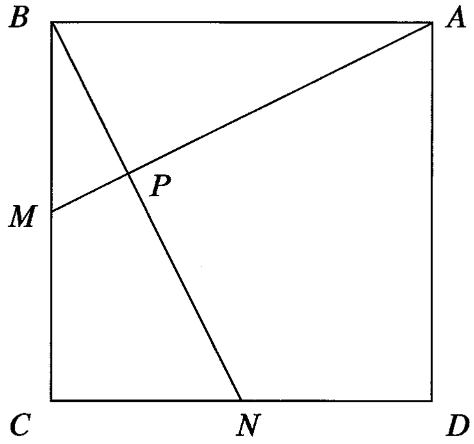
End of Question 5

- Start a new page

Marks

Question 6 (12 marks)

(a) *ABCD* is a square. *M* and *N* are the midpoints of *BC* and *CD*. *BN* and *AM* intersect at *P*.



Copy or trace the diagram onto your paper showing all given information.

(i) Prove $\Delta ABM \equiv \Delta BCN$. **3**

(ii) Prove that *AM* and *BN* are perpendicular. **2**

(b) Given the function $y = x^3 - x^2$. Find the value(s) of *x* for which $\frac{dy}{dx} = 8$. **2**

(c) (i) Graph $y = |\, x \,|$ and $y = 2x + 1$ on the same number plane labelling all important features. **2**

(ii) Using your graph, or otherwise explain why the equation $2x + 1 = |\, x \,|$ has only one solution. **1**

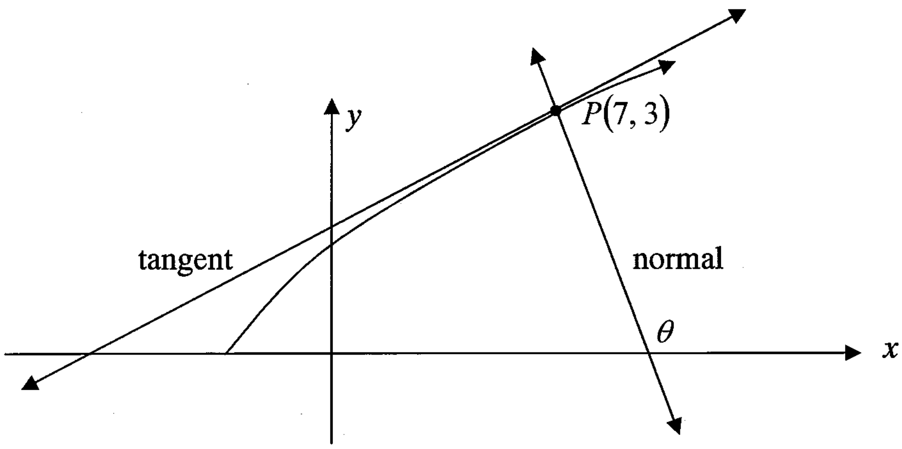
(iii) Solve $2x + 1 = |\, x \,|$. **2**

- Start a new page

Marks

Question 7 (12 marks)

(a) The diagram below is the graph of $y = \sqrt{x+2}$. The tangent and the normal at the point $P(7, 3)$ has also been drawn



(i) Find the equation of the normal at P. **3**

(ii) Calculate θ, the angle the normal makes with the position x-axis. **1**

(b) (i) On the same number plane graph $y = \sqrt{16 - x^2}$ and $2x + y = 4$. **2**

(ii) Shade the region on your diagram that satisfies the following: **1**

$$y \le \sqrt{16 - x^2}\text{, } 2x + y \ge 4 \text{ and } y \ge 0$$

(iii) Calculate the area of the shaded region. **1**

(c) (i) Show that $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$. **1**

(ii) Hence, or otherwise, solve **3**

$$\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta} = -1 \quad 0° \le \theta \le 360°$$

End of paper

Y11 MATHEMATICS SEMESTER 2 EXAMINATION – SOLUTIONS

Q1 a) 0.355 ✓ (1 for correct rounding)

- done well – some students need to revise significant figures

b) let $x = 0.64343....$ Reason – 2

$10x = 6.4343....$ ① ✓

$1000x = 643.4343....$ ②

② − ①

$990x = 637$

$x = \frac{637}{990}$ ✓

- Most students new the process but some got confused as to what quantity to multiply the x by.

c) $\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$ ✓

$= \frac{5+\sqrt{5}+\sqrt{5}+1}{5-1}$

$= \frac{6+2\sqrt{5}}{4}$

$= \frac{3+\sqrt{5}}{2}$

$\therefore a = \frac{3}{2} \quad b = \frac{1}{2}$ ✓

- most students knew to rationalise the denominator but some were confused with expressing the final answer as $a = \frac{3}{2}$ $b = \frac{1}{2}$

d) $\frac{x^2-144}{x^3+8} \div \frac{x+12}{x+2}$

$= \frac{\cancel{(x+12)}(x-12)}{\cancel{(x+2)}(x^2-2x+4)}$ ✓ $\times \frac{\cancel{x+2}}{\cancel{x+12}}$

$= \frac{x-12}{x^2-2x+4}$ ✓

- done fairly well

e) $\cos 210° = \cos(180°+30°)$ ✓

$= -\cos 30°$

$= -\frac{\sqrt{3}}{2}$ ✓

- students need to learn to express angles as either
$180-\theta$
$180+\theta$
$360-\theta$
this area needs review

f) $|x-2|<3$

$-3<x-2<3$ ✓

$-1<x<5$ ✓

- a number of students treated this as two separate inequalities you should state it as in the solutions

Q2 a) i) $y = 6 + 4x - 2x^2$

$\frac{dy}{dx} = 4 - 4x$ ✓

when $x = 1$

$\frac{dy}{dx} = 4 - 4 \times 1$ Calculus - 2

$= 0$ ✓

$\therefore$ gradient of the curve at $P(1, 8)$ is 0.

Some students didn't realise the y had to differentiate.

ii) Since the gradient of the curve at P is 0 then P is the vertex.

$\therefore$ Max value $= 8$ ✓ Reason - 1

Most students wrote the vertex max value is the y value only.

b) i) $f(-10) = 4 - (-10)$

$= 14$ ✓

ii) $f(-2) = 5$ ✓

iii) $f(m^2) = (m^2)^3 + 1$

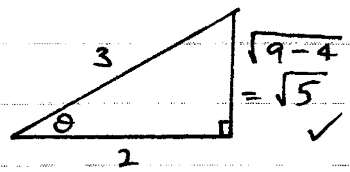
$= m^6 + 1$ ✓ Reason - 1

Well done (i) and (ii)

iii) Students identified the correct function but failed to sub m^2 in correctly

Index laws

c) $\cos\theta = -\frac{2}{3}$



$\cos\theta < 0 \quad \tan\theta > 0$

$\therefore \theta$ is in the 3rd quadrant

$\therefore \sin\theta < 0$ ✓

$\sin\theta = -\frac{\sqrt{5}}{3}$

$\therefore \csc\theta = -\frac{3}{\sqrt{5}}$ ✓ Reason - 3

On a whole well done

Students lost 1 mark for not identifying cosec θ as neg.

d) i) $\Delta = 0$ for equal roots

$\Delta = b^2 - 4ac$

$= (-(k+3))^2 - 4 \times 1 \times 4k$

$= k^2 + 6k + 9 - 16k$

$= k^2 - 10k + 9$ ✓

$\therefore k^2 - 10k + 9 = 0$

$(k-9)(k-1) = 0$

$k = 1$ or $k = 9$ ✓

A lot of students incorrectly substituted c, getting an incorrect quadratic.

$\Delta = 0$.

let $S(x,y)$ and $Q(2,4)$

$\therefore \frac{x+2}{2} = \frac{7}{2} \qquad \frac{y+4}{2} = 0$

$x+2 = \qquad y+4=0$

$x = 5 \qquad y=-4$

$\therefore S(5,-4)$ ✓ Reason - 2

vi) PQ: $3x-y-2=0 \quad R(7,2)$ ✓

$$d = \frac{|3\times 7-2-2|}{\sqrt{3^2+(-1)^2}}$$ ✓

$= \frac{17}{\sqrt{10}}$ units ✓

Learn the correct formula.

$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

vii) $A = bh$

$= 2\sqrt{10} \times \frac{17}{\sqrt{10}}$

$= 34 \text{ units}^2$ ✓ Reason - 1

Make sure you know area formula!!!

Parallelograms are easy.

$A=bh$.

b) $\sqrt{3}\tan\theta + 1 = 0$

$\sqrt{3}\tan\theta = -1$

$\tan\theta = \frac{-1}{\sqrt{3}}$

θ lies in the 2nd & 4th quadrants ✓

fqa $= 30°$

$\therefore \theta = 180° - 30° \qquad \theta = 360° - 30°$

$= 150° \qquad = 330°$ ✓

one mark for correct rearrangement and correct angle using either 30° or -30°.

second mark angles found in quadrants 2 & 4 since $\tan\theta < 0$.

Q4 a) i) $y = 5x^3 - \frac{1}{3}x^{-2}$ ✓

$\frac{dy}{dx} = 15x^2 + \frac{2}{3}x^{-3}$ ✓

$= 15x^2 + \frac{2}{3x^3}$

- expressing $\frac{1}{3x^2}$ as $\frac{1}{3}x^{-2}$ was a problem for many students

ii) $y = x(4x+3)^5$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (4x+3)^5 \times 1 + x \times 5(4x+3)^4 \times 4$ ✓ ✓

- done poorly because most students didn't recognise it was a product rule question

$\underset{u}{x}\underset{v}{(4x+3)^5}$

ii) $\alpha + (-\alpha) = -\frac{b}{a}$

$0 = \frac{k+3}{1}$

$k + 3 = 0$

$k = -3$ ✓ Reason - 1

Some students did not recognise equal in mag + opp in sign $= -\frac{b}{a}$

Q3 a) i) $PQ = \sqrt{(2-0)^2 + (4--2)^2}$

$= \sqrt{2^2 + 6^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$ units ✓

This part was well done. Leave your answer exact unless otherwise requested.

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ii) $M_{PQ} = \frac{4--2}{2-0}$

$= \frac{6}{2}$

$= 3$

using $y - y_1 = m(x - x_1)$

$y - -2 = 3(x - 0)$

$y + 2 = 3x$

$3x - y - 2 = 0$ ✓

Make sure you know your formulas.

<u>gradient</u>

$m = \frac{y_2 - y_1}{x_2 - x_1}$

<u>Equation of a line</u>

$y - y_1 = m(x - x_1)$

---

iii) $k \parallel PQ$

$\therefore M_k = 3$ ✓

$y - 2 = 3(x - 7)$

$y - 2 = 3x - 21$

$3x - y - 19 = 0$ ✓

parallel lines have equal gradients.

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iv) PQRS is a parallelogram as both pairs of opposite sides are parallel ✓ Communication - 1

<u>Note</u> You do not know the sides lengths unless you have found them all.

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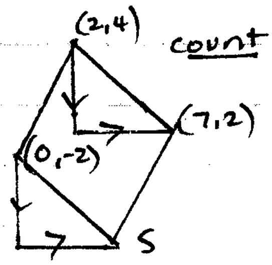
v) PR and QS has a common midpoint as the diagonals of a parallelogram bisect each other

$\therefore$ midpt PR is $\left(\frac{0+7}{2}, \frac{-2+2}{2}\right)$

$= \left(\frac{7}{2}, 0\right)$ ✓

$\therefore$ midpt QS $= \left(\frac{7}{2}, 0\right)$

You can just do part v) by counting.



<u>count</u> down 2

right 5

$\therefore S(5, -4)$

$= (4x+3)^4 [4x+3 + 20x]$

$= (4x+3)^4 (24x+3)$

iii) $y = \dfrac{6x-1}{4-2x}$ $\quad \dfrac{u}{v}$

$\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \dfrac{(4-2x)\times 6 - (6x-1)\times -2}{(4-2x)^2}$ ✓

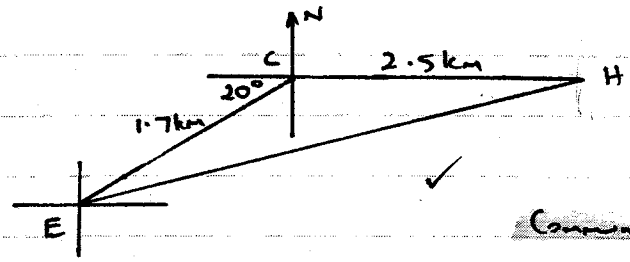
$= \dfrac{24 - 12x + 12x - 2}{(4-2x)^2}$

$= \dfrac{22}{(4-2x)^2}$ ✓

Calculus - 6

- done very well. A majority of students knew the rule and executed the process well.

b) i)



Communication - 1

- diagrams done well

ii) $\angle ECH = 250° - 90°$

$= 160°$

$\therefore EH^2 = 2.5^2 + 1.7^2 - 2\times 2.5\times 1.7 \times \cos 160°$ ✓

$= 17.127\ldots$

$EH = \sqrt{17.127\ldots}$

$= 4.1$ (to 1 dec. pl.) ✓

$\therefore$ the distance between Holly and Ellen is 4.1 km.

- this was done well the only problem was finding the angle.

try to remember to use units.

iii) Let $\angle CEH = \theta$

$\dfrac{\sin\theta}{2.5} = \dfrac{\sin 160}{4.1}$ ✓

$\sin\theta = \dfrac{2.5 \times \sin 160}{4.1}$

- it is important here not to use the rounded off value of 4.1
- use the memory fn on your calculator to use the most accurate answer for EH

$= 0.2066\ldots$

$\therefore \theta = \sin^{-1}(0.2066\ldots)$

$= 12°$ (to nearest degree) ✓

$\therefore$ bearing $= 70 + 12$

$= 82°$ ✓

$\therefore$ the bearing of Holly from Ellen is 082°T

Reasoning – 3

Q5 a) i) In Δ's ABC and DEF

$\angle ABC = \angle DEF$ (corresponding angles in parallel lines $AB \parallel DE$) ✓

$\angle ACB = \angle DFE$ (corresponding angles in parallel lines $AC \parallel DF$) ✓

$\therefore \triangle ABC \parallel\!| \triangle DEF$ (equiangular). ✓

Communication – 3

Some responses poorly set out

students who drew two separated triangles generally made errors

some students failed to state the relevant test.

ii) $\frac{AB}{DE} = \frac{BC}{EF}$ (corresponding sides in similar triangles are in the same ratio) ✓

$\frac{24}{16} = \frac{x+11}{x+6}$

$\frac{3}{2} = \frac{x+11}{x+6}$

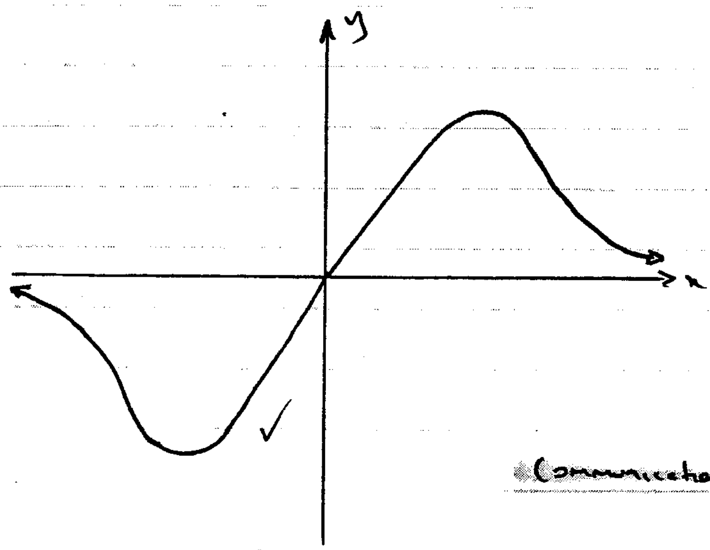
Reason – 2

$3x + 18 = 2x + 22$

$x = 4$ ✓

generally well done

b)



Communication – 1

allowed imperfect diagrams

Many students did not know the symmetry of odd functions

c) i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{(-2)}{1}$

$= 2$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{4}{1}$

$= 4$ ✓

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= (2)^2 - 2 \times 4$

$= 4 - 8$

$= -4$ ✓

Poor algebra cost marks.

iv) $\left(\alpha - \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$= \alpha\beta + 1 - 1 - \frac{1}{\alpha\beta}$ ✓

$= 4 - \frac{1}{4}$

$= 3\frac{3}{4}$ ✓

Reason – 2

Many students could not arrive at the second line.

Q6 a) i) In $\triangle ABM$ and $\triangle BCN$

$BC = AB$ (sides of a square are equal) ✓

$BM = CN$ (M & N are midpoints of equal sides)

$\angle ABM = \angle BCN = 90°$ (angles in a square are 90°) ✓

$\therefore \triangle ABM \equiv \triangle BCN$ (SAS) ✓

Communication – 3

2 facts correct = 1 mark
3 " " = 2 marks
Correct Test = 1 mark

Watch your reasons. Students make up reasons to suit themselves

PLEASE LEARN

ii) let $\angle NBC = x$

$\therefore \angle BAM = x$ (corr. angles in cong. triangles) ✓

Very poorly done.

$\angle ABN = 90 - x$ ($\angle ABC = 90°$)

$\therefore \angle BPA = 180 - (90 - x) - x$ (angle sum of a triangle is 180°) ✓

$= 90°$

$\therefore BN \perp AM$

Reason – 2

b) $y = x^3 - x^2$

$\frac{dy}{dx} = 3x^2 - 2x$ ✓

$\therefore 3x^2 - 2x = 8$

$3x^2 - 2x - 8 = 0$

$(3x+4)(x-2) = 0$

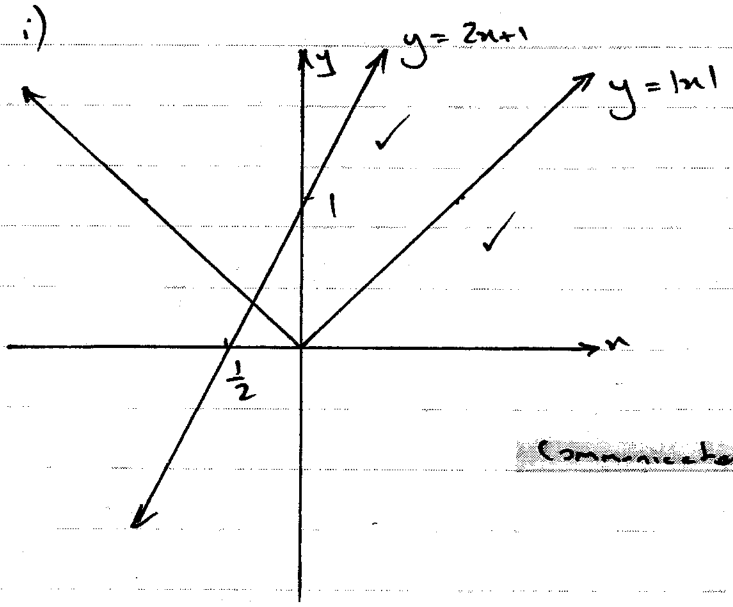
$x = 2 \quad x = -\frac{4}{3}$ ✓

Calculus 2

Differentiate was done ok but most students did not know what to do with the 8.

Alot of students don't know how to factorise.

c) i)



Communication - 2

Students need to draw neat graphs labelling all important features.

ii) Any pt of intersection between $y = |x|$ and $y = 2x+1$ is a solution to

$2x + 1 = |x|$ ✓

As there is one point of intersection

$\therefore$ one solution.

Communication - 1

Some students did not identify one point of intersection as one solution

iii) $2x + 1 = |x|$

$2x + 1 = x \quad$ or $\quad -2x - 1 = x$

$x = -1 \qquad\qquad -1 = 3x$

$\qquad\qquad\qquad x = -\frac{1}{3}$ ✓

test $x = -1$

LHS $= -1$

RHS $= 1$

$\therefore$ not a solution

test $x = -\frac{1}{3}$

LHS $= \frac{1}{3}$

RHS $= \frac{1}{3}$

$\therefore x = -\frac{1}{3}$ is only solution. ✓

Alot of students failed to check their solutions. Even though is part (ii) we know there is only one solution, students still wrote 2 here.

Watch setting out of checking LHS / RHS.

Q7 a) $y = (x+2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x+2)^{-1/2} \times 1$

$= \frac{1}{2\sqrt{x+2}}$ ✓

when $x = 7$ $\frac{dy}{dx} = \frac{1}{2 \times \sqrt{7+2}}$

$= \frac{1}{2 \times 3}$

$= \frac{1}{6}$

$\therefore M_{tang} = \frac{1}{6}$ at P

$\therefore M_{norm} = -6$ at P ✓ as tang $\perp$ normal

using $y - y_1 = m(x - x_1)$

$y - 3 = -6(x - 7)$

$y - 3 = -6x + 42$ Calculus - 3

$6x + y - 45 = 0$ ✓

- the differentiation was done well but the substitution was done poorly!! It was a shame that poor algebra skills rather than calculus skills that cost marks.

ii) $m = \tan\beta$

$\tan\beta = |-6|$

$= 6$

$\beta = 81°$

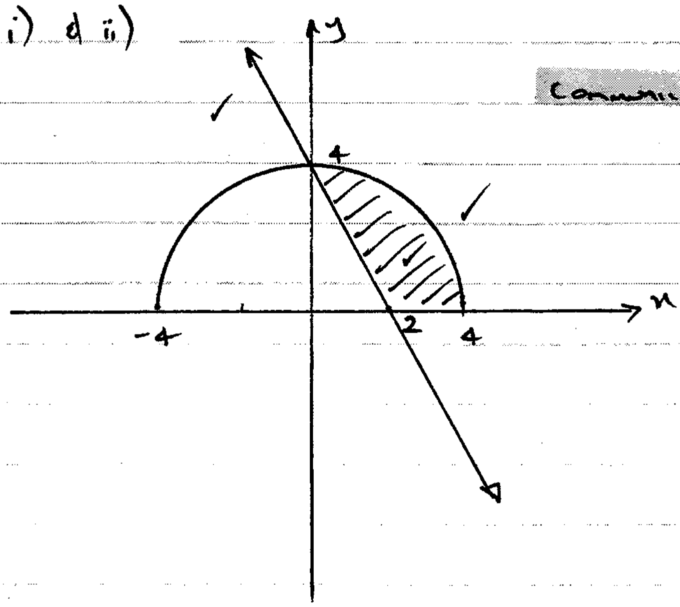
$\theta = 180° - 81°$

$= 99°$ ✓

- don't forget that a negative gradient means an obtuse angle

b) i) & ii)

Communication - 3



- the semi-circle was done well but the line was done poorly
- must learn to find x and y-intercepts

iii) Area $= \frac{1}{4} \times \pi \times 4^2 - \frac{1}{2} \times 2 \times 4$

$= (4\pi - 4)$ units^2 ✓ Reason - 1

- even though it wasn't stated an exact answer is better.

c) i) $LHS = \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

- done well

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ ✓

$= \frac{1}{\sin\theta\cos\theta}$

ii) $\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta}$

$= \frac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta\cos\theta}}$ ✓ from i)

$= \left(1 + \frac{\cos\theta}{\sin\theta}\right) \times \sin\theta - \frac{1}{\cos\theta}(\sin\theta\cos\theta)$

$= \sin\theta + \cos\theta - \sin\theta$

$= \cos\theta$

$\therefore \cos\theta = -1$ ✓

$\therefore \theta = 180°$ ✓ Reason - 3

- this was a hard question but it was algebra skills rather than trig skills that prevented getting the question out

- handy hint: although this doesn't always work it is a good idea to change $\tan\theta$, $\cot\theta$ to $\frac{\sin\theta}{\cos\theta}$ & $\frac{\cos\theta}{\sin\theta}$ and

$\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$