Centre Number

Student Number

SCEGGS Darlinghurst

2008

Preliminary Course
Semester 2 Examination

Mathematics

Outcomes Assessed: P2 – P8
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt **all** questions and show all necessary working
- Answer all questions on the pad paper provided
- Write your Student Number at the top of each page
- **Begin each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 78

- Attempt Questions 1 – 6

Question	Calc	Comm	Reasoning	Marks
1			/3	/13
2	/5	/1		/13
3		/2	/2	/13
4	/1	/1	/8	/13
5	/3	/2	/6	/13
6	/3	/3	/5	/13
TOTAL	**/12**	**/9**	**/24**	**/78**

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Marks

Question 1 (13 marks)

(a) Evaluate $\dfrac{542}{3.17 \times 10^{15}}$ expressing your answer in scientific notation correct to 3 significant figures. **2**

(b) Simplify $|-7| - |4|$ **1**

(c) A car salesman buys a second hand car and then sells it at a profit of 37.5%. If the car salesman sells the car for \$30 800, what price did he pay for the car originally? **2**

(d) Express $\dfrac{1}{4-\sqrt{3}} + \dfrac{1}{4+\sqrt{3}}$ in simplest form. **2**

(e) Factorise fully:

$$8 - x^3$$ **2**

(f) Solve:

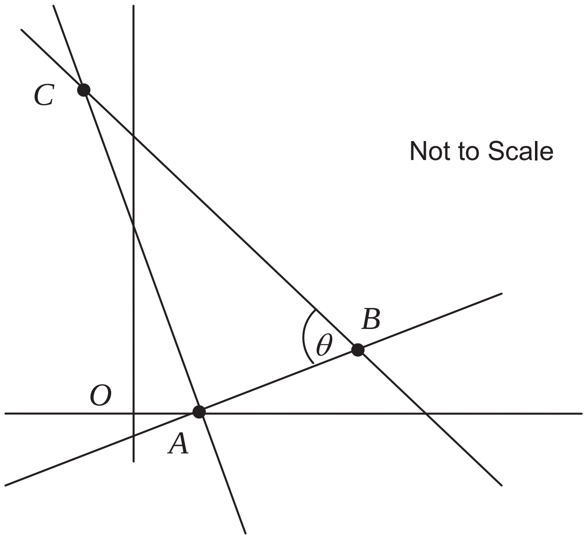
(i) $\dfrac{1}{2^x} = 8$ **1**

(ii) $|2x - 5| = 7 - 3x$ **3**

Marks

Question 2 (13 marks)

(a)



In the above diagram $A(1, 0)$, $B(4, 1)$ and $C(-1, 6)$ are points on the number plane.

Copy or trace this diagram into your writing booklet.

(i) Show that the equation of AC is $3x + y - 3 = 0$ **2**

(ii) Find the length of AB **1**

(iii) Show that $AB \perp AC$ **2**

(iv) Find $\tan \theta$ **2**

(v) On your diagram, shade the region satisfying the inequality $3x + y - 3 \leq 0$ **1**

(b) Differentiate:

(i) $2x^3 - \dfrac{1}{x} + 5$ **1**

(ii) $(3x^2 + 4)^5$ **2**

(iii) $x\sqrt{1 - x}$ **2**

Marks

Question 3 (13 marks)

(a) Solve $\sqrt{2}\sin\theta + 1 = 0$ for $0^\circ \le \theta \le 360^\circ$ **2**

(b) Solve $12 + 4x - x^2 > 0$ **2**

(c) A ship sails from point A on a bearing of 237º for a distance of 423 kilometres to point B. The ship then turns and sails due south to point C. The bearing of A from C is 041º.

(i) Draw a diagram showing this information **1**

(ii) Find the size of $\angle BAC$, to the nearest degree **1**

(iii) Calculate the distance (to the nearest km) the ship must travel back to point A from C. **2**

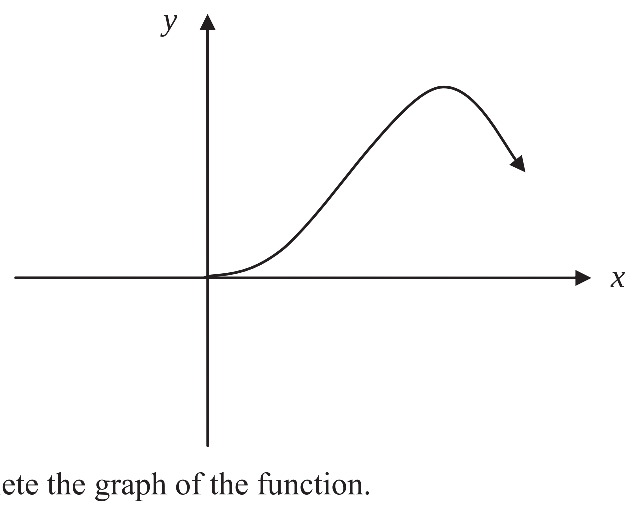
(d) If α and β are the roots of the quadratic equation $2x^2 - 6x + 3 = 0$ find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(e) The diagram below shows part of the graph of an even function **1**



Copy and complete the graph of the function.

Marks

Question 4 (13 marks)

(a) Find the quadratic equation whose roots are 2 and -3 **1**

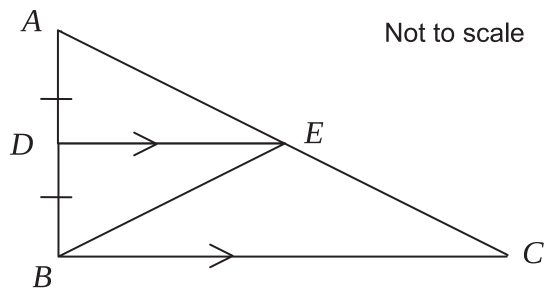
(b) Consider the function $y = \frac{1}{x} + 2$

(i) State the domain and range of the function **2**

(ii) Find $\frac{dy}{dx}$ **1**

(iii) Hence or otherwise explain why the gradient of $y = \frac{1}{x} + 2$ is always negative **1**

(c)



The triangle *ABC* has a right angle at *B*. *D* is the midpoint of *AB* and *DE* is parallel to *BC*. Copy this diagram into your writing booklet.

(i) Prove that *ADE* is a right angle **1**

(ii) Prove that triangle *AED* is congruent to triangle *BED* **2**

(iii) Prove that $BE = EC$ **2**

(d) Express $3x^2 - 7x - 2$ in the form $ax(x+1) + bx^2 + c(x+1)$ **3**

Marks

Question 5 (13 marks)

(a) The angles in a regular polygon are 156º each. Find the number of sides in the polygon. **2**

(b) Find the values of q for which the expression $2x^2 - x + 4q$ is positive definite **3**

(c) Differentiate from first principles **3**

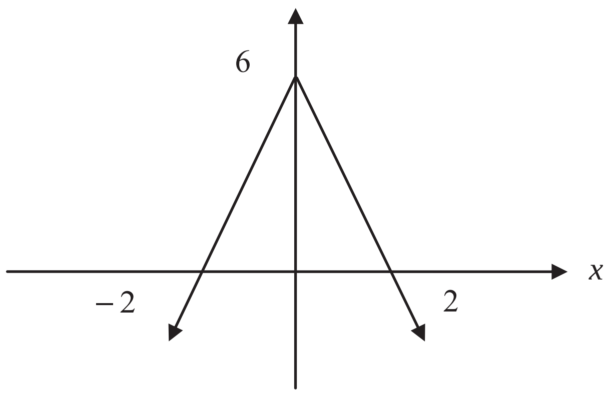
$$f(x) = 2x^2 - 6x$$

(d) Prove that: **3**

$$\sin\theta(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta) = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$$

(e) Maud was asked to sketch the graph of $y = \left| \, 6 - 3x \, \right|$ showing all important features **2**

Below is her solution:



The solution is incorrect.

Explain why this is incorrect and draw a correct solution.

Marks

Question 6 (13 marks)

(a) Simplify $\tan^2\theta\left(1-\sin^2\theta\right)$ **2**

(b) For the function defined by:

$$f(x)=\begin{cases}3-x^2 & \text{for} \quad -2\le x\le -1\\ 2x & \text{for} \quad -1<x<1\\ x^2+1 & \text{for} \quad 1\le x\le 2\end{cases}$$

(i) Evaluate: (α) $f(-2)$ **1**

(β) $f(1)$ **1**

(ii) Sketch the graph of the function in the given domain. **3**

(c) Find the value of x for which the curve $y=(3x-4)^3$ cuts the x-axis and find the gradient of the tangent at this point. **3**

(d) Solve $x^4-5x^2-36=0$ **3**

End of paper

2008 Mathematics Sem 2 Exam Solutions

Q1 SL

Reas 3

a) 1.71×10^{-13} ✓✓ (correct s.f.)

b) $7 - 4 = 3$ ✓

c) $137.5\% = \$30\,800$ ✓

$1\% = \$224$

$100\% = \$22\,400$ ✓

d) $\dfrac{1}{4-\sqrt{3}} + \dfrac{1}{4+\sqrt{3}}$

$= \dfrac{4+\sqrt{3} + 4-\sqrt{3}}{16-3}$ ✓

$= \dfrac{8}{13}$ ✓

e) $8 - x^3 = (2-x)(4+2x+x^2)$ ✓ ✓

f) i) $2^{-x} = 2^3$

$-x = 3$

$x = -3$ ✓

ii) $|2x-5| = 7-3x$ $\boxed{\text{Reas 3}}$

① $2x-5 = 7-3x$

$5x = 12$

$x = 2\frac{2}{5}$

② $-2x+5 = 7-3x$

$x = 2$

✓ both solutions

check

$LHS = |2 \times 2\frac{2}{5} - 5|$

$LHS = \frac{1}{5}$

$RHS = 7 - 3 \times 2\frac{2}{5}$

$RHS = -\frac{1}{5}$

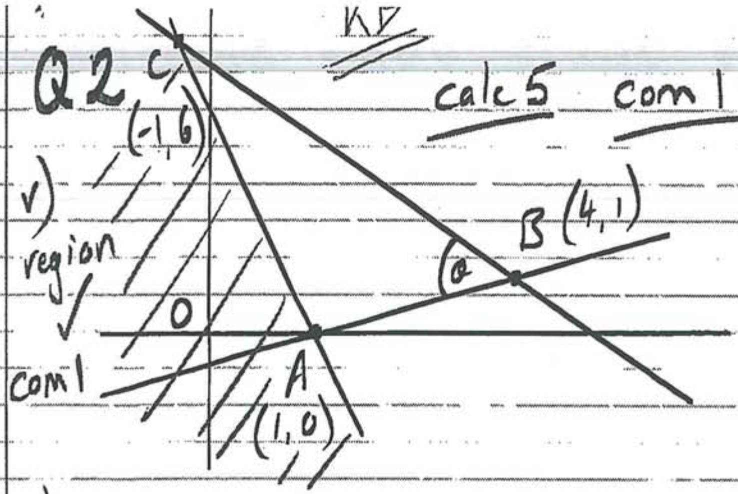
$LHS = |2\times 2 - 5| = 1$

$RHS = 7 - 3\times 2 = 1$

✓ checks

$\therefore$ only solution is $x = 2$ ✓ conclusion

Q2 calc 5 com 1



Diagrams often too small. Ruler and pencil is required.

i) AC: $m = \frac{6-0}{-1-1}$

$= -3$ ✓

eqn: $y - 0 = -3(x-1)$

$y = -3x + 3$ ✓

$3x + y - 3 = 0$

ii) $AB = \sqrt{(4-1)^2 + (1-0)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ ✓

iii) $M_{AB} = \frac{1-0}{4-1}$ $\quad M_{AC} = -3$

$= \frac{1}{3}$ ✓

$M_{AB} \times M_{AC} = \frac{1}{3} \times -3$

$= -1$ ✓

Must explain this step.

$\therefore AB \perp AC$

iv) $AC = \sqrt{(1--1)^2 + (0-6)^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$= 2\sqrt{10}$ ✓

$\tan\theta = \frac{AC}{AB}$

$= \frac{2\sqrt{10}}{\sqrt{10}}$

$\tan\theta = 2$ ✓

Must have $\tan\theta = 2$.

The angle was not requested. Read questions carefully.

b) i) $2x^3 - x^{-1} + 5$ $\boxed{\text{Calc 5}}$

$\frac{d}{dx} = 6x^2 + x^{-2}$

$\frac{d}{dx} = 6x^2 + \frac{1}{x^2}$ ✓

ii) $\frac{d}{dx} = 5 . 6x\left(3x^2+4\right)^4$ ✓

$\frac{d}{dx} = 30x\left(3x^2+4\right)^4$ ✓

Remember to differentiate $3x^2 + 4$

iii) $x\left(1-x\right)^{\frac{1}{2}}$

$u = x \qquad v = (1-x)^{\frac{1}{2}}$

$u' = 1 \qquad v' = -\frac{1}{2}\left(1-x\right)^{-\frac{1}{2}}$

$= \frac{-1}{2\sqrt{1-x}}$ ✓

This is a product Rule Question.

Very poor answers in many cases.

$\frac{d}{dx} = (1-x)^{\frac{1}{2}} + \frac{-x}{2\sqrt{1-x}}$ ✓

Q3

Com 2 reas2

a) $\sqrt{2}\sin\theta + 1 = 0$

$$\sin\theta = -\frac{1}{\sqrt{2}}$$

Learn this technique. and follow the steps

sin -ve in Q3, Q4

acute $\angle$: $\sin\theta = \frac{1}{\sqrt{2}}$

$$\theta = 45^\circ \checkmark$$

$\therefore \theta = 180 + 45,\ 360 - 45$

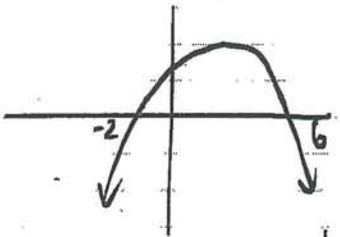
$\theta = 225^\circ,\ 315^\circ \checkmark$

b) $12 + 4x - x^2 > 0$

$-(x^2 - 4x - 12) > 0$

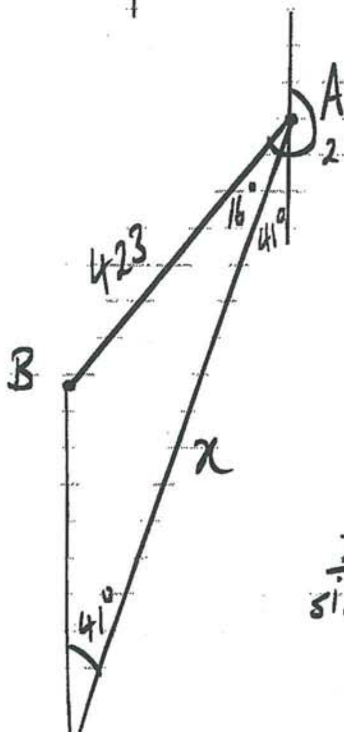
$-(x-6)(x+2) > 0 \checkmark$

$\boxed{\text{Reas 2}}$ Very badly done. Usually successful if a graph is drawn to solve the inequality. Factorise carefully.



$\therefore -2 < x < 6 \checkmark$

c)



(i) 237° ✓ $\boxed{\text{Com 1}}$

Diagrams often far too small. Otherwise mostly well done.

(ii)

$\angle BAC = 237 - 41 - 180$

$= 16^\circ \checkmark$

(iii) $\angle ABC = 123^\circ$

$$\frac{x}{\sin 123^\circ} = \frac{423}{\sin 41^\circ} \checkmark$$

$$x = 541\text{ km} \checkmark$$

d) $2x^2 - 6x + 3 = 0$

i) $\alpha + \beta = \frac{6}{2} = 3$ ✓

ii) $\alpha\beta = \frac{3}{2}$ ✓

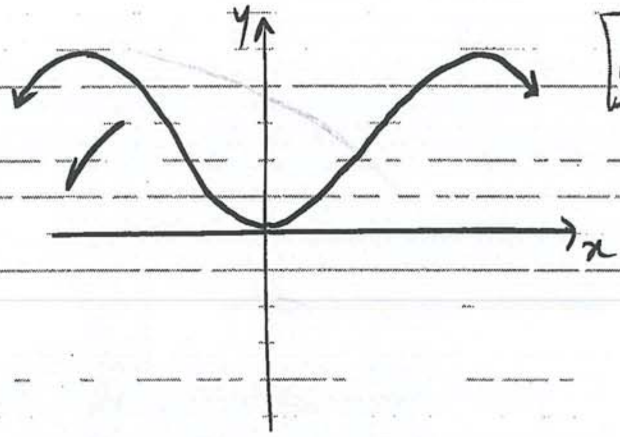
iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= 3^2 - 2 \times \frac{3}{2}$

$= 6$ ✓

Most students knew this works.

e)



$\boxed{\text{Com 1}}$

Some really bad graphs here! They did not really look symmetric though that was clearly the intention.

Q4 AV Calc 1 Com 1 Reas 8

a) $(x-2)(x+3) = 0$ ✓

well done, make sure you put $= 0$

b) $y = \frac{1}{x} + 2$

i) D: all real x, where $x \neq 0$ ✓
R: all real y, where $y \neq 2$ ✓

most were able to determine the asymptotes correctly although mixed up the x & y
please also include all real x or all real y for domain or range
not just domain: $x \neq 0$

ii) $y = x^{-1} + 2$

$\boxed{\text{Calc 1}}$

$\frac{dy}{dx} = -x^{-2}$ ✓

$= -\frac{1}{x^2}$

Well Done

iii) $-\frac{1}{x^2}$ is the gradient function

$\boxed{\text{Com 1}}$

Not well explained. Try to break ~~done~~ down the gradient function as shown.

since $x^2 > 0$

then $\frac{1}{x^2} > 0$

$\therefore -\frac{1}{x^2} < 0$ ✓

Or use a graph of the function to explain why the gradient is always negative.

hence the gradient must always be negative

c) i) $\angle ADE = \angle ABC$ (corresponding angles are equal on parallel lines) ✓
$= 90°$

$\boxed{\text{Reas 5}}$

Please learn your REASONS and clearly set out each step of your work.

ii) $\angle ADE = 90°$ (supplementary angles add to $180°$)
$\therefore \angle EDB = 90°$ ✓

For each statement you make you must have a reason

$AD = BD$ (given) ✓

NEVER ASSUME something is true unless you have proved it.

DE is common

$\therefore \triangle AED \equiv \triangle BED$ (SAS) ✓

iii) Prove $BE = EC$

$\angle EBC = \angle DEB$ (alternate angles are equal on parallel lines)

$\angle AED = \angle ECB$ (corresponding angles are equal on parallel lines)

$\angle AED = \angle BED$ (corresponding angles in congruent triangles are equal)

Some students used ratio of intercepts are equal on parallel lines which was also correct.

On the whole geometry was not well set out.

$\therefore \angle EBC = \angle ECB$

$\therefore BE = EC$ (equal sides in isos. triangle BEC)

d) $ax(x+1) + bx^2 + c(x+1)$ $\boxed{\text{Reas 3}}$

Those who had learnt their work had no problem getting to the solution.

$= ax^2 + ax + bx^2 + cx + c$

$= x^2(a+b) + x(a+c) + c$

Compare $3x^2 \quad - 7x \quad - 2$

$c = -2, \quad a+c = -7, \quad a+b = 3$

$a - 2 = -7 \qquad -5 + b = 3$

$a = -5 \qquad b = 8$

$\therefore 3x^2 - 7x - 2$

$= -5x(x+1) + 8x^2 - 2(x+1)$

Some lost 1 mark for not completing the question and writing in the form $ax(x+1) + bx^2 + c(x+1)$

MF

Q5 Calc 3 Com 2 Reas 6

a) int $\angle = 156°$

ext $\angle = 24°$ ✓

sides $= 360 \div 24$

$= 15$ ✓

OR use $\frac{(n-2) \times 180°}{n} = 156°$ and solve this equation.

This part was pretty well done.

b) pos. def. means $\boxed{\text{Reas } 3}$

① concave up $a>0$

② no real roots $\Delta < 0$

concave up since $a=2$, $a>0$ ✓

no real roots when $\Delta < 0$

$\therefore \quad b^2 - 4ac < 0$

$1 - 4 \times 2 \times 4q < 0$

$1 - 32q < 0$ ✓

$-32q < -1$

$q > \frac{1}{32}$ ✓

Think about the curve. It's easier to remember the facts.

concave up $a>0$

No roots $\Delta < 0$

You can get this mark if you correctly solve your inequality.

c) $f(x) = 2x^2 - 6x$ $\boxed{\text{Calc } 3}$

$f(x+h) = 2(x+h)^2 - 6(x+h)$

$= 2x^2 + 4xh + 2h^2 - 6x - 6h$ ✓

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- 6x} - 6h \cancel{- 2x^2} \cancel{+ 6x}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 6)}{\cancel{h}}$

$= 4x - 6$ ✓

Poorly set out even though it was in the last exam a few weeks ago!

Practise until you get it right & make sure you know the correct formula!!

|Reas 3|

d) $\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)=\dfrac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}$

$$LHS = \sin\theta + \frac{\sin^2\theta}{\cos\theta} + \cos\theta + \frac{\cos^2\theta}{\sin\theta} \quad ✓$$

$$= \frac{\sin^2\theta\cos\theta + \sin^3\theta + \cos^2\theta\sin\theta + \cos^3\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta(\cos\theta+\sin\theta)+\cos^2\theta(\sin\theta+\cos\theta)}{\cos\theta\sin\theta} \quad ✓$$

$$= \frac{(\cos\theta+\sin\theta)(\sin^2\theta+\cos^2\theta)}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta+\sin\theta}{\cos\theta\sin\theta} = RHS \quad ✓$$

Poorly done ☹

Here are some mistakes not to make next time.

$1+\tan\theta \neq \sec\theta$ ✗

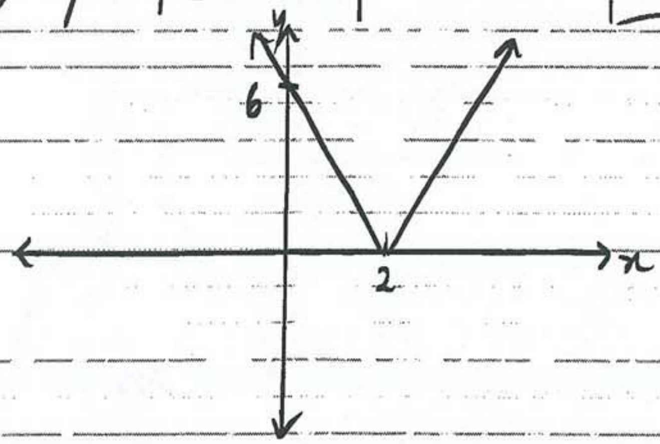
$1+\tan^2\theta = \sec^2\theta$ ✓

$1+\cot\theta \neq \operatorname{cosec}\theta$ ✗

$1+\cot^2\theta = \operatorname{cosec}^2\theta$ ✓

If $a^2+b^2=c^2$
$a+b \neq c$ Never, e
$\sqrt{a^2+b^2} \neq a+b$

e) $y = |6-3x|$ $\boxed{\text{Com } 2}$



✓

the correct solution is shown above.

Maud's solution cannot be correct since: (one of the following)

- $y=|6-3x|$ can never go below the x-axis since the absolute value sign makes it positive. ✓
- she has drawn $y = 6 - |3x|$

Poorly done.

Practise any questions in past papers that ask for explanations

You need to learn the correct wording.

The absolute value sign has the effect of making all the y values positive if the curve is of the form
$y = |f(x)|$

Q 6 ~~5~~ Calc 3 Com 3 Reas 5

a) $\tan^2\theta\,(1-\sin^2\theta)$ ✓ $\boxed{\text{Reas } 2}$

$= \tan^2\theta \cdot \cos^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$

$= \sin^2\theta$ ✓

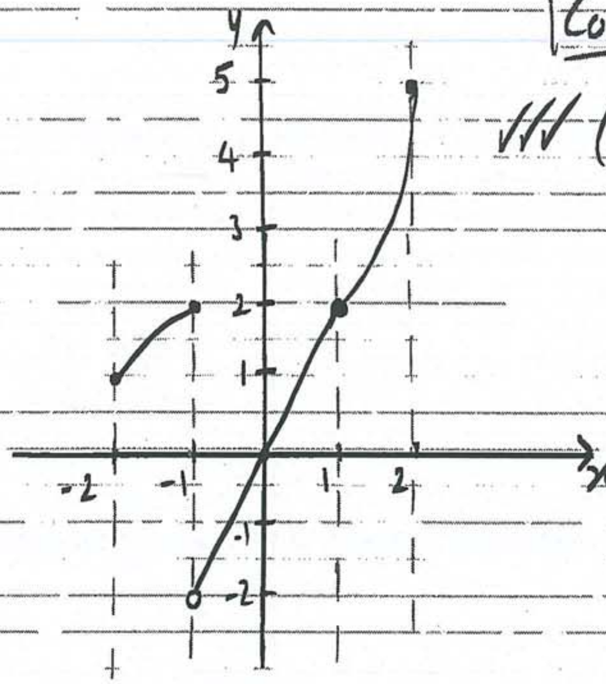
b) i α) $f(-2) = 3 - 4$

$= 1$

β) $f(1) = 1^2 + 1$

$= 2$

ii) $\boxed{\text{Com } 3}$



✓✓✓ (one per section)

c) cuts x-axis when $y=0$ $\boxed{\text{Calc } 3}$

$0 = (3x-4)^3$

$3x - 4 = 0$

$3x = 4$

$x = 1\frac{1}{3}$ ✓

$y = (3x-4)^3$

$y' = 3 \cdot 3(3x-4)^2$

$y' = 9(3x-4)^2$ ✓

at $(1\frac{1}{3}, 0)$

$m_T = 9(3 \cdot 1\frac{1}{3} - 4)^2$

$= 9(0)^2$

$= 0$ ✓

d) $x^4 - 5x^2 - 36 = 0$

let $u = x^2$

$u^2 - 5u - 36 = 0$

$(u-9)(u+4) = 0$

$u = 9, -4$ ✓

$\therefore 9 = x^2$ and $-4 = x^2$

$\pm 3 = x$ ✓ No solns ✓