

## SCEGGS Darlinghurst

2009<br>Preliminary Course<br>Semester 2 Examination

## Mathematics

Outcomes Assessed: P2 - P8
Task Weighting: 40\%

## General Instructions

- Reading time -5 minutes
- Working time -2 hours
- Write using blue or black pen
- Attempt all questions and show all necessary working
- Answer all questions on the pad paper provided
- Write your Student Number at the top of each page
- Begin each question on a new page
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical

Total marks - 78

- Attempt Questions 1 - 6

| Question | Calc | Comm | Reasoning | Marks |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ |  |  |  | $/ 13$ |
| $\mathbf{2}$ |  |  |  | $/ 13$ |
| $\mathbf{3}$ |  |  |  | $/ 13$ |
| $\mathbf{4}$ |  |  |  | $/ 13$ |
| $\mathbf{5}$ |  |  |  | $/ 13$ |
| $\mathbf{6}$ |  |  |  | $/ 13$ |
| TOTAL |  |  |  | $/ 78$ | equipment and scientific calculators may be used

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Question 1 (13 marks)
(a) Find the value of $\sqrt{\frac{5.36+31.98}{(4.75)^{2}}}$ to 3 significant figures.
(b) Factorise $8 x^{2}-10 x+3$.
(c) Express $0.1 \dot{8}$ in the form $\frac{a}{b}$, where $a, b$ are positive integers.
(d) After a discount of $20 \%$ off the original marked price. Jill paid $\$ 160$ for a sweater. What was the original price of this sweater?
(e) Rationalise the denominator $\frac{\sqrt{3}}{2+\sqrt{3}}$
(f) Find the exact value of $\cot 210^{\circ}$.
(g) Solve the equation $\frac{x}{3}-\frac{x+1}{2}=4$.

Question 2 (13 marks)
(a) On the number plane $A=(0,-3) B=(5,0) C=(-1,2)$.

(i) State the co-ordinates of $D$ such that $A B C D$ is a parallelogram.
(ii) Find the length of $A B$ as a surd.
(iii) Show that the equation of $A B$ is $3 x-5 y-15=0$.
(iv) Find as a surd the perpendicular distance of $C$ from $A B$.
(v) Hence find the area of this parallelogram.

## Question 2 continues on the next page

Question 2 (continued)
(b) Find a quadratic equation whose roots are -5 and 2 .
(c) Find the domain and range $y=\frac{2}{x-1}$. 2
(d) Solve $x^{2}>4$. 2

Question 3 (13 marks)
(a) If $f(x)=1-2 x$ :
(i) find the value of $f(-3)$.
(ii) for what value of $x$ does $f(x)=5$ ?
(b) A parabola has equation $(x-3)^{2}=16(y+1)$.

Find:
(i) the vertex.
(ii) the focus.
(iii) the equation of the directrix.
(iv) the equation of the axis of symmetry.
(c) If $\alpha$ and $\beta$ are the roots of the equation $x^{2}-2 x+4=0$, find the value of:
(i) $\frac{1}{\alpha}+\frac{1}{\beta}$
(ii) $\alpha^{2}+\beta^{2}$
(iii) $\frac{2-\alpha}{1+\beta}+\frac{2-\beta}{1+\alpha}$

Question 4 (13 marks)
(a) The point $P(x, y)$ moves such that its distance from the point $A(2,-4)$ is twice its distance from the point $B(-1,2)$.
(i) Show that the equation of the locus of the point $P(x, y)$ is

$$
x^{2}+4 x+y^{2}-8 y=0
$$

(ii) Find the centre and radius of this circle.
(b) Find the equation of the normal to the curve $y=\sqrt{x+2}$ at the point $(7,3)$.
(c) The diagonals of a quadrilateral $M N O P$ intersect at $T$.

Given $M T=O T$ and $N T=P T$, prove $M N=O P$.

(d) Solve for $x$ if $|x-2|=2 x-1$.

Question 5 (13 marks)
(a) (i) Write down the discriminant of $2 x^{2}+3 x+k$.
(ii) For what values of $k$ does $2 x^{2}+3 x+k=0$ have no real roots.

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(b) Differentiate each of the following.
(i) $y=3 x^{2}+9 x-5$
(ii) $y=\frac{3 x}{x^{2}-2 x}$
(iii) $y=(x-5)^{2}(2 x+3)$
(c) From a port $P$ a lighthouse $L$ is seen 2.4 km away on a bearing of $035^{\circ}$.

A boat leaves port and sails due East to a point $B, 4.2 \mathrm{~km}$ from the lighthouse.
(i) Draw a diagram showing this information
(ii) Calculate the size of $\angle L B P$, to the nearest degree.
(iii) Hence find the bearing of the boat from the lighthouse.
(d) If $\cos x>0$ and $\sin x=-\frac{8}{17}$, find the exact value of $\tan x$.

Question 6 (13 marks)
(a) (i) Write down the equation of this semicircle.

(ii) Prove that this is an even function.
(b) The interior angles of a regular polygon are $135^{\circ}$.

Find the number of sides in the polygon.
(c) (i) Draw a neat sketch showing the graphs $x^{2}+y^{2}=4$ and $y=2^{x}$.
(ii) Shade the region on your sketch where $x^{2}+y^{2} \leq 4$ and $y \leq 2^{x}$.
(d) Solve the equation $\sin x=-\frac{1}{2}$ for $0 \leq x \leq 360^{\circ}$.
(e) Prove $\frac{1+\cot x}{1+\tan x}=\cot x$.

## End of Paper

