Centre Number

Student Number

SCEGGS Darlinghurst

2010

Preliminary Course
Semester 2 Examination

Mathematics

Outcomes Assessed: P2 – P8
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt **all** questions and show all necessary working
- Answer all questions on the pad paper provided
- Write your Student Number at the top of each page
- **Begin each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 78

- Attempt Questions 1 – 6

Question	Calc	Comm	Reasoning	Marks
1				/13
2				/13
3				/13
4				/13
5				/13
6				/13
TOTAL				**/78**

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Marks

Question 1 (13 marks)

(a) Express $0.8\dot{3}$ in the form $\frac{a}{b}$, where a and b are positive integers. **2**

(b) The crowd attendance at a football game on the weekend was 15 660 which was a decline of 10% from the previous weekend. How many people attended the previous weekend? **2**

(c) Factorise fully: $3x^2 - 7x - 6$. **2**

(d) Find the exact value of $\cos 225°$. **2**

(e) Find the value of v in $v^2 = u^2 - 2as$ when $u = 6.4 \times 10^7$, $a = 21.5$ and $s = 5.7 \times 10^{13}$. Give your answer to 3 significant figures. **2**

(f) Find the values of a and b if $\frac{3}{\sqrt{5}-2} = \sqrt{a} + b$. **3**

- **Start a new page**

Marks

Question 2 (13 marks)

(a) Find the size of each interior angle of a regular nonagon (9-sided polygon). **2**

(b) Solve each of the following:

(i) $|2x-5| \leq 7$ **2**

(ii) $3^{2x} - 10 \times 3^{x} + 9 = 0$ **3**

(c) Solve $\tan(\theta + 60°) = \dfrac{1}{\sqrt{3}}$ for $0° \leq \theta \leq 360°$. **2**

(d) α and β are the roots of the quadratic equation $x^2 - 2x + 3 = 0$.
Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha^2\beta^2$ **1**

(iii) $\alpha^{-2} + \beta^{-2}$ **2**

- **Start a new page**

Marks

Question 3 (13 marks)

(a) Differentiate each of the following with respect to x:

(i) $5x^3 + \frac{2}{x} - 3$ **2**

(ii) $x(2x+3)^9$ **2**

(b) Find the equation of the tangent to the curve $y = \frac{x}{x+2}$ at the point where $x = -1$. Give your answer in general form. **4**

(c) Consider the quadratic function $y = kx^2 + 6x + k$.

(i) Show that the function is positive definite when $k > 3$. **3**

(ii) Sketch the graph of $y = 4x^2 + 6x + 4$ clearly showing any intercepts with the axes and the co-ordinates of the vertex. **2**

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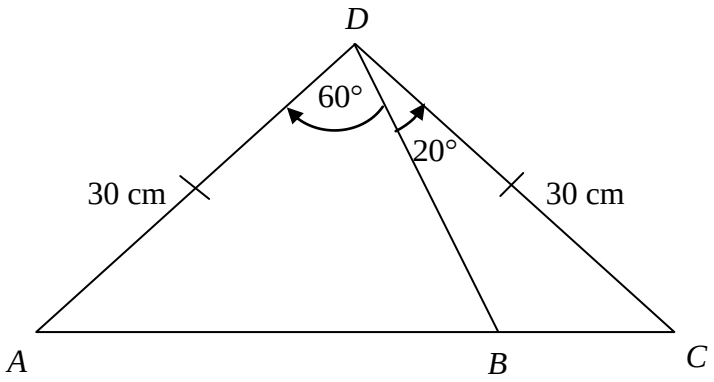
Marks

Question 4 (13 marks)

(a) In the diagram, AD and DC are equal to 30 cm.



NOT TO SCALE

(i) Find the size of $\angle DBA$ giving reasons. **2**

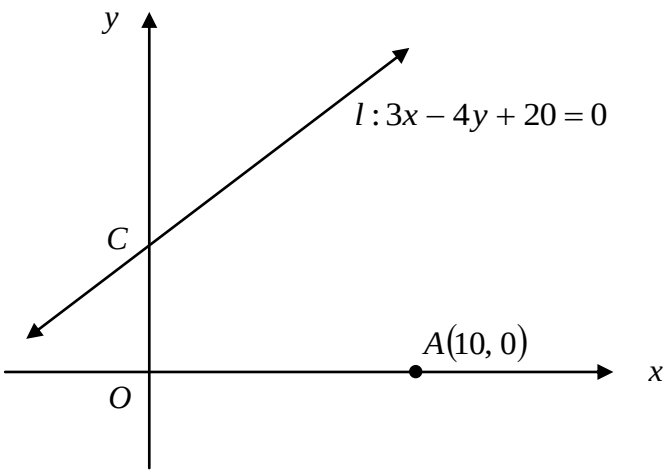
(ii) Hence find the length of AB to the nearest centimetre. **2**

Question 4 continues on the next page

Marks

Question 4 (continued)

(b)



In the diagram A is the point $(10, 0)$ and line l has the equation $3x - 4y + 20 = 0$.

(i) Find the co-ordinates of the point C where l intersects the y axis. **1**

(ii) B is on the line l such that AB is perpendicular to l. **2**
Find the distance AB.

(iii) Prove that $\Delta AOC \equiv \Delta ABC$. **2**

(iv) Find the area of the quadrilateral $OABC$. **1**

(v) Find the gradient of AB and hence find the angle which AB makes with the positive x axis. Give your answer to the nearest degree. **3**

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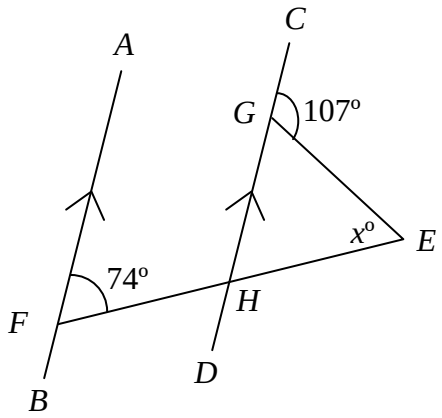
Marks

Question 5 (13 marks)

(a) **2**



Find the value of x giving reasons.

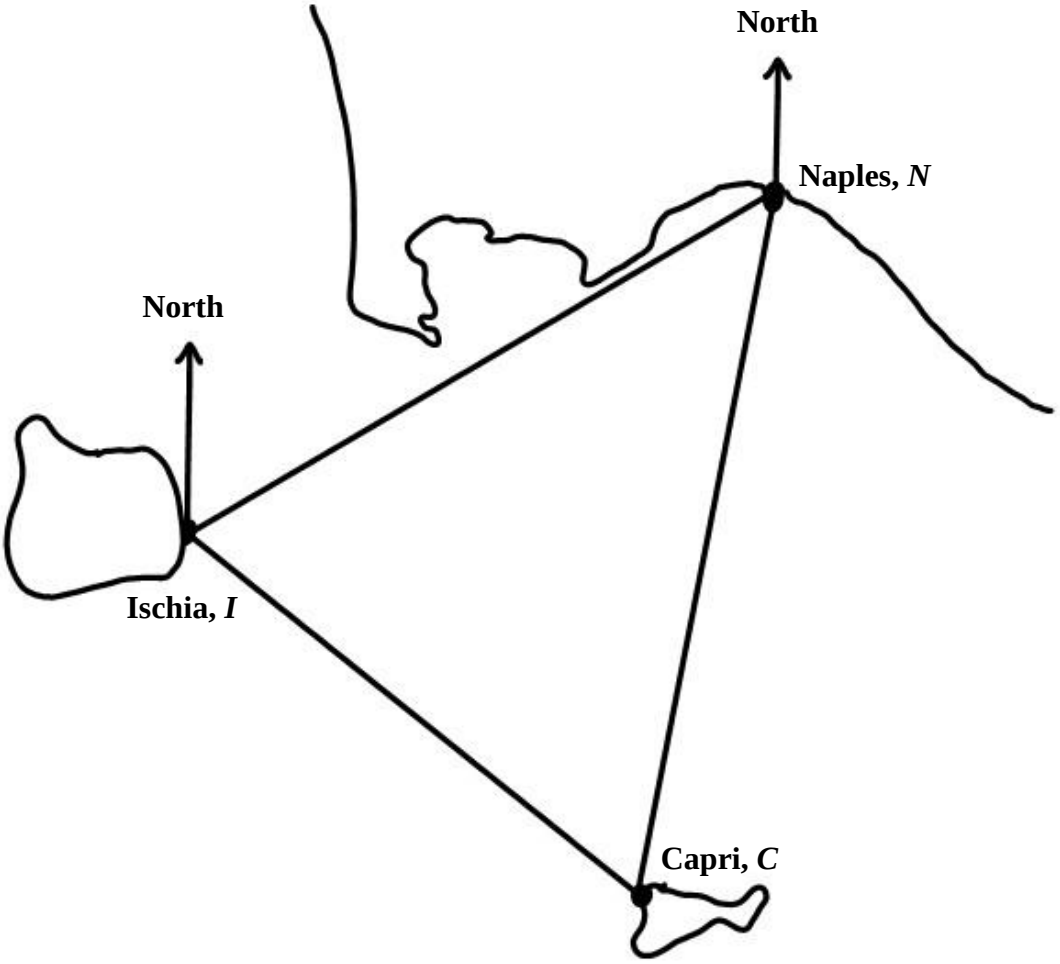
(b) Find a, b and c such that $x^2 - x + 3 \equiv a(x+1)^2 + b(x+1) + c$. **3**

Question 5 continues on the next page

Marks

Question 5 (continued)

(c) A boat leaves the Island of Ischia and travels 40 km on a bearing of $060°T$ to Naples. It then turns to travel a further 45 km to the Island of Capri on a bearing of $200°T$.



(i) Find the size of $\angle INC$. **1**

(ii) Find the distance between the islands of Ischia and Capri to the nearest kilometre. **2**

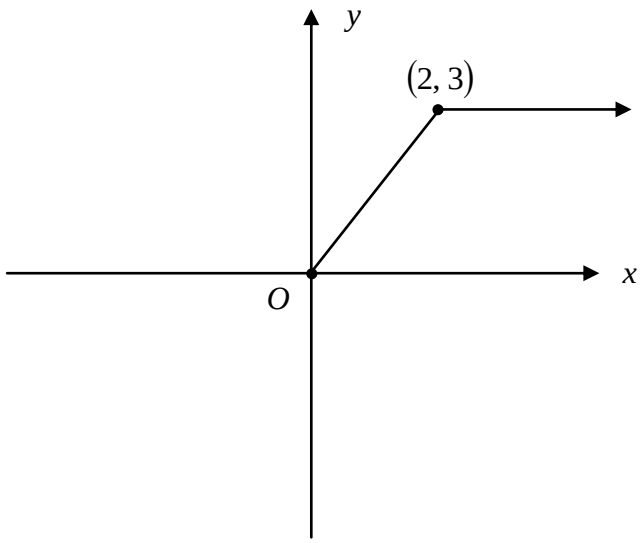
(iii) Find the area of ΔINC. Give your answer in square kilometres to 2 significant figures. **2**

Question 5 continues on the next page

Marks

Question 5 (continued)

(d)



(i) Copy and complete the sketch so that it is an odd function. **1**

(ii) Write down the equation of the piecewise function drawn in part (i). **2**

▪ **Start a new page**

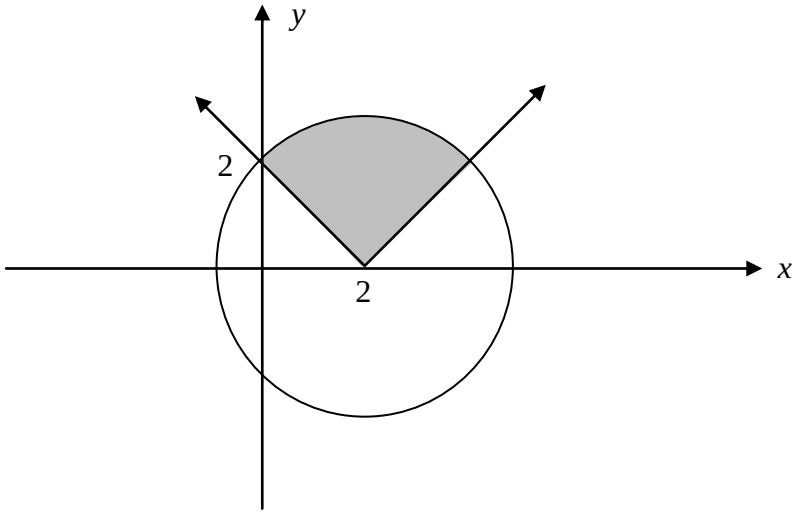
Marks

Question 6 (13 marks)

(a) (i) Write down the factorised form of $a^3 - b^3$. **1**

(ii) Hence show that $\dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - 1 = \sin\theta\cos\theta$. **2**

(b)



Write down a pair of inequations to describe the region shaded above. **4**

(c) $f(x) = \dfrac{1}{\sqrt{x}}$

(i) State the domain and range of $f(x)$. **2**

(ii) For what value of x does $f(x) = 2$? **1**

(iii) Find $f'(x)$ and hence calculate $f'(3)$ leaving your answer in simplified surd form with a rational denominator. **2**

(iv) Show that $(f(x))^3 = -2f'(x)$. **1**

End of Paper

Question 1

(a) let $x = 0.8333\ldots$

$10x = 8.3333\ldots$

$\Rightarrow 9x = 7.5$ ✓

$90x = 75$

$x = \frac{75}{90} = \frac{5}{6}$ ✓

(b) $90\% = 15660$ people ✓

$10\% = 1740$ people

$100\% = 17400$ people

$\therefore$ 17400 attended the previous weekend ✓

(c) $3x^2 - 7x - 6$

$= \frac{(3x-9)(3x+2)}{3}$ ✓

$= (x-3)(3x+2)$ ✓

one mark if signs incorrect.

(d) $\cos 225° = -\cos 45°$

$= -\frac{1}{\sqrt{2}}$ ✓ ✓

(e) $v^2 = (6.4 \times 10^7)^2 - 2 \times 21.5 \times 5.7 \times 10^{13}$

$= 1.645 \times 10^{15}$ ✓

Many students stopped at this point!

$v = \sqrt{1.645 \times 10^{15}}$

$= 40558599.58$ ✓

$\doteq 40600000$ (3 sig fig)

Many students rounded this to 406!!

(f) $\frac{3}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{3\sqrt{5}+6}{5-4}$ ✓ ✓

$= \sqrt{45} + 6$ ✓

$\therefore a = 45, \quad b = 6$

Question 2

(a) Angle sum $= 180 \times (n-2)$

Angle sum of 9 sided shape $= 180 \times (9-2)$

$= 1260°$ ✓

Each angle $= 1260° \div 9$

$= 140°$. ✓

Well done.

(b) (i) $|2x-5| \leq 7$

$-7 \leq 2x-5 \leq 7$

$-2 \leq 2x \leq 12$

$-1 \leq x \leq 6$ ✓✓

Be careful to rewrite solution

(ii) $3^{2x} - 10 \times 3^x + 9 = 0$

let $u = 3^x$

$u^2 - 10u + 9 = 0$

$(u-9)(u-1) = 0$

$u = 9$, $u = 1$ ✓

$3^x = 9$, $3^x = 1$

$x = 2$ ✓, $x = 0$ ✓

Learn this method

Those students that learnt method were awarded full marks.

(c) $\tan(\theta + 60°) = \frac{1}{\sqrt{3}}$

rel. $\angle = 30°$

S	A
T	C

✓ ✓

$\theta + 60° = 30°, 210°, 390°, \ldots$ ✓

$\theta = -30°, 150°, 330°, \ldots$

$\theta = 150°, 330°$ ✓ for $0 \leq \theta \leq 360°$.

Reas 3

Poorly done by most students you needed to -60 from your solutions, Not add to find θ.

(d) (i) $\alpha + \beta = \frac{-b}{a} = \frac{--2}{1} = 2$ ✓

(ii) $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

$\alpha^2\beta^2 = 9$ ✓

Well done (i) + (ii)

$\alpha^2\beta^2 \neq \alpha^2 + \beta^2$

(iii) $\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$ ✓

$= \frac{2^2 - 2 \times 3}{9} = \frac{-2}{9}$ ✓

Index Laws required

$\alpha^{-2} = \frac{1}{\alpha^2}$

Reas 2

Question 3

(a) (i) $15x^2 - 2x^{-2}$ ✓✓

(ii) $x \times 9(2x+3)^8 \times 2 + (2x+3)^9 \times 1$

$= 18x(2x+3)^8 + (2x+3)^9$ ✓✓ Calc 4.

$(uv' + vu')$

(b) $y = \frac{x}{x+2}$

$\frac{dy}{dx} = \frac{(x+2) \times 1 - x \times 1}{(x+2)^2}$ $\left(\frac{vu' - uv'}{v^2}\right)$

$= \frac{2}{(x+2)^2}$ ✓

$x = -1 \rightarrow m_T = \frac{2}{(-1+2)^2} = 2$ ✓

$y = \frac{-1}{-1+2} = -1$ ✓

Equation of tangent:

$y - -1 = 2(x - -1)$

$y + 1 = 2x + 2$

$2x - y + 1 = 0$ ✓ Calc 4

(c) (i) Positive definite $a>0$, $\Delta<0$.

$\Delta < 0$

$6^2 - 4 \times k \times k < 0$

$36 - 4k^2 < 0$ ✓

$(6-2k)(6+2k) < 0$

$k < -3, \quad k > 3$ ✓

Since $a>0$ for positive definite

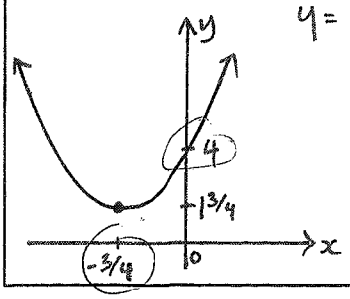
$k > 3$. ✓ Reas 3.

(ii) y int $\rightarrow x = 0 \quad y = 4$ ✓

Vertex $\rightarrow x = \frac{-b}{2a} = \frac{-6}{2 \times 4} = -\frac{3}{4}$

$y = 4\left(\frac{-3}{4}\right)^2 + 6\left(\frac{-3}{4}\right) + 4 = 1\frac{3}{4}$ ✓

(both labelled on graph)



Comm 2.

Question 4

(a) (i) $\angle DAC = \frac{180-80}{2} = 50°$ ✓

(angles opposite equal sides of a triangle are equal and angle sum of triangle is 180°)

$\angle DBA = 180 - 60 - 50 = 70°$ ✓

(angle sum of ΔDAB is 180°) Comm 2.

(ii) $\frac{AB}{\sin 60°} = \frac{30}{\sin 70°}$ ✓

$AB = \sin 60° \times \frac{30}{\sin 70°}$

$= 27.648\ldots$

$\doteq 28$ cm ✓

(b) (i) $C: x = 0$ in $3x - 4y + 20 = 0$

$3 \times 0 - 4y + 20 = 0$

$4y = 20$

$y = 5$

$\therefore C = (0, 5)$ ✓

(ii) $\perp d$ from $(10, 0)$ to $3x - 4y + 20 = 0$

$AB = \frac{|3 \times 10 - 4 \times 0 + 20|}{\sqrt{3^2 + (-4)^2}}$

$= 10$ units ✓✓

(iii) In ΔAOC & ΔABC

$\angle AOC = \angle ABC = 90°$ (given AB $\perp$ ℓ and axes are $\perp$)

AC is common

$AO = AB = 10$ (given $A = (10, 0)$ & Δ AB = 10 part ii)

$\therefore \Delta AOC \equiv \Delta ABC$ (RHS) ✓✓ Reas 2.

(iv) Area $= 2 \times (\frac{1}{2} \times 10 \times 5) = 50$ ✓ u^2

(v) m_ℓ: $3x - 4y + 20 = 0$

$y = \frac{3}{4}x + 5$

gradient $= 3/4$ ✓

$m_{AB} = -4/3$ ✓

$\tan\theta = -4/3$

related angle $= 53°$

$\theta = 180 - 53$

$= 127°$ ✓ Reas 3.

Question 5

(a) $\angle GHE = 74°$ (corresponding angles are equal when lines are parallel) ✓

$x° = 107° - 74° = 33°$ (exterior angle of a triangle is the sum of the two opposite interior angles) ✓

Comm 2.
Correct reason must have been given to be awarded 2 marks.

(b) $x^2 - x + 3 \equiv a(x+1)^2 + b(x+1) + c$

$\equiv a(x^2 + 2x + 1) + bx + b + c$

$\equiv ax^2 + (2a+b)x + (a+b+c)$

Learn this method.

Comparing coefficients:

x^2 : $a = 1$ ✓

x : $2a + b = -1$ (& $a = 1$)

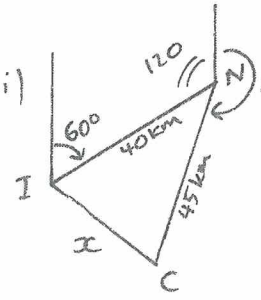
$\Rightarrow b = -3$ ✓

constant : $a + b + c = 3$ (& $a = 1$, $b = -3$)

$\Rightarrow c = 5$ ✓

Careless errors made when substituting.

(c) (i)



$\angle INC = 360 - 200 - 120$

$= 40°$ ✓

Advice is to redraw diagram + label information given.

(ii) $x^2 = 40^2 + 45^2 - 2 \times 40 \times 45 \times \cos 40$ ✓

$= 867.24$

$x = \sqrt{867.24} \doteq 29\text{km}$ ✓

Incorrect (i) did not result in penalties in part (ii) + (iii) as long as method was correct.

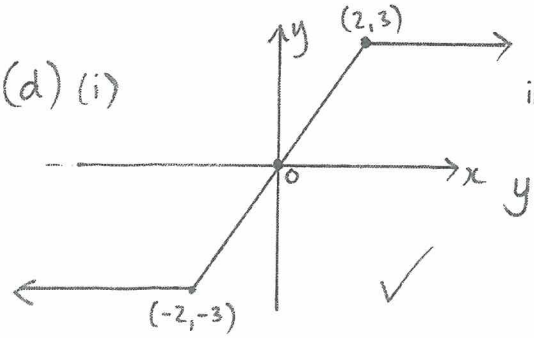
(iii) Area $= \frac{1}{2} \times 40 \times 45 \times \sin 40°$ ✓

$\doteq 578.5088\ldots$

$\doteq 580\ \text{km}^2$ (2 sig fig) ✓

Must have 2 sig fig

(d) (i)



✓

ii) $y = \begin{cases} 3 & x \geq 2 \\ \frac{3x}{2} & -2 < x < 2 \\ -3 & x \leq -2 \end{cases}$

✓✓

Label graph (No marks deducted this time!)

Comm 1
Reas 2.
Piecemeal function very poorly done.

Question 6

(a) (i) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ ✓

(ii) LHS $= \dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta}$ -1

$= \dfrac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{(\sin\theta - \cos\theta)}$ ✓R -1

$= \sin^2\theta + \cos^2\theta + \sin\theta\cos\theta - 1$

$= 1 + \sin\theta\cos\theta - 1$ ✓R

$= \sin\theta\cos\theta$ = RHS

Reas 2.

(b) Absolute value: $y = |x-2|$ ✓R

Circle: radius $= \sqrt{2^2+2^2} = \sqrt{8}$ ✓R

centre $= (2,0)$

$(x-2)^2 + y^2 = 8$ ✓R

Inequations: $y \geq |x-2|$

and $(x-2)^2 + y^2 \leq 8$ ✓R

Reas 4

(c) (i) D: $x > 0$ ✓ R: $y > 0$ ✓

(ii) $f(x) = 2$

$\dfrac{1}{\sqrt{x}} = 2$

$1 = 2\sqrt{x}$

$\dfrac{1}{2} = \sqrt{x}$

$x = 1/4$ ✓

- Many errors at this line.

(iii) $f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$ ✓C

$= \dfrac{-1}{2x\sqrt{x}}$

$f'(3) = \dfrac{-1}{2\times 3\sqrt{3}}$

$= \dfrac{-\sqrt{3}}{18}$ ✓C

Calc 2

LHS $= (f(x))^3$

$= \left(\dfrac{1}{\sqrt{x}}\right)^3$

$= \dfrac{1}{x\sqrt{x}}$

RHS $= -2 \times f'(x)$

$= -2 \times \dfrac{-1}{2x\sqrt{x}}$

$= \dfrac{1}{x\sqrt{x}}$

$\therefore$ LHS = RHS ✓R

Reas 1

Setting out often very poor.