Shore

Examination Number:

Set:

Year 11
Mathematics
Yearly Examination
September 2013

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Answer Questions 1–5 on the Multiple Choice Answer Sheet provided
- Start each of Questions 6–10 in a new writing booklet
- In Questions 6–10, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 80

Section I Pages 2–3

5 marks

- Attempt Questions 1–5
- Each question is worth 1 mark
- Allow about 10 minutes for this section

Section II Pages 4–8

75 marks

- Attempt Questions 6–10
- Each question is worth 15 marks
- Allow about 1 hour and 50 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1–5.

1 Which one of the following is equivalent to $\frac{x}{8}-\frac{x+1}{4}$?

(A) $\frac{1-x}{8}$

(B) $\frac{-1-x}{8}$

(C) $\frac{2-x}{8}$

(D) $\frac{-2-x}{8}$

2 Which one of the following is equal to $\frac{1+\sqrt{5}}{7-2\sqrt{5}}$?

(A) $\frac{5\sqrt{5}-3}{29}$

(B) $\frac{5\sqrt{5}-3}{39}$

(C) $\frac{9\sqrt{5}+17}{29}$

(D) $\frac{9\sqrt{5}+17}{39}$

3 The roots of a quadratic equation are –3 and 4.

Which one of the following could be the quadratic equation?

(A) $x^2 - x - 12 = 0$

(B) $x^2 + x - 12 = 0$

(C) $x^2 - x + 12 = 0$

(D) $x^2 + x + 12 = 0$

4 What are the solutions to $\sqrt{3}\tan x = -1$ for $0° \le x \le 360°$?

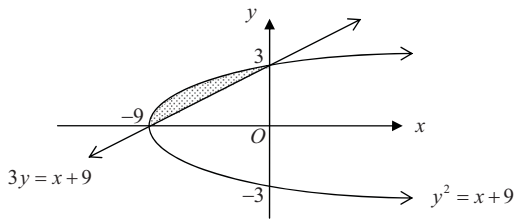
(A) 120° and 240°

(B) 120° and 300°

(C) 150° and 210°

(D) 150° and 330°

5 The diagram shows the region enclosed by $3y = x + 9$ and $y^2 = x + 9$.



Which one of the following pairs of inequalities describes the shaded region in the diagram?

(A) $3y \ge x + 9$ and $y^2 \ge x + 9$

(B) $3y \ge x + 9$ and $y^2 \le x + 9$

(C) $3y \le x + 9$ and $y^2 \ge x + 9$

(D) $3y \le x + 3$ and $y^2 \le x + 9$

Section II

75 marks
Attempt Questions 6–10
Allow about 1 hour and 50 minutes for this section

Start each of Questions 6–10 in a new writing booklet.

In Questions 6–10, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks) Use a SEPARATE writing booklet.

(a) The selling price of a television is \$1056. This includes a 10% tax on the original price. Calculate the original price of the television. **2**

(b) Find the perpendicular distance from the point $(-2, 5)$ to the line $4x + 3y = 12$. **2**

(c) Factorise fully $x^2 - y^2 + 2x + 2y$. **2**

(d) Solve $|3x - 1| < 7$. **2**

(e) Find the value of m if the line $6x + my + 7 = 0$ is parallel to the line $3x + 2y - 4 = 0$. **2**

(f) A regular polygon has an interior angle sum of 3960°. **2**

How many sides does the polygon have?

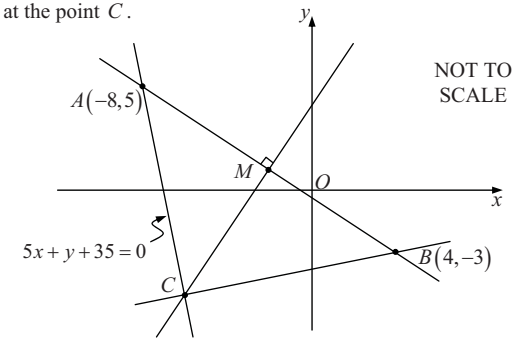
(g) A function is given by $f(x) = x^3 - 2x^2 + x$.

Find the values of x for which $f'(x) = 0$. **3**

Question 7 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate $\left(1-4x^2\right)^5$ with respect to x. **2**

(b) The diagram shows the points $A(-8,5)$ and $B(4,-3)$. The point M is the midpoint of the interval AB. The line $5x+y+35=0$ meets the perpendicular bisector of AB at the point C.



(i) Find the coordinates of M. **1**

(ii) Find the gradient of AB. **1**

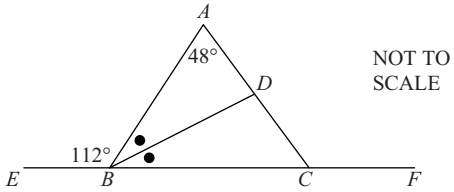
(iii) Show that the equation of the perpendicular bisector of AB is $3x-2y+8=0$. **2**

(iv) Show that the coordinates of the point C are $(-6,-5)$. **2**

(v) Prove that ΔABC is right-angled at C. **2**

(vi) Find the area of ΔABC. **2**

(c) In the diagram, $\angle ABE = 112°$, $\angle BAC = 48°$ and BD bisects $\angle ABC$. **3**



Copy or trace the diagram into your writing booklet.

Find the size of $\angle BDC$, giving reasons for your answer.

Question 8 (15 marks) Use a SEPARATE writing booklet

(a) Simplify $\tan\theta\sqrt{1-\sin^2\theta}$. **2**

(b) Find $f'(x)$ if $f(x)=\dfrac{1}{2x^4}$. **2**

Write your answer in simplest form with positive integer indices.

(c) Find $\lim\limits_{x\to -4}\left(\dfrac{x^2+3x-4}{2x+8}\right)$. **2**

(d) Consider the curve $y=x^3-3x^2$. **3**

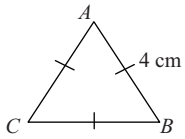
Find the equation of the tangent at the point on the curve where $x=-1$.

(e) For what values of k does $3x^2-2kx+k=0$ have real roots? **3**

(f) Sketch the curve $y=\dfrac{1}{x-3}+1$ clearly labelling any intercepts with the coordinate axes and other important features. **3**

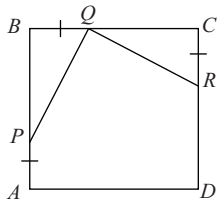
Question 9 (15 marks) Use a SEPARATE writing booklet.

(a) Find the exact area of an equilateral triangle with side length 4 centimetres. **2**



NOT TO SCALE

(b) In the diagram, $ABCD$ is a square. The points P, Q, and R lie on AB, BC, and CD respectively such that $AP = BQ = CR$.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Prove that $\Delta PBQ \equiv \Delta QCR$. **3**

(ii) Prove that $\angle PQR$ is a right angle. **2**

(c) Solve $2\sin^2 x = \sin x$ for $0° \le x \le 360°$. **2**

(d) Let α and β be the roots of the quadratic equation $4 - 8x - x^2 = 0$.

(i) Find $\alpha + \beta$. **1**

(ii) Find $\alpha\beta$. **1**

(iii) Find $\alpha^2 + \beta^2$. **2**

(iv) Find $(\alpha - \beta)^2$. **2**

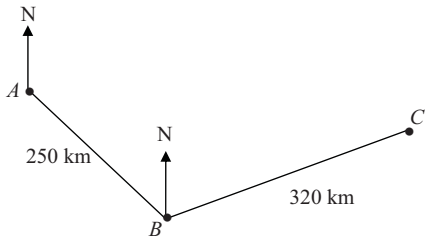
Question 10 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $4 + 23x^2 - 6x^4 = 0$. **2**

(b) Determine whether the function $f(x) = 2x - x^3$ is even, odd or neither. **2**

(c) Write x^2 in the form $A(2-x)^2 + B(2-x) + C$. **3**

(d) A ship sailed 250 kilometres from Port A on a bearing of 153° and arrived at Port B to pick up some passengers. It then progressed 320 kilometres to Port C on a bearing of 071°.



NOT TO SCALE

Copy or trace this diagram into your writing booklet.

(i) Find the distance AC correct to the nearest kilometre. **3**

(ii) Hence, find the bearing of Port A from Port C correct to the nearest degree. **2**

(e) Find $\dfrac{d}{dx}\left(\dfrac{4x^2}{\sqrt{3x-2}}\right)$. **3**

Write your answer in simplest form with positive integer indices.

End of Paper

Examination Number:

Set:

Mathematics 2 Unit

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

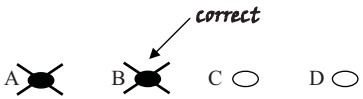
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.



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1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

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1. D
2. C
3. A
4. D
5. B

Question 6

(a) Original Price $= \frac{\$1056}{1.1}$

$= \$960$

(b) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|4(-2) + 3(5) - 12|}{\sqrt{4^2 + 3^2}}$

$= \frac{|-5|}{5}$

$= 1 \text{ unit}$

(c) $x^2 - y^2 + 2x + 2y$

$= (x-y)(x+y) + 2(x+y)$

$= (x+y)(x-y+2)$

(d) $|3x-1| < 7$

$-7 < 3x - 1 < 7$

$-6 < 3x < 8$

$-2 < x < 2\frac{2}{3}$

(e) $-\frac{6}{m} = \frac{-3}{2}$

$-12 = -3m$

$m = 4$

(f) $(n-2) \times 180^\circ = 3960^\circ$

$n - 2 = 22$

$n = 24$

(g) $3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ or $x = 1$

Question 7

(a) $\frac{d}{dx}(1-4x^2)^5$

$= 5(1-4x^2)^4 \times -8x$

$= -40x(1-4x^2)^4$

(b) (i) $M = \left(\frac{-8+4}{2}, \frac{5+-3}{2}\right)$

$= (-2, 1)$

(ii) $m_{AB} = \frac{-3-5}{4+8}$

$= \frac{-8}{12}$

$= \frac{-2}{3}$

(iii) $m_{MC} = \frac{3}{2}$ $M(-2, 1)$

$y - 1 = \frac{3}{2}(x+2)$

$2y - 2 = 3x + 6$

$3x - 2y + 8 = 0$

(iv) $5x + y + 35 = 0$ (1)

$3x - 2y + 8 = 0$ (2)

Solve:

$10x + 2y + 70 = 0$ (1a)

$3x - 2y + 8 = 0$ (2)

(1a) + (2) $13x + 78 = 0$

$13x = -78$

$x = -6$

Sub $x = -6$ into (1)

$5(-6) + y + 35 = 0$

$y + 5 = 0$

$y = -5$

$\therefore$ C is $(-6, -5)$

(v) $m_{AC} = \frac{-5-5}{-6+8}$ $m_{BC} = \frac{-5+3}{-6-4}$

$= \frac{-10}{2}$ $= \frac{-2}{-10}$

$= -5$ $= \frac{1}{5}$

$m_{AC} \times m_{BC} = -5 \times \frac{1}{5}$

$= -1$

$\therefore$ AC $\perp$ BC

(vi) $AC = \sqrt{(-8+6)^2 + (5+5)^2}$

$= \sqrt{4 + 100}$

$= \sqrt{104}$

$BC = \sqrt{(4+6)^2 + (-5+3)^2}$

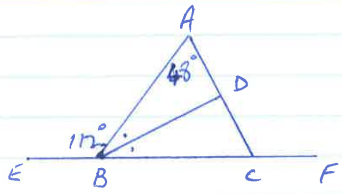
$= \sqrt{100 + 4}$

$= \sqrt{104}$

$\therefore$ Area $= \frac{1}{2} \times \sqrt{104} \times \sqrt{104}$

$= 52 \text{ units}^2$

(c)



$\angle ABC = 68^\circ$ straight $\angle EBC$

$\angle ABD = 34^\circ$ BD bisects $\angle ABC$

$\angle BDC = 82^\circ$ Sum of 2 interior $\angle$s equals opp. exterior $\angle$

Question 8

(a) $\tan\theta\sqrt{1 - \sin^2\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \sqrt{\cos^2\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$

$= \sin\theta$

(b) $f(x) = \frac{1}{x^4}$

$f'(x) = -2x^{-5}$

$= \frac{-2}{x^5}$

(c) $\lim_{x \to -4} \frac{x^2 + 3x - 4}{2x + 8}$

$= \lim_{x \to -4} \frac{(x+4)(x-1)}{2(x+4)}$

$= -\frac{5}{2}$

(d) $y = x^3 - 3x^2$

$y' = 3x^2 - 6x$

at $x = -1$ $\quad y = (-1)^3 - 3(-1)^2$

$= -1 - 3$

$= -4$

$m_T = 3(-1)^2 - 6(-1)$

$= 3 + 6$

$= 9$

$\therefore$ equation is: $\quad y + 4 = 9(x + 1)$

$y + 4 = 9x + 9$

$y = 9x + 5$

(e) For real roots $\Delta \geq 0$

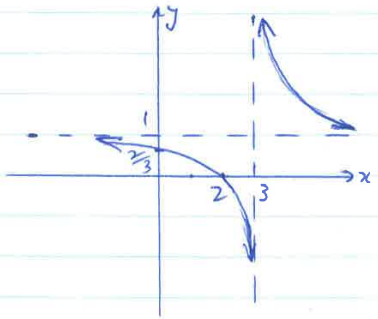
$(-2k)^2 - 4 \times 3 \times k \geq 0$

$4k^2 - 12k \geq 0$

$4k(k - 3) \geq 0$

$\therefore \quad k \leq 0$ or $k \geq 3$

(f)



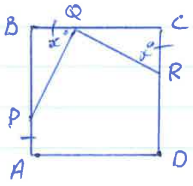
Question 9

(a) Area $= \frac{1}{2} \times 4 \times 4 \times \sin 60°$

$= 8 \times \frac{\sqrt{3}}{2}$

$= 4\sqrt{3}$ units^2

(b)



(i) $\angle B = \angle C = 90°$ angles in square

$BQ = CR$ given

$BP = QC$ subtraction of equal lengths from sides of square

$\therefore \triangle PBQ \equiv \triangle QCR$ SAS

(ii) $\angle BQP = \angle QRC = x°$ corresp. $\angle$s in congruent $\triangle$s.

$\angle CQR = 90° - x°$ $\angle$ sum $\triangle QCR$

$\angle PQR = 180° - (x° + 90° - x°)$ st. $\angle BQC$

$= 90°$

(c) $2\sin^2 x = \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x (2\sin x - 1) = 0$

$\sin x = 0 \qquad \sin x = \frac{1}{2}$

$x = 0°, 180°, 360°$ or $x = 30°, 150°$

(d) (i) $\alpha + \beta = \frac{-b}{a}$

$= -\frac{-8}{-1}$

$= -8$

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{4}{-1}$

$= -4$

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-8)^2 - 2(-4)$

$= 72$

(iv) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 72 - 2(-4)$

$= 80$

Question 10

(a) $4 + 23x^2 - 6x^4 = 0$

$(4 - x^2)(1 + 6x^2) = 0$

$4 - x^2 = 0 \qquad 1 + 6x^2 = 0$

$x^2 = 4 \qquad x^2 = -\frac{1}{6}$

$x = \pm 2 \qquad$ no solution

(b) $f(x) = 2x - x^3$

$f(-x) = 2(-x) - (-x)^3$

$= -2x + x^3$

$= -(2x - x^3)$

$= -f(x)$

$f(-x) = -f(x) \quad \therefore f(x)$ is odd

(c) $x^2 \equiv A(2 - x)^2 + B(2 - x) + C$

$= A(4 - 4x + x^2) + 2B - Bx + C$

$= Ax^2 + (-4A - B)x + (4A + 2B + C)$

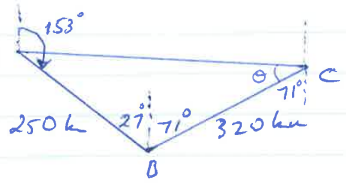
$A = 1 \qquad -4A - B = 0 \qquad 4A + 2B + C = 0$

$-4 - B = 0 \qquad 4 - 8 + C = 0$

$B = -4 \qquad C = 4$

$\therefore x^2 \equiv (2 - x)^2 - 4(2 - x) + 4$

(d)



(i) $AC^2 = 250^2 + 320^2 - 2 \times 250 \times 320 \cos 98°$

$= 187\,167.6962$

$AC \doteq 433$ km

(ii) $\frac{\sin\theta}{250} = \frac{\sin 98°}{433}$

$\sin\theta = \frac{\sin 98°}{433} \times 250$

$\sin\theta = 0.5717\ldots$

$\theta \doteq 35°$

$\therefore$ bearing $= 180° + 71° + 35°$

$= 286°$

(e) $\frac{d}{dx}\left(\frac{4x^2}{\sqrt{3x-2}}\right) = \frac{(3x-2)^{\frac{1}{2}} \cdot 8x - 4x^2 \cdot \frac{3}{2}(3x-2)^{-\frac{1}{2}}}{3x-2}$

$= \frac{(3x-2) \cdot 8x - 6x^2}{(3x-2)^{\frac{1}{2}}(3x-2)}$

$= \frac{18x^2 - 16x}{(3x-2)^{\frac{3}{2}}}$